Electromagnetic Scattering by a Periodic Array of Thick-Walled Parallel Plate Waveguides

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Abstract

Electromagnetic wave scattering by a periodic array of semi-infinite thick-walled parallel plate waveguides is studied in this paper. The cases of TE and TM polarization of an incident plane harmonic wave are considered separately. The scattered field above the waveguides is sought in the form of a series of spatial harmonics in accordance with the Floquet’s theorem, whereas in the waveguide regions it is sought in the form of parallel plate waveguide modes. To satisfy the boundary and edge conditions by field components in the free space above the array the Fourier expansion for spatial harmonics amplitudes with corresponding coefficients being properly chosen Legendre functions is exploited. The unknown coefficients are the solutions of certain doubly infinite systems of linear equations. The approximate solution is found numerically.

1 Introduction

Electromagnetic scattering and radiation by periodic systems is a classical problem of diffraction theory. It has been investigated by many authors and it is still a problem of current importance from theoretical and practical points of view. In this paper we solve the problem of plane wave scattering in an infinite array of parallel plate waveguides for oblique incidence. This is a complicated 3D problem never considered in literature where mainly 2D problems were considered regarding the incidence in the plane normal to the array aperture and parallel to the direction of periodicity. Exact closed-form solutions has been obtained only in few cases of waveguides with infinitesimally thin walls [1], [2]. In the case of thick waveguide walls different methods from Wiener-Hopf [3] and variational technique [4] to purely numerical treatment of integral equation [5] and finite-element time-domain method [6] were used to obtain an approximate solution.

In this paper the approach to the analysis of electromagnetic scattering by periodic array of thick-walled parallel plate waveguides presented in [7] and [8] is further extended on the case of TM polarization of incident wave. The scattered field is sought in the form of spatial harmonics in the space above the array and in the form of parallel plate waveguide modes in the waveguide regions. A Fourier series expansion with coefficients being properly chosen Legendre functions is then applied for spatial harmonic amplitudes representation in the free-space region. This assures that the boundary and edge conditions are satisfied. The problem is reduced to solving the doubly infinite system of linear equations for unknown expansion coefficients. The exact solution can not be obtained, however, and numerical calculations are required to solve this system approximately.

2 Field Solutions

Let us consider an infinite system of perfectly conducting thick-walled parallel plate waveguides shown in Fig.1. The period of the structure is $A$ and the waveguide aperture is $d$. The system is homogeneous.
in the z-direction, and periodic in the x-direction. Waveguides occupy the lower half-plane \( y < 0 \). An incident plane harmonic wave of the angular frequency \( \omega \) impinges on the system at the angle \( \theta \) counted from the y-axis. In what follows, the term \( \exp(j\omega t) \) will be omitted. The case of TM and TE incidence will be considered. For the case of TE incidence the only nonzero component of the magnetic field vector is \( H_z \), whereas \( E_z \) is the only nonzero component of the electric field in the case of TM polarization of the incident wave. The total field resulting from the Maxwell equations can be represented as follows:

\[
\{H, E\}_z, \{E, H\}_x = \pm \frac{1}{j\omega \{\epsilon, \mu\}_0} \frac{\partial \{H, E\}_z}{\partial y}, \{E, H\}_y = \mp \frac{1}{j\omega \{\epsilon, \mu\}_0} \frac{\partial \{H, E\}_z}{\partial x}.
\]  

(1)

The first symbol in the curl brackets and the upper sign in (1) correspond to the case of TE polarization and the second symbol and lower sign to TM polarization of the incident wave. The same notation regarding the curl brackets will hold throughout the text whereas the one concerning the signs is adopted in this chapter only. The total field can be represented in the following form:

\[
\{H, E\}_z = \{H, E\}^+_z + \{H, E\}^-_z, \{E, H\}_i = \{E, H\}^+_i + \{E, H\}^-_i, y > 0,
\]

\[
\{H, E\}_z = \{H, E\}^+_z, \{E, H\}_i = \{E, H\}^+_i, y < 0,
\]  

(2)

where \( i = x, y \) and \( I \) denotes the field of given incident wave:

\[
\{H, E\}^+_z = e^{-j(rx-sy)}, \{E, H\}^+_z = \pm \frac{s}{\omega \{\epsilon, \mu\}_0} \{H, E\}^+_z, \{E, H\}^+_y = \pm \frac{r}{\omega \{\epsilon, \mu\}_0} \{H, E\}^+_z,
\]

\[
r = k \sin \theta, s = k \cos \theta, k = 2\pi/\lambda,
\]  

(3)

and the superscripts +,− denote the scattered field in the free-space region and in the waveguide region respectively. The problem is to find the scattered filed. Consider first the region \( y > 0 \). Due to the system periodicity the scattered field can be represented by a series of spatial harmonics according to the Floquet’s theorem, yielding:

\[
\{H, E\}^+_z = \sum_n A_n e^{-j(r_n^+x+s_n^+y)},
\]

\[
\{E, H\}^+_x = \pm \frac{1}{\omega \{\epsilon, \mu\}_0} \sum_n s_n A_n e^{-j(r_n^+x+s_n^+y)},
\]

\[
\{E, H\}^+_y = \pm \frac{1}{\omega \{\epsilon, \mu\}_0} \sum_n r_n A_n e^{-j(r_n^+x+s_n^+y)},
\]  

(4)

Figure 1: Infinite array of semi-infinite thick plates
where $A_n$ are unknown amplitudes; $n \in \mathbb{Z}$ throughout the paper, unless otherwise stated. The wave numbers of spatial harmonics $r_n^+$ and $s_n^+$ are defined as follows:

$$r_n^+ = r + nK, \quad K = \frac{2\pi}{\Lambda}, \quad s_n^+ = \begin{cases} (k^2 - (r_n^+)^2)^{1/2} & \text{for real } s_n^+, \\ -j ((r_n^+)^2 - k^2)^{1/2} & \text{for imaginary } s_n^+. \end{cases}$$  

(5)

The field in the waveguide regions is represented by a series of the parallel plate waveguide modes as follows (for one period $-\Lambda/2 < x < \Lambda/2$):

$$\{H, E\}_x = \sum_p B_p \left\{ \begin{array}{c} \cos \\ \sin \end{array} \right\} (r_p^- (x + d/2)) e^{js_p^- y},$$

$$\{E, H\}_x = \frac{+1}{\omega \{\epsilon, \mu\}_0} \sum_p s_p^- B_p \left\{ \begin{array}{c} \cos \\ \sin \end{array} \right\} (r_p^- (x + d/2)) e^{js_p^- y},$$

(6)

$$\{E, H\}_y = \frac{-j}{\omega \{\epsilon, \mu\}_0} \sum_p r_p^- B_p \left\{ \begin{array}{c} \sin \\ \cos \end{array} \right\} (r_p^- (x + d/2)) e^{js_p^- y},$$

where $B_p$ are unknown mode amplitudes; $p \in \mathbb{N}$ for TE and $p \in \mathbb{N} \cup 0$ for TM incidence throughout the paper, unless otherwise stated. The propagation constants $r_p^-$ and $s_p^-$ are defined below:

$$r_p^- = p\pi/d, \quad \gamma_p = \begin{cases} (k^2 - (r_p^-)^2)^{1/2} & \text{for real } s_p^- , \\ -j ((r_p^-)^2 - k^2)^{1/2} & \text{for imaginary } s_p^- . \end{cases}$$

(7)

It is easy to verify that it is sufficient to consider the field distribution in one period only. The mode amplitudes in different waveguides are related to those in (6) by simple relation:

$$B_p^m = B_p e^{-jrm\Lambda}, \quad m \in \mathbb{Z}.$$  

(8)

3 **Boundary Conditions**

The tangential component of total electric field vector must vanish on the perfectly conducting walls of the waveguides. This yields:

in the free-space region:

$$\{E_x, E_z\} = 0, \quad d/2 < |x| < \Lambda/2, \quad y = +0,$$

(9)

in the waveguide region:

$$\{E_y, E_z\} = 0, \quad x = \pm d/2, \quad y < 0.$$  

(10)

Besides, near the edges of the waveguide walls the tangential components of the electric (TE) and magnetic (TM) field vectors exhibit singular behavior:

$$\{E, H\}_x = O(\varrho^{-1/3}), \quad \{E, H\}_y = O(\varrho^{-1/3}), \quad \varrho = \sqrt{(x \pm d/2)^2 + y^2}, \quad \varrho \to 0.$$  

(11)

Condition (10) is satisfied directly by (6). To obey the boundary condition (9) in the free-space region, we make use of the following expansion [12]:

$$\sum_n P_n^\mu (\cos \Delta) e^{-jnKx} = \begin{cases} C e^{jKx/2} & |x| < d/2, \\ (\cos(Kx) - \cos \Delta)^{\mu+1/2}, & d/2 < |x| < \Lambda/2, \end{cases}$$  

(12)

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where the constant terms
\[ C = (\pi/2)^{1/2} \left( \sin \Delta \right)^{\mu} / \Gamma(1/2 - \mu) \]
and \( \Delta = \pi d/\Lambda \) are introduced to shorten notation; \( P^\mu_n \) denotes the associated Legendre polynomials, \( \Gamma \) is the gamma-function. Multiplying (12) by \( \exp(-jmKx) \), where \( m \) is some integer, and taking linear combination of the resulting equations after straightforward algebraic manipulations, we obtain:
\[
\sum_{m,n} \alpha_m P^\mu_n (\cos \Delta) e^{-jnKx} = \begin{cases} 
C e^{jKx/2} \sum_m \alpha_m e^{-jmKx} & |x| < d/2, \\
(\cos(Kx) - \cos \Delta)^{\mu+1/2} & d/2 < |x| < \Lambda/2, 
\end{cases}
\]
where \( \alpha_m \) are unknown constants. A similar expansion was used for modeling of an infinite array of infinitesimally thin strip [10]. Expansion (13) represents the Fourier series of certain \( \Lambda \)-periodic function as required by Floquet’s theorem, vanishing in certain domains as required by the boundary condition (9) and having singular behavior at the bounds of the above domains (at the edges of the waveguide apertures) in accordance with the edge conditions (11), if \( \mu = -1/6 \) is applied. The above properties of the function given by expansion (13) will be exploited to obey the boundary conditions in the free-space region. We first consider the case of TE polarization. Noting the similar behavior of the expansion (13) and \( E_x(x,0) \) we can rewrite the expression for the tangential component of the total electric field vector in the free region in the form, satisfying conditions (9) and (11) as follows:
\[
E_x(x,0) = \frac{e^{-jrx}}{\omega \epsilon_0} \sum_{n,m} \alpha_m P^\mu_n (\cos \Delta) e^{-jnKx},
\]
where \( \mu = -1/6 \) hereinafter. Comparing (14) with (2)-(4) we obtain the following simple relation between the corresponding amplitudes of spatial harmonics \( A_n \) and the expansion coefficients \( \alpha_m \):
\[
A_n = \delta_{n0} - \frac{1}{s_n} \sum_m \alpha_m P^\mu_{n-m} (\cos \Delta).
\]
where \( \delta_{nI} \) is Kronecker delta. In the case of TM polarization we apply a similar consideration to the \( x \)-derivative of the \( E_z \) component of the electric field vector:
\[
\frac{\partial E_z(x,0)}{\partial x} = e^{-jrx} \sum_{n,m} \alpha_m P^\mu_n (\cos \Delta) e^{-jnKx},
\]
\[
E_z(x,0) = 0, \quad x = k\Lambda, k \in \mathbb{Z}.
\]
In (16) the additional condition is applied to obey the boundary condition (9) for \( E_z \). Taking into account (2)-(4) we obtain a similar relation between \( A_n \) and \( \alpha_m \) for TM polarization:
\[
A_n = -\delta_{n0} + \frac{1}{t_n} \sum_m \alpha_m P^\mu_{n-m} (\cos \Delta).
\]

4 Evaluation of Expansion Coefficients

To evaluate the unknown coefficients \( \alpha_m \) we use the continuity conditions of the tangential field components:
\[
\{H,E\}_x^I + \{H,E\}_x^+ = \{H,E\}_x^-, \quad \{E,H\}_x^I + \{E,H\}_x^+ = \{E,H\}_x^-.
\]
We first substitute (3),(4) and (5) into (18). Next, we multiply the resulting equations by \( \{\cos, \sin\}(p\pi (x/d + 1/2)) \)
and integrate with respect to $x$ from $-d/2$ to $d/2$ to obtain

$$B_p = \frac{4e^{j\pi/2}}{d(1+\delta_p)} \sum_n \left\{ \frac{r_n^+, - jr_p^-}{(r_n^+)^2 - (r_p^-)^2} \right\} \left( \frac{r_n^- d - p\pi}{(r_n^-)^2 - (r_p^-)^2} \right) (\delta_{n0} + A_n),$$

$$B_p = \frac{4e^{j\pi/2}}{d(1+\delta_p)} \sum_n \frac{s_n^+}{s_p^-} \left\{ \frac{r_n^+, - jr_p^-}{(r_n^+)^2 - (r_p^-)^2} \right\} \left( \frac{r_n^- d - p\pi}{(r_n^-)^2 - (r_p^-)^2} \right) (\delta_{n0} - A_n),$$

(19)

where $p \in \mathbb{N}$ for TE and $p \in \mathbb{N} \cup 0$ for TM polarization respectively. Subtracting the second equation from the first in (19), after rearrangement of terms we obtain:

$$\sum_n \left\{ \frac{r_n^+, - jr_p^-}{(r_n^+)^2 - (r_p^-)^2} \right\} \left( 1 + \frac{s_n^+}{s_p^-} \right) A_n = - \left( 1 - \frac{s_n^+}{s_p^-} \right) \left\{ \frac{r_n^- - jr_p^-}{r_n^- d - p\pi} \right\} \left( \frac{1}{r^2 - (r_p^-)^2} \right).$$

(20)

Taking into account (15) and (17), after simple algebraic manipulations we obtain the following system of linear equations for unknown $\alpha_m$:

$$\sum_{m,n} \alpha_m P_{n-m}^{\mu}(s_n^+ + s_p^-) \sin \left( \frac{1}{2} (r_n^- d - p\pi) \right) \left\{ \frac{r_n^+}{s_n^+ s_p^-}, \frac{1}{r_n^-} \right\} = \frac{2(r, s)}{r^2 - (r_p^-)^2}. \sin \left( \frac{1}{2} (r d - p\pi) \right).$$

(21)

The above doubly infinite systems of linear equations (21) can be only solved numerically (see Sec.5). If the coefficients $\alpha_m$ are known the scattered field in both regions can be evaluated:

$$B_p = \frac{4e^{j\pi/2}}{d\gamma_p(1+\delta_p)} \sum_{m,n} \alpha_m P_{n-m}^{\mu} \left( s_n^+ + s_p^- \right) \sin \left( \frac{1}{2} (r_n^- d - p\pi) \right) \left\{ \frac{r_n^+}{s_n^+ s_p^-}, \frac{1}{r_n^-} \right\}$$

(22)

where $p \in \mathbb{N}$ for TE and $p \in \mathbb{N} \cup 0$ for TM polarization of the incident wave.

5 Numerical Results

The problem of the uniqueness and existence of the solution of the systems similar to (20) was discussed in [13]. In order to find an approximate solution of the system (20) we reduce the number of equations taking into account only the modes transporting energy from the plane $y = 0$ and a finite number of lower order evanescent modes. The higher order vanishing harmonics are ignored in the solution for the scattered field. To find the unknown coefficients $\alpha_m$ and consequently the partial wave amplitudes $A_n$ and $B_p$ we consider the truncated systems of linear equations (21):

$$\sum_{m=-M}^{M} G_{pm} \alpha_m = g_p,$$

(23)

where the corresponding coefficients of the truncated system are:

$$G_{pm} = \sum_{n=-N}^{N} P_{n-m}^{\mu} \left( s_n^+ + s_p^- \right) \sin \left( \frac{1}{2} (r_n^- d - p\pi) \right) \left\{ \frac{r_n^+}{s_n^+ s_p^-}, \frac{1}{r_n^-} \right\} g_p = \frac{2(r, s)}{r^2 - (r_p^-)^2} \sin \left( \frac{1}{2} (r d - p\pi) \right),$$

(24)

and $p = 0...2M$ for TE or $p = 1...2M + 1$ for TM polarization. It should be noted, that the infinite series in (21) has been truncated at some $n = N$ (large enough to assure convergence) in order to evaluate approximate values of the matrix coefficients $G_{pm}$. Generally, the series in (24) is fast converging and it is sufficient to take $N$ about one order of magnitude higher than $2M$ to assure an acceptable accuracy of
Figure 2: Matrix elements (24) asymptotic behavior for a) TE and b) TM polarization of the incident wave. $\Lambda/d = 2, \Lambda/\lambda = 3.2, \theta = 10^\circ$.

numerical calculations. The convergence of matrix elements $G_{pm}$ was studied numerically. In Fig. 2 their asymptotic behavior is shown for large values of $p$ and $m$. The two considered cases correspond to $G_{p0}$ for large $p$ (solid line) and $G_{0m}$ for large $m$ (dashed line). To estimate the accuracy of numerical calculation we consider the relative error in the power relation resulting from the complex Poynting’s theorem. Without going into details we shall give here the final expression for the propagating modes derived for one period of the structure [13]:

$$\sum_{n(s_n \text{ real})} |A_n|^2 |s_n| + \frac{d}{2\Lambda} \sum_{p(s_p^- \text{ real})} |B_p|^2 |s_p^-| \{1 + \delta_p\} = s. \tag{25}$$

This condition for the squares of amplitudes of propagating modes is equivalent to the energy conservation law for the scattered field. It should be satisfied by the field components in both regions. The relative error in the power relation is defined as follows:

$$\delta P = \left|\frac{(P^I - P^+ - P^-)}{P^I}\right| 100\%,$$

where $P^\pm$ denotes the power of scattered field in the upper (reflected filed) and lower (transmitted field) half-planes marked by superscripts + and − respectively; $P^I$ is the power of the incident wave field. For instance, Table 1 shows the relative error in the power relation (25) for the following case: $\Lambda/d = 2, \Lambda/\lambda = 3.8, \theta = 10^\circ$ and $N = 1000$ in (24).

<table>
<thead>
<tr>
<th>$M$</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta P$ (%), TE</td>
<td>3.40</td>
<td>2.37</td>
<td>1.71</td>
<td>1.22</td>
<td>0.78</td>
</tr>
<tr>
<td>$\delta P$ (%), TM</td>
<td>7.02</td>
<td>2.12</td>
<td>0.048</td>
<td>0.03</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Table 1: Relative error in the power relation for different number of equations in (24)

It is worth noting that using (15), (17) and (22) we can evaluate the amplitudes of spatial harmonics $A_n$ and waveguide modes $B_p$ for $|n| > M$ and $p > 2M$ respectively, having solved the truncated system (24) for unknown $\alpha_m, |m| \leq M$. This allows us to improve the numerical accuracy of the scattered field.
evaluation. In Table 2 the relative error in the power relation (25) is shown for different number \( N_{\text{harm}} \) of spatial harmonics \( A_n \) and waveguide modes \( B_p \) in both half-planes: \(-N_{\text{harm}} \leq n \leq N_{\text{harm}}, 0 \leq p \leq 2N_{\text{harm}}\), for fixed \( M = 50, N = 1000 \) in (24) and \( \Lambda/d = 2, \Lambda/\lambda = 3.8, \theta = 10^\circ\).

<table>
<thead>
<tr>
<th>( N_{\text{harm}} )</th>
<th>50</th>
<th>100</th>
<th>500</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta P ) (%), TE</td>
<td>0.78</td>
<td>0.48</td>
<td>0.14</td>
<td>0.075</td>
</tr>
<tr>
<td>( \delta P ) (%), TM</td>
<td>0.022</td>
<td>0.021</td>
<td>0.021</td>
<td>0.021</td>
</tr>
</tbody>
</table>

Table 2: Relative error in the power relation for different number of space harmonics

As it is seen from Table 2 the numerical accuracy improvement takes place mainly in the case of TE polarization. The TM case is almost insensitive to the \( N_{\text{harm}} \) value increase. In Fig. 3 the dependence of the transmitted field power versus the normalized period is shown. It is defined as the real part of the complex Poynting flux (the second sum in (25)). In Fig. 3 the resonant phenomena (the peaks of the transmitted field power) are clearly observable for the values of \( \Lambda/\lambda \geq 0.5 \). The inflection points of the curves correspond to the critical values of wavelength \( \lambda_k \) of the harmonics in the upper and lower half-planes. From (5) in the upper half-plane we have

\[
\Lambda/\lambda_k = n \frac{S_n + \sin \theta}{\cos^2 \theta}, \quad n = \pm 1, \pm 2, \ldots
\]
where $S_n = 1$ for $n > 0$ and $-1$ for $n < 0$; $+$ and $-$ signs correspond to the backward and forward propagating harmonics respectively. In the waveguide regions from (7) we have

$$\Lambda/\lambda_k = p, \quad p = 1, 2, \ldots,$$

which holds for the considered case $\Lambda/d = 2$. Thus, for example, in the case of $\theta = 30^\circ$ the inflection points are located at $\Lambda/\lambda = 2n/3$ for the backward propagating modes and $\Lambda/\lambda = 2n$ for the forward propagating ones in the upper half-plane, whereas the waveguide modes start propagating at $\Lambda/\lambda = p$.

In Figs.4(a) and 4(b) the dependence of the transmitted field power versus the incidence angle are shown for TE and TM polarization respectively.

![Graph](image)

Figure 4: Dependence of the transmitted field power versus the incident angle for a) TE and b) TM polarization of the incidence wave and different relative aperture $d/\Lambda$; $\Lambda/\lambda = 0.5$ (TE) and 2.5 (TM) cases.

The above example shows that in the case of TE polarization the transmitted field power for $d/\Lambda > 0.5$ is almost independent of $\theta$ in wide range of incident angles, approaching some limit for $\theta \to 0$. For smaller values of relative aperture, we observe slight increase of $P^-/P_I$ in this range, which is wider for larger values of $d/\Lambda$. For instance, in the case of $d/\Lambda = 0.5$ the transmitted field power approaches the limit value $P^-/P_I \approx 0.89$ for incidence close to normal and is almost constant for $\theta < 60^\circ$. With further increase of $\theta$ the transmitted field power decrease rapidly, approaching 0 for $\theta \to 90^\circ$ for all values of $d/\Lambda$. In the case of TM polarization the transmitted field power for $d/\Lambda > 0.5$ varies within some limits in the range of incident angles $0 < \theta < 40^\circ$ and falls down to 0 beyond this range almost linearly. For smaller relative apertures the value of $P^-/P_I$ decreases in almost linear manner for all values of $\theta$ except for the narrow range near $\theta \to 0$ where it is almost constant.

The surface current density distribution defined as:

$$j = n \times H,$$

where $n$ is unity vector along $y$-direction, is shown in Fig. 5 for one period of the structure: $d/2 < x < \Lambda - d/2, \ y = +0$. The number of equations $M = 15$ in (23). This example is similar to those given in [10] (compare with Fig. 4 on p.102) and [11] (compare with Fig. 8 on p.422). Qualitative comparison of current distribution presented in the cited work agrees well with those shown in Fig. 5(a) for $\Lambda/\lambda = 3.7$). This is reasonable since the main difference between both cases in the plane $y = 0$ is only due to the order of singularity of the electric and magnetic fields near the edges of the waveguide walls.

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Finally, we shall give an example analogous to those considered in the work of Kent and Lee [15] who studied the problem of diffraction by an infinite array of parallel strips. In their work for the limiting case of semi-infinite plates of infinitesimal thickness the authors give the expression for the magnitude of the reflection coefficient as follows:

\[ |A_0| = \frac{1 - \cos \theta}{1 + \cos \theta} \]  

(26)

provided \( \Lambda < \lambda/2 \). In Fig. 6 the magnitude of the reflection coefficient \( |A_0| \) is shown for three different small values of the parameter \( d \) describing the thickness of waveguide walls.

![Figure 6: Dependence of the magnitude of the reflection coefficient versus the incident angle \( \theta \) for small values of the waveguide wall thickness \( d \); \( \Lambda = \lambda/4 \). Dashed line corresponds to the case of infinitely thin strips (see (26)).](image)

It is easy to observe that the curves approach the limiting case of infinitely thin walls (the dotted line in Fig. 6) which corresponds to (26) as \( d \) decreases.

Unfortunately, the presented method fails for exactly normal incidence because the matrix elements
become singular in (23). But it works well for very small angles, up to \( \theta \sim 10^{-10} \) for TE and \( \theta \sim 10^{-6} \) for TE polarization respectively. Therefore, high precision calculations are required for incidence very close to normal. In our numerical results the calculations were performed in double precision arithmetic. To solve the system of equations (23), which in general is badly conditioned, the Matlab® SVD (singular value decomposition) routine together with the technique of zeroing of small singular values was applied [14]. The form of the matrix elements (24) is quite straightforward and easy for numerical evaluation. The corresponding series (see (24)) is fast converging and even for \( M = 10 \div 25 \) the obtained results are quite accurate.

6 Conclusions

In this paper we have presented a method for analyzing the electromagnetic wave scattering by an infinite array of thick-walled parallel plate waveguides. Its key point is the application of Fourier expansion with coefficients being properly chosen Legendre functions for field representation in free-space above the array to satisfy the boundary and edge conditions. Although, in contrast to [4] and [3], our method cannot be applied for normal incidence, but it works well for an incidence close to normal with machine accuracy. The presented method is quite straightforward due to the simple form of matrix elements (see (24)) being fast converging series, and can be easily implemented for an approximate evaluation of the scattered field. We have considered separately the cases of plane electromagnetic wave scattering for TE and TM polarization of the plane incident wave, but the method can be generalized and applied to the case of oblique incidence too. The problem of radiation by the array of thick-walled parallel plate waveguides can be also approached in a similar way. The method can be generalized and applied to a number of practically important problems of electromagnetic wave propagation and scattering like the scattering by the system of periodic conducting strips of finite thickness.

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