ELSEVIER

Journal of Materials Processing Technology 60 (1996) 469–474

Journal of Materials Processing Technology

A study on the influence of friction evolution on thickness changes in sheet metal forming

W. Sosnowski, M. Kleiber

Institute of Fundamental Technological Research, Polish Academy of Science, 00-021 Warszawa, Świętokrzyska 21

Abstract

In this paper previous study on the influence of frictional contact effects on velocity field [2], is extended to the case of blank thickness variation with respect to some parameters characterising the friction evolution law. The algorithm presented can be used in sheet metal forming for the thickness distribution optimisation.

Keywords: metal forming, frictional contact, sensitivity, optimisation

1. Introduction

In [2] the influence of frictional contact effects on variations of some nonlinear metal forming response characteristics were studied.

A crucial factor in accurate modelling of realistic technological problems is the way of dealing with frictional contact effects.

A rigid - viscoplastic material model is assumed. The equations describing the rigid-viscoplastic (or, as a special case, rigid-plastic) material behavior are identical to those of a non-Newtonian fluid. This justifies the use of the flow approach in which the main variables are the velocities of the deforming body defined in an Eulerian frame typical of fluid flow problems. Also, the strain rates are linearly dependent on the velocities in a standard manner while the constitutive equation relates stress and strain rate.

In this paper the previous sensitivity study is extended to the case of blank thickness variation with respect to some parameters characterising the friction evolution law. The simplistic models of friction used by the authors so far and based on numerically motivated minor modifications of the standard Coulomb law have proved to be insufficient. Thus, a more advanced description of friction should be used in which the friction coefficient is changing in the course of the process and depends on the surface finish, lubrication properties and contact forces.

The importance of this study is belived to originate from the broadly accepted conviction that the blank thickness distribution is a major factor in evaluating the quality of the forming process. Thus, the sensitivity of the thickness to friction evolution parameters appears crucial for industrial applications.

Large scale numerical computations illustrate the theoretical development.

2. Basic formulation of sheet metal forming

The flow approach to metal forming problems based on the rigid-viscoplastic material model is used as the basis in this paper [3, 5].

After spatial discretization the 'secant' stiffness matrix $\mathbf{K}$ depends on the solution $\dot{\mathbf{q}}$ through the viscosity parameter μ^* so that an iterative process is generally needed to find the solution vector $\dot{\mathbf{q}}$.

$$\mathbf{K}^{(i)}\dot{\mathbf{q}}^{(i+1)} = \mathbf{Q} \qquad i = 0, 1, 2, \dots \tag{1}$$

in which $\mathbf{Q}$ denotes the external force, for instance pressure (usually constant),

$$\mathbf{K}^{(i)} = \mathbf{K}(\mu^*(\dot{\mathbf{q}}^{(i)})) \tag{2}$$

and the constitutive function (viscosity) μ^* is defined as

$$\mu^* = \frac{\bar{\sigma}}{3\dot{\bar{\varepsilon}}} = \frac{\sigma_y + (\frac{\dot{\bar{\varepsilon}}}{\gamma})^{\frac{1}{n}}}{3\dot{\bar{\varepsilon}}} \tag{3}$$

Here, σ_y is the current static tensile yield limit, $\bar{\sigma}$ is the equivalent stress $\bar{\sigma} = \left(\frac{3}{2}s_{ij}s_{ij}\right)^{\frac{1}{2}}$, $\dot{\bar{\varepsilon}}$ is the effective inelastic strain rate, $\dot{\bar{\varepsilon}} = \left(\frac{2}{3}d_{ij}d_{ij}\right)^{\frac{1}{2}}$ and γ , n are parameters of the model. For strain hardening materials the yield limit σ_y is a function of the effective inelastic strain $\bar{\varepsilon}$, $\sigma_y = \sigma_y(\bar{\varepsilon})$; $\bar{\varepsilon}$ has to be computed as the time integral of $\dot{\bar{\varepsilon}}$.

Using the Newton-Raphson scheme the i-th residual is defined as

0924-0136/96/$15.00
PII 0924-0136 (96) 02372-2

$$\mathbf{R}^{(i)} = \mathbf{Q} - \mathbf{K}^{(i)}\dot{\mathbf{q}}^{(i)} \quad (4)$$

while the iterative correction $\delta\dot{\mathbf{q}}^{(i)}$ such that

$$\dot{\mathbf{q}}^{(i+1)} = \dot{\mathbf{q}}^{(i)} + \delta\dot{\mathbf{q}}^{(i)} \qquad i = 0, 1, 2, ... \quad (5)$$

is computed from

$$\mathbf{K}_T^{(i)}\delta\dot{\mathbf{q}}^{(i)} = \mathbf{R}^{(i)} \quad (6)$$

where $\mathbf{K}_T$ is the tangent stiffness matrix.

Let us consider a single slave node 's' coming into contact with a master segment i.e. belonging to the finite set of active nodes $S_A, s \in S_A$

The velocity vector $\dot{\mathbf{q}}_s$ is assumed to be related to the vector of the element nodal velocities by means of the transformation

$$\dot{\mathbf{q}}_{s_{2\times 1}} = \mathbf{A}^{(e)}_{s_{2\times N_e}} \dot{\mathbf{q}}_{N_e\times 1} \quad (7)$$

The quantities

$$_{(e)}\mathbf{K}_{c s N_e\times N_e} = \epsilon \mathbf{A}_s^{(e)^T}{}_{N_e\times 2}\mathbf{n}_{2\times 1}\mathbf{n}^T_{1\times 2}\mathbf{A}_s^{(e)}(\theta\Delta t)^2 \quad (8)$$

$$_{(e)}\mathbf{R}^{(i)}_{c_s\, N_e\times 1} = \epsilon \mathbf{A}_s^{(e)^T}{}_{N_e\times 2}\mathbf{n}_{2\times 1}g_s{}^{(i)}\theta\Delta t \quad (9)$$

are the contributions to the element tangent stiffness matrix and tangent residual force vector due to contact [3, 5, 4]. $\mathbf{n}$ denotes the unit normal to the tool (master) segment, ϵ is the parameter, g_s is gap function and θ is a parameter of the integration scheme.

Limiting ourselfes to the linear Coulomb friction law we postulate the tangent force residual in the form, cf eq. (9)

$$_{(e)}\mathbf{R}_{f_s} = -\mu\epsilon \mathbf{A}_s^{(e)^T}{}_{N_e\times 2}\mathbf{t}'_{2\times 1}g_s\theta\Delta t \quad (10)$$

in which

$$\mathbf{t}' = \mathbf{t}sgn(\dot{\mathbf{q}}_s) = \frac{\dot{\mathbf{q}}^{slip}}{||\dot{\mathbf{q}}^{slip}||} \quad (11)$$

$$\mathbf{K}_{f_s} = -\mu\epsilon(\theta\Delta t)^2\mathbf{A}_s^{(e)^T}\mathbf{t}'\mathbf{n}^T\mathbf{A}_s^{(e)^T} \quad (12)$$

is the non-symmetric contribution to the element stiffness matrix due to friction at the node 's'. After transformation of the secant stiffness matrix to the tangent one the final system of FEM equations becomes

$$(\mathbf{K}_T^{(i)} + \mathbf{K}_c^{(i)} + \mathbf{K}_f^{(i)})\delta\dot{\mathbf{q}}^{(i+1)} = \mathbf{R}^{(i)} + \mathbf{R}_c^{(i)} + \mathbf{R}_f^{(i)} \quad (13)$$

3. Sensitivity analysis with tangent stifness matrix

By using the definition of the residual given in eq. (4) and differentiating it with respect to h where h is any parameter entering the theory we obtain, [2]

$$\mathbf{K}_T\frac{d\dot{\mathbf{q}}}{dh} = \frac{d\mathbf{R}}{dh}\,|_{\dot{\mathbf{q}}\neq\dot{\mathbf{q}}(h)} \quad (14)$$

in which the notation on the right-hand side is meant to indicate that the derivative should be computed under the assumption of the current velocities $\dot{\mathbf{q}}$ independent of the parameter h.

As a consequence, the right-hand side vector can be computed provided the primary (equilibrium) problem has been solved; the velocity sensitivity vector $\frac{d\dot{\mathbf{q}}}{dh}$ then follows by solving eq. (14) which is linear and does not require iteration. Clearly, the latter property has fundamental significance in terms of computational efficiency.

The direct differentiatiom method (DDM) based sensitivity equation (14) can be extended to account for contact and friction effects by observing eq. (12). We obtain the equation

$$(\mathbf{K}_T + \mathbf{K}_c + \mathbf{K}_f)\frac{d\dot{\mathbf{q}}}{dh} = \frac{d}{dh}[\mathbf{R} + \mathbf{R}_c + \mathbf{R}_f]\,|_{\dot{\mathbf{q}}\neq\dot{\mathbf{q}}(h)} \quad (15)$$

which can be effectively used to compute sensitivity of any functional. In the above equation the following notation was used

$$\mathbf{R}_c = \sum_{e=1,..,E}\sum_{s\in_{(e)}S_A} -\epsilon\mathbf{A}_s{}^{(e)T}\mathbf{n}g_s\theta\Delta t = \sum_{s\in S_A} {}_{(e)}\mathbf{R}^{(i)}_{c_s\, N_e\times 1} \quad (16)$$

$$\mathbf{R}_f = \sum_{e=1,..,E}\sum_{s\in_{(e)}S_A} -\mu\epsilon\mathbf{A}_s{}^{(e)^T}\mathbf{t}'g_s\theta\Delta t = \sum_{s\in S_A} {}_{(e)}\mathbf{R}^{(i)}_{f_s\, N_e\times 1} \quad (17)$$

where E is the number of finite elements in the system.

By specifying the parameter h we can readily derive explicit expressions for the right-hand side vector in eq. (15) or eq. (16).

For $h = \mu$ (coefficient of friction) we have, for instance

$$\frac{d\mathbf{R}_c}{d\mu}\,|_{t+\Delta t}\dot{\mathbf{q}}\neq\dot{\mathbf{q}}(\mu) = \sum_{e=1,..,E}\sum_{s\in_{(e)}S_A} -\epsilon\mathbf{A}_s{}^{(e)^T}\mathbf{n}\{\mathbf{n}[\frac{d^t\mathbf{x}_s}{d\mu} + (1-\theta)\Delta t\mathbf{A}^e\frac{d^t\dot{\mathbf{q}}}{d\mu}]\}\theta\Delta t \quad (18$$

$$\frac{d\mathbf{R}_f}{d\mu}\,|_{t+\Delta t}\dot{\mathbf{q}}\neq\dot{\mathbf{q}}(\mu) = -\sum_{e=1,..,E}\sum_{s\in_{(e)}S_A} (\epsilon\mathbf{A}_s{}^{(e)^T}\mathbf{t}'g_s\theta\Delta t + \epsilon\mathbf{A}_s{}^{(e)^T}\mathbf{t}'\{\mathbf{n}[\frac{d^t\mathbf{x}_s}{d\mu} + (1-\theta)\Delta t\mathbf{A}^e\frac{d^t\dot{\mathbf{q}}}{d\mu}]\}\theta\Delta t) \quad (19)$$

4. Sensitivity analysis with secant stifness matrix

In [2] the sensitivity solution was obtained only once, at the end of calculations, by replacing the secant stiffness matrix used in the whole analysis of the basic problem by the tangent stiffnesss matrix. Such an algorithm had serious limitations - it was very difficult, and in the majority of analysed examples even impossible to get sensitivity solution at the early stages of the simulation process. The reason was the ill-conditioning of the tangent stiffness matrix as explained in [1]. It was stated there that the Newton-Raphson iteration scheme is applicable (i.e. it converges) only for markedly rate-dependent materials (steel in hot working conditions, for instance), for which $n < 2$ in eq. 3.

Let us consider again eq. (1) , i. e.

$$\mathbf{K}[\mu^*(\dot{\mathbf{q}},h)]\dot{\mathbf{q}}(h) = \mathbf{Q}(h) \tag{20}$$

Differentiating eq. (20) with respect to h gives

$$\frac{d\mathbf{K}}{dh}\dot{\mathbf{q}} + \mathbf{K}\frac{d\dot{\mathbf{q}}}{dh} = \frac{d\mathbf{Q}}{dh} \tag{21}$$

$$\mathbf{K}\frac{d\dot{\mathbf{q}}}{dh} = \frac{d\mathbf{Q}}{dh} - \frac{d\mathbf{K}}{dh}\dot{\mathbf{q}} \tag{22}$$

$$\mathbf{K}\frac{d\dot{\mathbf{q}}}{dh} = \frac{d\mathbf{Q}}{dh} - (\frac{d\mathbf{K}}{d\mu^*}\frac{d\mu^*}{dh})\dot{\mathbf{q}} \tag{23}$$

The above equations have to be solved with respect to $\frac{d\dot{\mathbf{q}}}{dh}$ iteratively, using the value $\frac{d\dot{\mathbf{q}}^{(i-1)}}{dh}$ calculated previously at the iteration (i-1).

$$\mathbf{K}\frac{d\dot{\mathbf{q}}^{(i)}}{dh} = \frac{d\mathbf{Q}}{dh} - \frac{d\mathbf{K}}{d\mu^*}(\frac{\partial\mu^*}{\partial h} + \frac{\partial\mu^*}{\partial\dot{\mathbf{q}}}\frac{d\dot{\mathbf{q}}^{(i-1)}}{dh})\dot{\mathbf{q}} \tag{24}$$

The right-hand side vector in eq. (24) can be presented more explicitely by noting that

$$\frac{d\mathbf{K}}{d\mu^*}_{N\times N} = \int_\Omega 2\mathbf{B}^T\mathbf{B}d\Omega = \sum_{e=1,..,E}\int_{\Omega_e} 2\mathbf{B}_e{}^T{}_{6\times 5}\mathbf{B}_{e5\times 6}d\Omega_e \tag{25}$$

$$\frac{\partial\mu^*}{\partial h} = \frac{\partial\mu^*}{\partial\dot{\mathbf{q}}}\frac{d\dot{\mathbf{q}}}{dh} + \frac{d\mu^*}{dh} \tag{26}$$

$$\frac{\partial\mu^*}{\partial\dot{\mathbf{q}}^{(i)}} = \frac{\partial\mu^*}{\partial\dot{\bar{\varepsilon}}}\frac{\partial\dot{\bar{\varepsilon}}}{\partial\mathbf{d}}\frac{d\mathbf{d}}{d\dot{\mathbf{q}}} \tag{27}$$

where

$$\frac{\partial\mu^*}{\partial\dot{\bar{\varepsilon}}} = \frac{[-\sigma_y + (\frac{1}{n}-1)(\frac{\dot{\bar{\varepsilon}}}{\gamma})^{\frac{1}{n}}]}{3\dot{\bar{\varepsilon}}^2} \tag{28}$$

$$(\frac{\partial\dot{\bar{\varepsilon}}}{\partial\boldsymbol{\varepsilon}})_{6\times 1} = \frac{2\boldsymbol{\varepsilon}^T}{3\dot{\bar{\varepsilon}}} = \frac{2\mathbf{B}\dot{\mathbf{q}}^T}{3\dot{\bar{\varepsilon}}}_{1\times 6} \tag{29}$$

is the row vector, and

$$(\frac{d\mathbf{d}}{d\dot{\mathbf{q}}})_{6\times N} = \frac{d(\mathbf{B}\dot{\mathbf{q}})}{d\dot{\mathbf{q}}} = \mathbf{B} \tag{30}$$

$$\mathbf{K}\frac{d\dot{\mathbf{q}}^{(i)}}{dh} = \frac{d\mathbf{Q}}{dh} - \frac{d\mathbf{K}}{d\mu^*}\frac{\partial\mu^*}{\partial h}\dot{\mathbf{q}} - \frac{d\mathbf{K}}{d\mu^*}(\frac{\partial\mu^*}{\partial\dot{\mathbf{q}}}\frac{d\dot{\mathbf{q}}^{(i-1)}}{dh})\dot{\mathbf{q}} \tag{31}$$

$$\mathbf{K}_{N\times N}(\frac{d\dot{\mathbf{q}}^{(i)}}{dh})_{N\times 1} = (\frac{d\mathbf{r}}{dh})_{N\times 1} - (\frac{d\mathbf{K}}{d\mu^*})_{N\times N}[(\frac{d\mu^*}{d\dot{\mathbf{q}}})_{1\times N}(\frac{d\dot{\mathbf{q}}^{(i-1)}}{dh})_{N\times 1}]\dot{\mathbf{q}}_{N\times 1} \tag{32}$$

with N denoting the number of the degrees of freedom for the whole structure.

The term $\frac{d\mathbf{r}}{dh}_{[N\times 1]}$ in eq. (32) on the right hand side of eq. (15) has the form

$$\frac{d\mathbf{r}}{dh}_{N\times 1} = \frac{d}{dh}[\mathbf{R} + \mathbf{R}_c + \mathbf{R}_f]\ |_{\dot{\mathbf{q}}\neq\dot{\mathbf{q}}(h)} \tag{33}$$

used for the sensitivity solution with the tangent stiffness matrix. The secant stiffness employed in eq.(32) yields sensitivity solutions for all steps, (i.e. after each step of the equilibrium problem solution), thus giving information about evolution of the sensitivities in the whole process of deformation.

5. Thickness sensitivity with respect to friction

The thickness of the blank is calculated using the incompressibility condition. For an incompressible material we assume

$$\varepsilon_3 = -(\varepsilon_1 + \varepsilon_2) \tag{34}$$

Thickness of the blank $b^{(e)}$ for element No. 'e' (index (e) is later dropped for brevity) is calculated as the function of initial thickness b_0 and strain component in the thickness direction calculated at time $t + \Delta t$

$$\int_0^{t+\Delta t}\frac{\dot{b}}{b}dt = \int_0^{t+\Delta t}\dot{\varepsilon}_3 dt \tag{35}$$

$$\varepsilon_3^{t+\Delta t} = \log b|_0^{t+\Delta t} \tag{36}$$

$$\varepsilon_3^{t+\Delta t} = \log b^{t+\Delta t} - \log b_0 \tag{37}$$

$$\varepsilon_3^{t+\Delta t} = \log\frac{b^{t+\Delta t}}{b_0} \tag{38}$$

$$b^{t+\Delta t} = b_0 exp(\varepsilon_3)^{t+\Delta t} \tag{39}$$

The strain at time $(t + \Delta t)$ can be obtained using the deformation from previous iteration plus the corresponding increment:

$$\boldsymbol{\varepsilon}^{t+\Delta t}{}_{5\times 1} = \boldsymbol{\varepsilon}^t{}_{5\times 1} + \dot{\boldsymbol{\varepsilon}}_{5\times 1}^{t+\Delta t}\Delta t = \boldsymbol{\varepsilon}^t{}_{5\times 1} + \mathbf{B}_{5\times 6}\dot{\mathbf{q}}_{6\times 1}^{t+\Delta t}\Delta t \tag{40}$$

Using eqs. (34) and (40) equation (39) becomes

$$b^{t+\Delta t} = -b_0 exp[\varepsilon_1^t + (\mathbf{B}\dot{\mathbf{q}})_1^{t+\Delta t}\Delta t + \varepsilon_2^t + (\mathbf{B}\dot{\mathbf{q}})_2^{t+\Delta t}\Delta t] \quad (41)$$

The thickness sensitivity with respect to friction can be obtained under the assumption that sensitivity of the velocity field with respect to friction is known. We obtain

$$\frac{db^{t+\Delta t}}{d\mu} = -b_0[\frac{d\varepsilon_1^t}{d\mu} + (\mathbf{B}\frac{d\dot{\mathbf{q}}}{d\mu})_1^{t+\Delta t}\Delta t + \frac{d\varepsilon_2^t}{d\mu} + (\mathbf{B}\frac{d\dot{\mathbf{q}}}{d\mu})_2^{t+\Delta t}\Delta t] \; exp[\varepsilon_1^t + (\mathbf{B}\dot{\mathbf{q}})_1^{t+\Delta t}\Delta t + \varepsilon_2^t + (\mathbf{B}\dot{\mathbf{q}})_2^{t+\Delta t}\Delta t] = -b^{t+\Delta t}\Delta t[(\mathbf{B}\frac{d\dot{\mathbf{q}}}{d\mu})_1^{t+\Delta t} + (\mathbf{B}\frac{d\dot{\mathbf{q}}}{d\mu})_2^{t+\Delta t}] \quad (42)$$

6. Example - stretching of circular blank with hemispherical punch

The stretching of a thin circular, isotropic sheet with a hemispherical punch is considered.

In [2] the sensitivity of the horizontal nodal velocities with respect to the friction coefficient was calculated by using the direct differentiation method (DDM) and tangent stiffness matrix for punch travel of $q_{1y} = 1.17$ inches was presented. Sensitivity values for the last step of analysis were only calculated due to the fact, that the analysis with tangent stiffness matrix at the beginning of the process simulation was not possible - due to singularity

The geometrical configuration of the problem, tools geometry, deformed sheet shape at the punch travel of 1.17 inches and its node numbers are shown in Fig. 1. Limits of the no-contact area are marked in this figure as well. The blank has the initial overall radius of 2.2 inches. The coefficient of friction for the basic problem is $\mu = 0.04$.

50 uniformly distributed linear axisymmetric shell elements are used for the analysis, [3]. The finite element program MFP2D described by [3]. and extended by the first author of this paper is employed.

The uniaxial stress- strain curve for the matrix material is given by

$$\sigma = 5.4 + 27.8\bar{\varepsilon}^{0.504}, \quad \frac{ton}{in^2}, \quad for \quad \bar{\varepsilon} \leq 0.36 \quad (43)$$

$$\sigma = 5.4 + 24.4\bar{\varepsilon}^{0.504}, \quad \frac{ton}{in^2}, \quad for \quad \bar{\varepsilon} > 0.36 \quad (44)$$

In this paper sensitivity of the nodal velocities with respect to the friction coefficient could be calculated at any step. Then thickness sensitivity was obtained according to the formulae given in section 5. For the

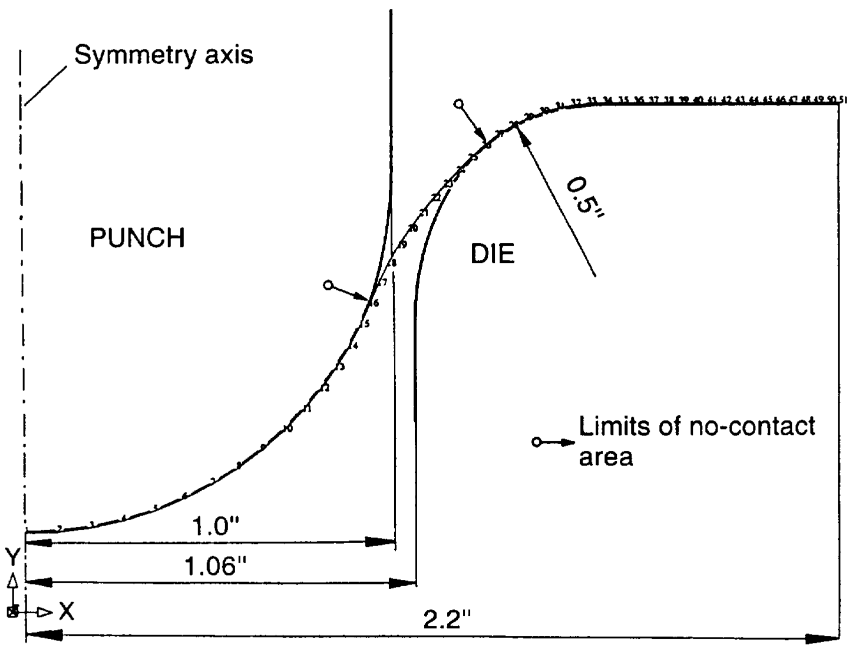

Figure 1: Hemispherical punch stretching problem. Tools geometry. Deformed shape and nodal points numbers.

time being only friction evolution due to changing area of contact was accounted for. Any friction evolution due to any friction parameter changes can be easily included as only very small changes to the program are necessary in such a case.

Similarly as in [2] sensitivity values corresponding to the sheet node numbers given in the previous figure are shown for four different punch travels q in Fig. 2. One can observe that the maximal sensitivity of t with respect to friction corresponds to the points being in contact with the punch or die surface, depending on punch travel. Also a negative sensitivity area in the no-contact zone can be observed.

Similarly as in [2] the results for advanced process (for punch travels of 0.927 inches and 1.227 inches) indicate that the response sensitivity decreases with the process development.

At the very beginning of the process for the punch travel $q = 0.327$ inches, due to sticking conditions on the rigid tools (punch and die) surfaces, very small sensitivity of thickness with respect to friction was observed.

All solutions have been obtained using the secant stiffness matrix as described in Sec. 4 and have converged very well at all stages of the process.

7. Conclusions

1. In the paper an important, but so far very rarely treated in the computational mechanics literature, area of nonlinear parameter sensitivity studies has been identified and discussed.
2. A practical approach to the sensitivity analysis for contact / friction problems has been developed and tested numerically. The lubrication effects on thickness changes can be easily investigated using the algorithm presented. One can optimize process parameters or lubrication conditions in order to minimize thickness sensitivity with respect to friction. It would be difficult to overestimate the practical usefulness of such a numerical tool.

Acknowledgments

The research on which this paper is based was partly supported by the Polish Committee for Scientific Research (KBN) under grant No 7 T08B 007 08 to the Institute of Fundamental Technologial Research, Polish Academy of Sciences, and by the International Center

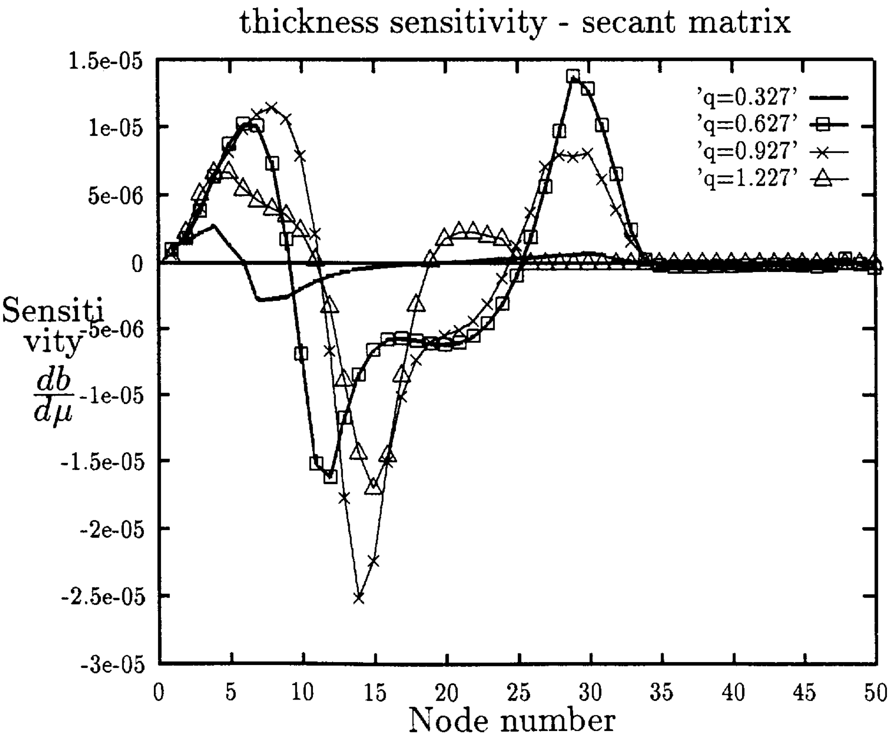

Figure 2: Hemispherical punch stretching problem. Sensitivity of thickness of the blank w.r.t. friction by direct differentiation method (DDM) for different punch travels

for Numerical Methods in Engineering (CIMNE), University Polytechnics of Catalunya, Barcelona, Spain. The support is gratefully acknowledged.

The authors appreciate the work of Professor C. Agelet de Saracibar of CIMNE who developed the program MFP2D used after some extensions in this paper.

References

[1] H. J. Antunez and S. R. Idelsohn. Topics in numerical solution of isothermal and thermal - coupled forming processes. *Latin Amer. Appl. Res.*, 20:69–89, 1990.

[2] M. Kleiber and W. Sosnowski. Parameter sensitivity analysis in frictional contact problems of sheet metal forming. *Comput. Mech.*, 16:297–306, 1995.

[3] E. Oñate and C. Agelet de Saracibar. *Numerical modelling of sheet metal forming problems in P. Hartley et al. (Eds.).* Numerical Modelling of Material Deformation Process, Springer-Verlag (1992).

[4] P. Wriggers, J.C. Simo, and R.L. Taylor. Penalty and augmented lagrangian formulations for contact problems. *Proc. NUMETA'85 pp 97-106*, 1985.

[5] O.C. Zienkiewicz and P.N. Godbole. Flow of plastic and viscoplastic solids with special reference to extrusion and forming processes. *Int. J. Num. Meth. Engng.*, 8:3–16, 1979.