Torque Detection using Brownian Fluctuations

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We report the statistical analysis of the movement of a submicron particle confined in a harmonic potential in the presence of a torque. The absolute value of the torque can be found from the auto- and cross-correlation functions of the particle’s coordinates. We experimentally prove this analysis by detecting the torque produced onto an optically trapped particle by an optical beam with orbital angular momentum.

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The way in which deterministic perturbations affect the random walk of a small particle immersed in a fluid bath (Brownian motion) underlies many physical, chemical, and biological phenomena: examples include Brownian motors [1], molecular transport processes [2], thermally activated transitions in a potential landscape [3], and thermal fluctuations in an optical trap [4]. In a harmonic potential well, a Brownian particle oscillates around the equilibrium position. A statistical analysis of these fluctuations allows one to measure pico- to femto-Newton forces [5–11]. Another type of perturbation is a mechanical torque which can be exerted on the Brownian particle. This has previously been studied by measuring the rotation rate of the particle itself [12–22] and a statistical description is available for the case of rotational Brownian motion (the Brownian movement of the particle around its own axis) [23]. However, the precise manner in which the torque affects the statistics of the translational Brownian trajectories (the Brownian movement of the center of mass of the particle) is still unknown. Here we report the analysis and measurements of the movement of a submicron particle confined in a harmonic potential in the presence of a torque. The absolute value of the torque can be found from the auto- and cross-correlation functions of the particle’s coordinates. We anticipate our study to be a starting point for the development of new techniques to measure the torque produced, for example, by biomolecules, molecular motors, or by optical beams with angular momentum.

We consider a sphere of mass $m$ and radius $R$ suspended in a liquid medium and confined within a harmonic potential well, where it moves randomly due to the thermal excitation [Fig. 1(a)]. We suppose an external torque is exerted on the sphere. In the absence of the potential well due to the friction the sphere rotates around the $z$ axis with a constant angular velocity $\Omega$, whose value results from a balance between the torque applied to the sphere and the drag torque: $\tau_{\text{drag}} = r \times F_{\text{drag}} = \gamma r \times v = \gamma r \times (r \times \Omega)$, where $r$ is the sphere’s position, $v$ is its linear velocity, $\gamma = 6\pi R \eta$ is the friction coefficient, and $\eta$ is the viscosity. Hence, the force acting on the sphere from the torque source is given by $F = \gamma r \times \Omega$, which depends on the position of the sphere. A time average of the torque exerted on the particle can then be expressed as $\langle \tau \rangle = \gamma (r \times (r \times \Omega)) = \gamma (\Omega r^2) = \gamma \Omega (r^2)$, where $\langle r^2 \rangle$ is the mean square displacement of the sphere in the plane orthogonal to the torque.

The Einstein-Ornstein-Uhlenbeck equations [24] for the Brownian motion of the sphere in the plane perpendicular to the rotation axis can now be presented as:

FIG. 1 (color online). (a) Harmonic potential well $U(x, y) = \frac{k}{2} (x^2 + y^2)$ ($k$ is the restoring force constant or stiffness of the harmonic oscillator) with a Brownian particle inside and with an external torque acting on the particle. (b) Experimental setup: 1—trapping 785 nm laser beam, 2—532 nm beam, 3—holographic mask, 4—Dove prism, 5—100 × 1.3 NA objective, 6—collimating 40× objective, 7—quadrant photodetector. (c) Position of the sphere in the chamber when only the 532 nm LG propagates, and (d), when both the 532 nm and the 785 nm beams propagate in the chamber. The arrows show the propagation direction of the trapping and LG beams.