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ABSTRACT

The paper focuses on some issues regarding the utilization of small rectangle-shaped piezoelectric transducers as both sensors and actuators in active vibration and vibroacoustic control systems of beam, plate and panelled structures with arbitrary (non-homogeneous) boundary conditions. A new form of description of a simple proportional active control system with multiple independent feedback loops is proposed. The modal sensitivity functions of sensors and the modal selectivity functions of actuators are introduced to describe their ability for sensing and exciting specific structural modes of the structures. Basing on the assumed form of cost function and the derived equations of control system the influence of the modal characteristics of transducers on the stability of the system and on the performance of the active control is analyzed. The results of analytical solutions and numerical simulations are compared with the results of the experiments carried out on various beam and plate structures made up of aluminium or composite materials including the actual materials used in aviation, proving usefulness of the presented approach.

1. INTRODUCTION

Active vibration and vibroacoustic control methods are of great interest in many industrial applications and have been the topic of numerous scientific investigations over the past several decades. The necessary elements of control systems developed for such applications are sensors and actuators: the sensors allow to determine the current state of vibrating structures (or some parameters of the generated acoustic field), whereas the actuators are used to apply the control loads. Among a variety of available techniques of implementation, one of the most commonly used are piezoelectric transducers attached to the surface of structures under control. Such solution preserves compactness of the controlled system while providing good electro-mechanical properties. The electric signals from sensors are processed by a control unit and, based on the results, the optimal parameters of the excitation signals driving actuators are determined. However, assuming that the parameters of the external excitation – which is the primary source of the vibrations – are unknown, the information obtained from the sensors is never complete, that is: as the number of sensors and their areas are limited to some reasonable (finite) values, the gathered data would not allow to posses the complete knowledge about the parameters of the vibrations of the considered structure in any possible case. On the other hand, for the very similar reasons, the control system is not able to excite any arbitrarily chosen form of vibrations using the finite number of actuators.

Many of the studies devoted to the field of active vibration and vibroacoustic control focus on specific types of structures, which may be accurately described using analytic formulas – like, for example, beams or simply supported plates. Solutions obtained for such cases allowed to design piezoelectric sensors and actuators sensitive only to specific sets of structural modes [1] or even to a single structural mode [2, 3] by changing the shapes, sizes and/or locations of the transducers. However, the results of these investigations cannot be easily generalized into a more general case of plates with arbitrary boundary conditions of support.

Considering active vibroacoustic control systems, the most commonly investigated case of acoustic conditions is the far-field acoustic radiation of structures placed in an infinite, rigid baffle (see, for example [1, 4, 5]). The radiation characteristics of the vibrating structure in such case can be computed using the Rayleigh's integral. In case of the other acoustic boundary conditions or for the near-field computations, numerical techniques have to be implemented in order to determine the amplitude of the radiated acoustic pressure field. Typically it is the Finite Element Method (FEM) or the Boundary Element Method (BEM). The approach using FEM (see for example [6]) allows for straightforward coupling with the vibrating structure, yet it involves a huge number of degrees of freedom and means that some non-reflective boundary conditions must be properly applied since only a finite sub-domain of the whole infinite acoustic domain can be meshed. This final problem is non-existent in case of BEM. The method applicable to the most general cases is the indirect variational Boundary Element Method. However, it involves relatively high computational cost when compared with the other versions of BEM, and requires special integration schemes to deal with the singularities in the integrands [7–9].

Taking into account the parameters of the closed-loop feedback control system it is desirable to use collocated piezoelectric sensor-actuator pairs. Two different solutions which ensure this feature can be found in literature. The first one – which is simpler and more practical, yet not always feasible due to the possible lack of access to both sides of a structure – is to attach the transducers symmetrically to the both surfaces of a thin beam or plate [1, 10]. The second solution involves the use of a single piezoelectric element as sensor and actuator simultaneously [11–15]. The advantages of such a solution with respect to the functionality of the control system are significant, but the necessary complications of the corresponding electronic circuits together with a requirement to meet very stringent parameters make it impractical.

Optimization algorithms for the placement of sensors and actuators may be based on various cost functions depending on the type of structure, its purpose, and also some restrictions related with the usage of various types of transducers. The state of the art in this field is well documented in corresponding review papers (see, for example [16–18]). Again, the majority of relevant scientific investigations is focused on thin beams [19, 20] and plates with specific boundary conditions (simply supported [21, 22], clamped [23, 24], cantilevered [23, 25]). Other approaches also usually impose some restrictions on the structure mounting parameters, like, for example, plates with arbitrary but homogeneous along the edges boundary conditions [26]. The optimization problem is usually solved numerically with different iterative algorithms.

The purpose of the present paper is to investigate the modal characteristics of small rectangle-shaped piezoelectric transducers attached to surfaces of beam, plate and panelled structures in terms of their utilization for active control of vibrations. A new form of theoretical description is proposed. The modal sensitivity functions of sensors and the modal selectivity functions of actuators are introduced to describe their ability for sensing and exciting specific forms of vibrations of the structures. The presented approach – in contrast to most studies described in literature – is elaborated and tested for plates with arbitrary (non-homogeneous) boundary conditions; moreover, due to the high level of generality of the proposed form of description it should work for structures of more complex geometries than rectangular plates. It has been shown in the paper how such relatively simple approach may be utilized to deal with sandwich composites used in aviation – one of the tested sample structures is actually made from the same material as the fuselage of a small gyrodyne, provided by the aircraft manufacturer. Therefore, the main original contribution of the paper is not the introduced theoretical approach by itself, but rather in relation with the results of experiments. The comparison between predicted and obtained values of modal characteristics provides many useful conclusions regarding the utilization of small rectangle-shaped piezoelectric transducers in active control of vibrations of various real-life structures. Some results of the related preliminary investigations have already been presented in [27].

2. THEORETICAL CONSIDERATIONS

2.1. Problem statement

Beam and plate structures with arbitrary boundary conditions are investigated. It is assumed, that *N* small piezoelectric transducers are attached to the surfaces of the considered structures. Some of the transducers are used as sensors while the remaining serve as actuators in a closed-loop active feedback control system. It is assumed that the structures and the piezoelectric transducers attached to their surfaces are rectangle in shape and that their edges are parallel to the axes of the global coordinate system. The typical geometry of the problem is depicted in Figure 1. Vibrational motion of the structures is assumed to occur only in the z direction, so only one, corresponding component of the displacement field is considered, namely, the deflection $w = w(x,\ y,\ t)$.

In case of the so-called beam structures it is assumed that the length a of a structure is much greater than its width b and its thickness h_s. The flexural waves propagate along the x direction only and the deflection w is constant along the y direction, that is: $\dfrac{\partial w}{\partial y} = 0$. The vibrations of the beams are modeled using the classical Euler-Bernoulli beam theory.

Similarly, plate and panelled structures considered in this study are thin in the sense of the classical Kirchhoff's plate theory. They are considered to be made of homogeneous, isotropic material (thus, in case of composites, such approach can be applied provided that the relevant effective material constants are known). The equations of motions for the considered structure models can be found, in example, in [28].

It is assumed, that each of the considered structures is subjected to an external harmonic excitation with arbitrary spatial distribution. The system is linear and the structural damping is neglected, therefore the response of the structure is also harmonic, with the same frequency and phase as the excitation. The present study

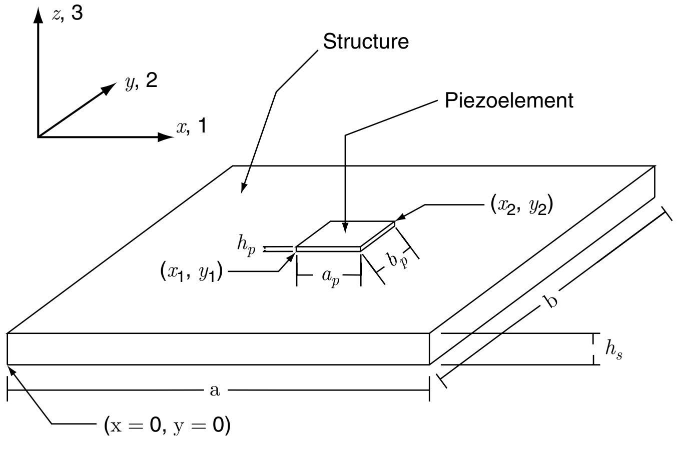


Figure 1. Geometry of the considered problem.

focuses only on a low-frequency range (up to about 400 Hz), since higher frequency vibrations can be rather easily suppressed using the well-known passive techniques - like thin soft liners, a porous core of panel (see for example [29, 30]). Taking all these assumptions into account, the response of a structure can be approximated by a finite sum of N structural modes as follows:

$$w(x,t) \cong \sum_{n=1}^{N} \Phi_n(\boldsymbol{x}) w_n(t) = e^{\mathrm{i}\omega t} \sum_{n=1}^{N} \Phi_n(\boldsymbol{x}) W_n \cong e^{\mathrm{i}\omega t} \tilde{w}(x), \tag{1}$$

where $(x) \equiv (x)$ in case of beam structures and $(x) \equiv (x,\ y)$ in case of plate structures, while Φ_n is the normalized shape function of mode n with w_n as the corresponding time-varying coefficient. When a harmonic motion is considered - with $\omega = 2\pi f$ as the angular frequency of the external excitation force (f being the frequency) - these coefficients are time-harmonic and can be expressed as $w_n(t) = e^{\mathrm{i}\omega t} W_n$, where W_n are the (frequency-dependent) modal amplitudes; $\tilde{w}(x)$ in the (frequency-dependent) amplitude function of harmonic vibrations. Here and below, it is understood that when the time-harmonic term $e^{i\omega t}$ is involved, eventually only real (or imaginary) part of the whole expression has physical meaning (and should be eventually taken as the final result).

Modal shape functions Φ_n are found by solving the corresponding eigen-problems. In the case of beams, regardless of their boundary conditions (and, as a matter of fact, because of their 'unidimensional' simplicity), it is always possible to find analytical solution consisting of a sum of trigonometric and hyperbolic functions [28]. In the case of plate structures, however, even when they are rectangular in shape, the analytical solutions can be found only for some specific ('geometrically-homogeneous') boundary conditions and - in general - it is required to use numerical methods, such as the Finite Element Method, in order to solve such problems.

In order to control the vibrations of structures, piezoelectric sensors and actuators are connected in control loops. In the present study control performance and stability is evaluated on the example of a relatively simple, decentralized proportional feedback system. There are, of course, many other well-known control algorithms that can deal with relevant problems (see, for example, [1, 4, 10, 13]). However, taking into account the fact that steady-state harmonic vibrations are considered, the control processes in most of the cases can be easily reduced and modeled with the proposed approach. It is assumed, that the control system consists of M independent feedback loops. The electric signal from the piezoelectric sensors is fed back to the corresponding actuators through the amplifiers with adjustable gains. Under such conditions, the optimal control strategy is to find such a vector of gains with values belonging to the available control space, for which the desired cost function should be minimized. The form of this function depends on the scopes of the control. Operating of the active control system causes changes of the vibrational mode components, which may lead to minimization of the global vibration level or radiated acoustic pressure in a given sub-space.

The acoustic pressure field generated (in air) by a vibrating structure is, in general, complex and varies strongly with the frequency, the distance from the source, and the boundary conditions. Due to the linearity of the considered system the sought values in the specified points of the space surrounding considered structure can be regarded as linear functions of the modal amplitudes. That leads to the following, general formula:

$$p(\mathbf{R}) = p(\mathbf{R}, W_1, \ldots, W_N) = p_{\mathrm{re}}(\mathbf{R}, W_1, \ldots, W_N) + jp_{\mathrm{im}}(\mathbf{R}, W_1, \ldots, W_N) \tag{2}$$

where p_{re} and p_{im} denote the real and imaginary parts of the complex amplitude of acoustic pressure and can be written as:

$$p_{\mathrm{re}}(\mathbf{R}, W_1, ..., W_N) = \sum_{n=1}^{N} P_n^{\mathrm{re}}(\mathbf{R}) W_n \tag{3}$$

and

$$p_{\mathrm{im}}(\mathbf{R}, W_1, ..., W_N) = \sum_{n=1}^{N} P_n^{\mathrm{im}}(\mathbf{R}) W_n. \tag{4}$$

Here, $P_n^{\mathrm{re}}(\mathbf{R})$ and $P_n^{\mathrm{im}}(\mathbf{R})$ are the modal radiation coefficients at a given point of space $\mathbf{R}$; they can be computed either analytically ((using, for example, Rayleighs integral), or numerically using, for example, one of the methods described briefly in previous section). Equations (2)–(4) link the parameters of the generated acoustic pressure field with the vibrational characteristics of the considered structure.

The shapes of the piezoelectric transducers and their locations on the surface of structure determine the sets of the vibrational mode components available by changing the gains of the feedback loops in a specified, limited range of values. Therefore, in the considered cases of the external excitation forces and the boundary and acoustic conditions the control performance is strongly affected by the parameters and distribution of sensor/actuator pairs. Due to the fact that the piezo-transducers are permanently bound to the surfaces of the controlled structures and their locations have to be chosen at the stage of the control system design, it is necessary to analyze in advance their parameters and probable control strategies.

2.2. Piezoelectric sensors and actuators

The behavior of piezoelectric transducers is governed by the constitutive equations which include coupling between mechanical and electrical phenomena. Assuming that the summation convention is used (i.e., the summation is carried out over the repeating indices i, j, k, $l = 1, 2, 3$) these equations can be presented as follows, for example, in the so-called stress-charge form:

$$T_{ij} = c_{ijkl} S_{kl} - e_{kij} E_k, \tag{5}$$

$$D_k = e_{kij} S_{ij} + \epsilon_{ki} E_i \tag{6}$$

where $T_{ij}\left[\frac{N}{m^2}\right]$ is the second-order stress tensor, $S_{ij}\left[\frac{m}{m}\right]$ is the second-order strain tensor, $c_{ijkl}\left[\frac{N}{m^2}\right]$ is the fourth-order elasticity tensor, $e_{kij}\left[\frac{C}{m^2}\right]$ is the third-order tensor of piezoelectric coefficients (for the so-called stress-charge form), $D_k\left[\frac{C}{m^2}\right]$ is the electric displacement vector, $E_k\left[\frac{V}{m}\right]$ is the electric field vector, and $\epsilon_{ki}\left[\frac{F}{m}\right]$ is the second-order tensor of dielectric constants.

It is assumed that a sensor electrode covers the whole relevant surface S of the transducer and that the polarization of the material is constant. The electric charge which appears on the electrodes of a piezoelectric sensor fixed to the surface of a vibrating thin plate or beam structure is computed as follows

$$Q = -\iint_S D_i n_i dS = -\iint_S D_3 dS \tag{7}$$

where n_i are the components of the unit vector normal to the surface of structure and at the same time identical with the direction of polarization of the piezoelectric sensor ($n_1 = n_2 = 0$, $n_3 = 1$ in Figure 1), while – in the absence of external electric field, and assuming that the piezoelectric transducers are made up of transversely-isotropic piezo-ceramics (which involves that: $e_{311} = e_{322}$ which now will be denoted by e_3, whereas $e_{312} = e_{321} = 0$) – the relevant component of the dielectric displacement vector (6) (having also noticed that $S_{33} \sim 0$) reads

$$D_3 = e_{3ij} S_{ij} = e_{311} S_{11} + e_{322} S_{22} + e_{333} S_{33} \simeq e_3 (S_{11} + S_{22}). \tag{8}$$

It is assumed that (because of a very good bonding) the in-plane deformation of piezoelectric element is consistent with the deformation of the underlying structure, thus, the relevant components depend on the corresponding curvatures and the distance between the mid-planes of the piezo-element and the structure, namely:

$$S_{11} = \frac{h_p + h_s}{2} \frac{\partial^2 w}{\partial x^2}, \quad S_{22} = \frac{h_p + h_s}{2} \frac{\partial^2 w}{\partial y^2}. \tag{9}$$

Here, h_p and h_s are the thickness of the piezo-element and the structure, respectively. Obviously, in the case of beam structures $S_{22} = 0$. Now, the electric charge induced on the shunted piezoelectric sensor attached to the surface of the plate structure can be expressed as follows

$$Q = \frac{-\left(h_p + h_s\right)}{2} e_3 \int_{x_1}^{x_2} \int_{y_1}^{y_2} \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} dx dy. \tag{10}$$

We would like to obtain the sensitivity function of piezoelectric sensor to specific structural modes. To this end, we first compute the amplitude of the electric charge induced on a transducer by substituting the time-harmonic form (1) into equation (10) to obtain:

$$Q = e^{\mathrm{i}\omega t} \tilde{Q} = e^{\mathrm{i}\omega t} \frac{-\left(h_p + h_s\right)}{2} e_3 \sum_{n=1}^{N} W_n \left[\int_{x_1}^{x_2} \int_{y_1}^{y_2} \frac{\partial^2 \Phi_n}{\partial x^2} + \frac{\partial^2 \Phi_n}{\partial y^2} dx dy \right]. \tag{11}$$

Here, $\tilde{Q}$ denotes the amplitude of the harmonically varied sensor charge.

It is assumed that the piezoelectric sensors are connected to the charge-to-voltage transducers circuits. Hence, the resulting voltage signal which is fed to the active control system is proportional to the charge given by equation (11), and so the desired sensitivity function $\tilde{S}_m \left[\frac{V}{m}\right]$ of a sensor to the structural mode m can be defined as follows:

$$\tilde{S}_m = R \frac{\tilde{Q}_m}{W_m} = R \frac{-\left(h_p + h_s\right)}{2} e_3 \left[\int_{x_1}^{x_2} \int_{y_1}^{y_2} \frac{\partial^2 \Phi_m}{\partial x^2} + \frac{\partial^2 \Phi_m}{\partial y^2} dx dy \right], \tag{13}$$

where $\tilde{Q}_m$ is the electric charge amplitude induced by the mode m, and $\mathrm{R}\left[\frac{V}{C}\right]$ is the gain of the signal conditioning circuit attached to the piezoelectric transducer.

The external loading introduced by the rectangle-shaped piezoelectric actuator situated in such a way that its edges are in parallel with the relevant axes of the global coordinate system (see Figure 1) can be approximated by linear (i.e., per length) moments acting along these edges. The excitation function can be then expressed as follows [1]:

$$F_S(x,y) = EIK^f s_a\Big[\delta'(x-x_1)-\delta'(x-x_2)\Big]\Big[H(y-y_1)-H(y-y_2)\Big] \\ +EIK^f s_a\Big[\delta'(y-y_1)-\delta'(y-y_2)\Big]\Big[H(x-x_1)-H(x-x_2)\Big], \tag{13}$$

where $\delta'(\cdot)$ is the derivative of the Dirac delta function, $H(\cdot)$ is the Heaviside step function, E is the Young's modulus of the structure, K^f is the material-geometric constant dependent of material properties of the piezo-ceramics and type of actuator (symmetric or antisymmetric) [1] and s_a is the strain of the actuator (the same in the x- and y-direction, because of the transversal-isotropy in the xy-plane) caused by the applied driving voltage V which generates within the piezo-element a uniform electric field in the z-direction, $E_3 = V/h_p$, therefore:

$$s_a = \frac{d_3 V}{h_p} \tag{14}$$

where d_3 is the relevant piezoelectric material constant ($d_3 \equiv d_{311} = d_{322}$ from the strain-charge form of piezoelectric constitutive relation). The effects of added mass and stiffness introduced by the actuator as well as a longitudinal strain of the structure (resulting from the transverse asymmetry of the actuator) are neglected in the present considerations.

While considering response of a structure to an external harmonic excitation, it is very convenient to perform the decomposition of the loading force into the eigenmodes of the structure. Due to the orthogonality property of the mode shape functions Φ_n, the amplitude of mode number m excited by the external force F_S can be expressed as:

$$W_m = \frac{\iint_S F_S(x,y)\Phi_m dS}{\rho_s h_s\left(\omega_m^2-\omega^2\right)\iint_S \Phi_m^2 dS}, \tag{15}$$

where ρ_s is the density of the structure and ω_m is the eigenfrequency of the considered mode m. To compute the modal decomposition coefficients of the excitation introduced by the actuator driven with the harmonic voltage V relations (13) and (14) are used for equation (15) and yield the following result:

$$A_m\left(V,\omega\right) = \frac{EIK^f d_3 V}{h_p\rho_s h_s\left(\omega_m^2-\omega^2\right)\iint_S \Phi_m^2 dS}\left[\int_{x_1}^{x_2}\int_{y_1}^{y_2} \frac{\partial^2\Phi_m}{\partial x^2}+\frac{\partial^2\Phi_m}{\partial y^2}dxdy\right]. \tag{16}$$

We now introduce the actuator selectivity function to the structural mode m, defined as:

$$\tilde{A}_m = \tilde{A}_m(\omega) = \frac{A_m(V,\omega)}{V}. \tag{17}$$

The selectivity of a piezo-actuator to the structural mode m describes the amplitude of mode m excited by the actuator driven with a harmonic signal of unit voltage amplitude with angular frequency ω (in absence of other excitation forces).

It should be pointed out that – in contrast to the modal sensitivity function of a piezo-sensor, defined previously – the modal selectivity is a function of the frequency of driving signal and strongly depends on the difference between this frequency and the eigenfrequency of mode m. The singularity in equation (16) occurring in case when $\omega_m = \omega$ results obviously from the assumptions of negligible damping and linearity of the system, which are not valid for large amplitude vibrations. When the equations (16) and (17) are compared with the relation defining the sensor sensitivity function (12) an important remark should be made, namely: the surface integrals are the same and depend only on the transducer's coordinates $(x_1,\ y_1)$ and $(x_2,\ y_2)$. This means that the efficiency of a piezoelectric transducer with respect to a particular structural vibration mode is similar both for the mode sensing or actuating. It will be shown below that these conclusions – which result from the reciprocal principle regarding the direct and inverse piezoelectric effects – are of a great importance for the process of development of the active control system.

2.3. Active vibroacoustic control

The pairs of piezoelectric sensors and actuators are connected in feedback loops via amplifiers with adjustable gains $-G$, where $0 \leqslant G \leqslant G_{\max}$ and $G_{\max}$ is the maximum available gain value. Through the operation of the control system the parameters of vibrations of the structure are modified. The amplitudes of vibrational modes can be linked with the distribution of the acoustic pressure by utilization of separate acoustic analysis and equations (2)-(4). Depending on the chosen objective function different gain values may be optimal. Equations describing general relations between modal amplitudes and the parameters of the control system are derived and presented below.

In the first step a simplified active control system with a single feedback-loop is analyzed. It is assumed that the considered electro-mechanical system is linear. There are two sources of vibrations. The primary source is an external disturbance described with spatial force distribution $F_S\ (x,\ y)$, acting with angular frequency ω. The modal amplitudes of the vibrating structure excited by the external force itself – i.e. in the absence of the forces introduced by the control system – can be computed using the equation (15):

$$\mathfrak{F}_n = \frac{\iint_S F_S(x,y)\Phi_n dS}{\rho_s h_s(\omega_n^2 - \omega^2)\iint_S \Phi_n^2 dS}. \tag{18}$$

The other source of vibrations of excited in the structure is the influence of the actuator (to be used for active control). Modal decomposition of the introduced loading is given by equation (16). The driving, harmonic voltage V is equal:

$$V = -Ge^{\mathrm{i}\omega t}\sum_{n=1}^{N}\tilde{S}_n W_n. \tag{19}$$

where $\tilde{S}_n$ is the sensor sensitivity (12).

Due to the assumption of system linearity the transverse vibrations of the structure may be written as a sum of the responses to both mentioned excitation sources:

$$\begin{aligned} w(x,y,t) &= e^{\mathrm{i}\omega t}\sum_{n=1}^{N}\Phi_n(x,y)W_n = e^{\mathrm{i}\omega t}\left[\left(\sum_{n=1}^{N}\mathfrak{F}_n\Phi_n\right)+\left(\sum_{n=1}^{N}A_n\Phi_n\right)\right] \\ &= e^{\mathrm{i}\omega t}\left[\left(\sum_{n=1}^{N}\mathfrak{F}_n\Phi_n\right)-G\left(\sum_{n=1}^{N}\tilde{S}_nW_n\right)\left(\sum_{n=1}^{N}\tilde{A}_n\Phi_n\right)\right]. \end{aligned} \tag{20}$$

After using the orthogonality property of the modal shape functions and some transformations, the following relation for resulting amplitude of structural mode a is eventually obtained:

$$W_a = \frac{\mathfrak{F}_a}{1+G\tilde{S}_a\tilde{A}_a} + \frac{G\tilde{A}_a}{1+G\tilde{S}_a\tilde{A}_a}\sum_{\substack{m=1\\ m\neq a}}^{N}\tilde{S}_m W_m. \tag{21}$$

Equation (21) implies some important remarks that should be taken into account while developing the active vibroacoustic control system. First part of the right-hand side of the equation $\frac{\mathfrak{F}_a}{1+G\tilde{S}_a\tilde{A}_a}$ represents the well-known relation describing the resultant gain of the closed-loop feedback controller. If we would be able to create a single-mode in-phase sensor/actuator pair, the system would remain unconditionally stable and the amplitude of the selected mode reaches zero as the feedback gain reaches infinity. Method of creating modal sensors/actuators has been described by Lee and Moon [2]. However, the practical implementation of such transducers is limited to simple one-dimensional beam structures and only to few lowest-order structural modes. Another important disadvantage of single-mode sensors/actuators is the fact, that we would need one separate pair of transducers for every mode we would like to control, which would lead to very complex, multi-layered structure.

Another remark, that can be concluded from equation 21 is that one of the conditions of the stability of the active control system is $G\tilde{S}_a\tilde{A}_a \neq -1$ for every mode number a in the whole considered frequency range. The sensor should also be sensitive to the structural modes excited by the corresponding actuator. To provide the described features the collocated sensor/actuator pairs can be used. There are two ways of implementing this solution. First, we can use a single piezoelectric transducer, working simultaneously as a sensor and an actuator. Second method requires two piezoelectric elements, mounted symmetrically on the both sides of the controlled structure. The examples of implementations of the first method have been described i.e. by Dosch [12], Anderson and Hagood [11] and Vipperman [13, 14]. One of the main disadvantages of this solution is the presence of the high actuator driving signal and the low sensing signal simultaneously at the input of the signal conditioning circuit. This implies the requirement of very high range of linear operation of the first stage of the amplifier. The second described solution is much less complicated and commonly used, but it requires the access to the both sides of the structure, which may not always be possible.

Next, the more general case of an active control system with a number of K independent control loops, consisting of sensor/actuator pairs and feedback amplifiers of gain G_k, where $k \in \langle 1;\ K\rangle$ is the number of a control loop, is considered. To compute the modal amplitudes of vibrations of a plate structure with such system attached we have to modify equation 21 and - upholding the assumption of linearity - include influence of all of the control loops:

$$W_a = \mathfrak{F}_a - \sum_{k=1}^{K}\left[G_k\tilde{A}_{ka}\left(\sum_{n=1}^{N}\tilde{S}_{kn}W_n\right)\right]. \tag{22}$$

where $\tilde{A}_{ka}$ is the selectivity function of actuator k to mode a (see equation (17)) and $\tilde{S}_{kn}$ is the sensitivity function of sensor k to mode n (see equation (12)). The equation (22) reveals the complex relation between all of the control loops acting each on each other, coupled via all of the considered vibrational modes. However, the conclusion that the collocation of the single sensor/actuator pairs guarantees the stability of the considered system is still valid. The detailed analysis of dependencies between modal amplitudes and feedback gain values is complex and fall beyond the scope of the paper, as it would require separate, extensive description – however, with one exception. In case when excitation frequency is equal to one of the eigenfrequencies of the considered structures then – if damping is low – the equations (21)-(22) can be trivialized to describe only one, corresponding form of vibrations. The derived formulas are thus simplified to well-known relations describing simple closed-loop feedback system. The amplitude of vibrations is monotonic function of every single feedback gain G_k. The higher are the relevant modal sensitivity and selectivity values of involved sensor/actuator pairs, the higher reduction of vibrations (and generated sound pressure) can be obtained. It should be emphasized, that this conclusion has significant practical implications as many real-world structures reveal problems with high noise emission or level of vibrations only when excited at one of their eigenfrequencies. The presented approach allow in such case for fast and simple computation of the control performance, based on the simplified forms of equations (21)-(22).

3. NUMERICAL AND EXPERIMENTAL INVESTIGATIONS

Modal sensitivity/selectivity functions of small, rectangle-shaped piezoelectric transducers attached to the surfaces of beam, plate and panelled structures are investigated in this section. The solutions obtained using analytical formulas and numerical simulations are compared to the results of experiments.

The numerical finite-element analyses were used to solve eigen-problems of investigated plate and panelled structures, however, it must be emphasized here that in the proposed line of investigation only eigenmode shapes were of interest since the paper subject is the modal sensitivity and selectivity. Thus, the mass and stiffness properties of structures were not important when caring out these analyses, and in case of composite structures very approximative values could be taken. Such approach, however, requires that the investigated plate and beam composite structures can be considered as macroscopically homogeneous and macroscopically isotropic (in their planes), so that the mode shapes should be the same whatever are the stiffness and mass density, and they depend only on the structure geometry and conditions of support. This entails also that the effect of small piezoelectric patches fixed to their faces can be neglected. However, this latter assumption – important also for aluminium structures and usually valid at lower frequencies – is rather standard and should be also valid in case of stiffer composites.

The assumption of in-plane isotropy may at first appear as disputable in case of composites, however, one should notice that although the carbon-fibers for the composite plate faces were woven in an orthogonal pattern (see Figure 3), exactly the same fibers were used in both mutually perpendicular directions, and that results in the so-called structural isotropy (in plane) of both faces. In other words, the carbon fabric is a plain weave and thus isotropic in the plane of the weave. The honeycomb core is also isotropic in the plane of the cell pattern under three loading mechanisms as explained in [31], since it is formed from cells of regular hexagons.

Nevertheless, the final confirmation of the validity of both assumptions of macroscopic homogeneity and isotropy is confirmed by the results of the proposed approach which compares and utilises in conjunction numerical and experimental investigations.

3.1. Beam structures

Due to the undertaken assumptions the classical Euler-Bernoulli thin beam theory is used to describe the vibrational motion of the considered beam structures. Under such conditions the vibration mode shapes can be computed analytically, as the sum of harmonic and hyperbolic functions, with coefficients depending on the boundary conditions [28]. Basing on such a formula, the modal sensitivity function was computed for a piezoelectric sensor (of known dimensions) attached to the clamped beam structure. Some results of these computations, obtained for a 3 cm long piezo-element on a 58 cm long beam (with one end clamped and the other free) are shown in Figure 2. According to the considerations discussed in section 2.2, piezoelectric sensors and actuators are bounded with reciprocal relation, which implies that the sensitivity and ability of exciting specific structural modes depend only on the location of the transducer on the surface of structure.

The presented results were used for positioning piezoelectric transducers on thin beams made of aluminium and glass-fiber, which were examined during further experimental research. For homogeneous beams the modal shape functions do not depend on the material; they are the same for every thin beam of the same length and boundary conditions and the material properties affect only the eigenfrequencies. In the presented case, the piezo-element location that allows to sense or excite every mode is close to the clamped end of the beam. The transducers may be positioned such that they will not respond or induce some specific structural modes, but they still will be sensitive to most of the modes in the considered low-

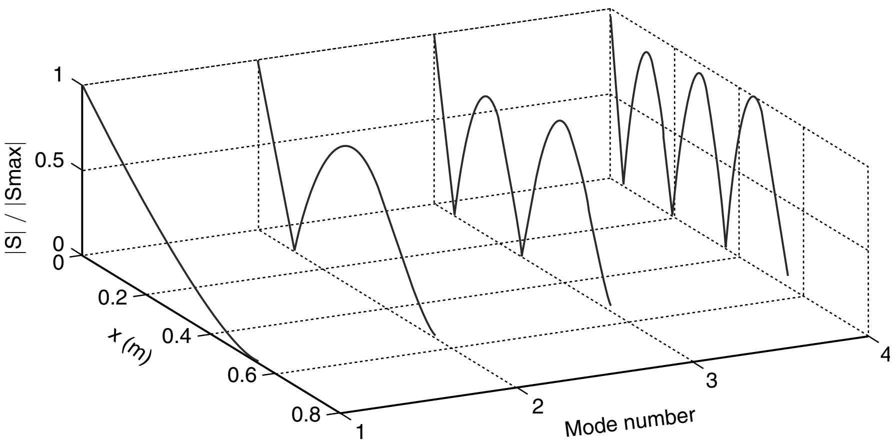


Figure 2: Normalized sensitivity functions for the rectangle-shaped piezoelectric sensor attached to a cantilevered beam of length 58 cm as a function of the structural mode number and the distance of the sensor from the clamped end of the beam.

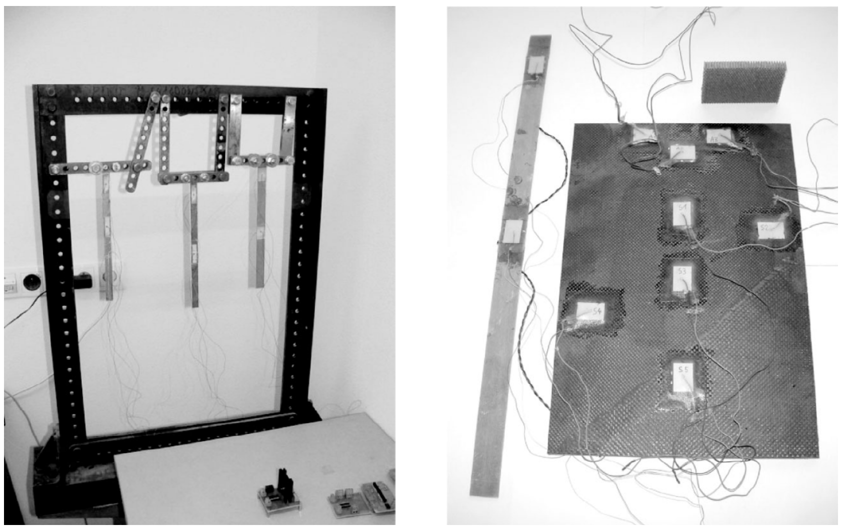


Figure 3: Three thin aluminium beams fixed to an experimental stand (left); a glass-fiber composite beam and a sandwich panel made up from the carbon-fiber composite liners with the Nomex-honeycomb core (right) used in experimental investigations.

frequency range.

The experimental investigations were performed using 1 mm thick, 28 cm long and 2 cm wide aluminium beams and a single glass-fiber composite beam 58 cm long, 3 cm wide and 2.3 mm thick. The piezo-elements were made of Pz29 piezoceramics and were 2 cm wide, 3 cm long, and 0.3 mm thick. The values of important relevant parameters of the utilized piezoceramic material are as follows: $e_3 = 21,2\frac{C}{m^2}$, $d_3 = 5,74e^{-10}\frac{C}{N}$. The beam structures that were used in experimental investigations are shown in Figure 3.

Experimental examination of the vibrations of beam structures revealed an excellent agreement with the theoretical predictions. The vibrations were excited by a single piezoelectric actuator positioned close to the clamped end of beam, while the other piezo-elements, fixed at different distances along the beam, were used as sensors. The electrodes of the sensors were connected to a charge-to-voltage converters. It is worth to notice that for aluminium beams – for which the material constants are known – the predicted and measured first three eigenfrequencies of the bending modes (i.e. all eigenfrequencies of the bending modes in the considered low-frequency range below 400 Hz) at 11, 65 and 185 Hz agreed with an accuracy better than 1 Hz. That observation justifies the assumption to neglect the stiffness and mass influence of the attached piezoelectric elements to the vibration characteristics of beam structures.

In case of the beam made of glass-fiber composite, no material constants were known. Two rectangle-shaped piezoelements were attached to the surface of the structure: the first one – fixed 4 cm from the clamped end – served as actuator simulating external source of vibrations. The second transducer was located 29 cm from the clamped end and it was used as sensor. Due to the numerical simulations, the sensor should be insensitive to the structural modes No. 3 and 5. The resonant frequencies were found experimentally and the mode shapes were identified using a laser vibrometer. The results are presented in Table 1; the modes No. 3 and 5 were not sensed by the sensor which agrees with the theoretical predictions.

If a thin beam (of length L and the rectangular cross-section of height h_s) is elastic, isotropic and homo- geneous - or can be approximately treated as such - its eigenfrequencies can be calculated using the following formula [28]:

$$f_n = \frac{\beta_n^2 h_s}{2\pi L^2 2\sqrt{3}} v_b, \tag{23}$$

where $v_b = \sqrt{E_b / \rho_b}$ is the velocity of plane wave in the (supposedly elastic and isotropic) material of the beam (E_b and p_b are the Young's modulus of the material and its density, respectively) and β_n is the coefficient dependent on the boundary conditions and the mode number [28]. Equation (23) and the results of measurements given in Table 1 were used to estimate the ("effective", average)

Table I.

Measured resonant frequencies and the corresponding parameters of structural mode shapes for the glass-fiber composite beam

Frequency [Hz]	Number of nodes in the mode shape function	Identified mode number
95.8	3	4
235.5	5	6
318.6	6	7
448.5	7	8
562.1	8	9

speed of sound for the composite material of which the examined beam was made. The mean value found using the measured eigenfrequencies listed in Table 1 was 2553 m/s. Then, this value was used in equation (23) with the coefficient $\beta_2^2 =$ 22,034 [28] appropriate for the 2nd mode (not used in the previous calculations) to estimate the eigenfrequency of this mode. The computed result of 17.7 Hz agrees well with the resonant frequency of 18.2 Hz measured for this mode.

The presented results clearly conclude that in case of thin beams made from different materials the ability of sensing or exciting specific forms of vibrations with small, rectangle shaped piezoelectric transducers can be accurately determined with simple analytical formulas. The optimal locations of sensors and actuators should be chosen in order to maximize (or minimize) the modal sensitivity and selectivity values for modes most significant in considered cases. However, as it can be seen from figure 2, such transducers will always be sensitive to most of the forms of vibrations. This conclusion is especially important when considering off-resonant vibrations with many modal components involved, as it has been described in section 2.3.

3.2. Plate and panelled structures

The modal sensitivity and selectivity functions (12) and (17) of small rectangle-shaped piezoelectric transducers attached to the surfaces of plate or sandwich-panel structures are investigated in this section. In general, for arbitrary (non-homogeneous) boundary conditions of support, the rectangle plate mode-shape functions Φ_n cannot be found analytically. Therefore, the finite element analysis was applied to determine the eigenfrequencies and the corresponding eigenvectors of the investigated structures. Experimental investigations were carried out on the sandwich composite panel made up of two carbon-fiber faces and a *Nomex*-honeycomb core (see Figure 3) and a thin aluminium plate (see Figure 5). The aluminium plate was 300 mm long, 200 mm wide and 1 mm thick, while the sandwich plate was 402 mm long, 272 mm wide and 5 mm thick. The structures were clamped by a part of their shorter edges and all the other edges were free.

Eight 0.3 mm-thick rectangle-shaped piezoelectric transducers with dimensions 20 mm x 30 mm were attached to one face of the sandwich panel. Three of them were fixed close to the clamped boundary and served as actuators which simulated the external excitation sources. The other five acted as sensors. In case of the aluminium plate, five pairs of such piezotransducers were used. In each pair, the two piezotransducers were attached symmetrically to both sides of the plate, with polarization and wires connected in that way so that an asymmetric bimporh actuator/sensor was formed. From five pairs one served as the source of the excitation force while the others were used as sensors.

The COMSOL Multiphysics software was used for numerical simulations. Two different models of the considered structures were developed and compared: a simple two-dimensional thin plate model and a three dimensional model of plate with five pairs of asymmetrically-attached piezoelectric sensors/actuators. In the

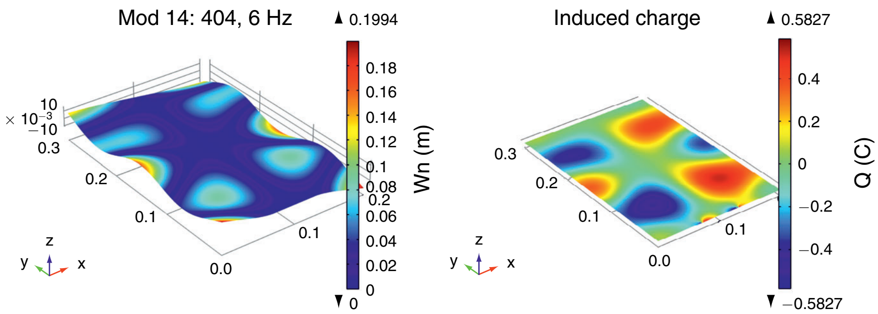


Figure 4: Results of the numerical simulations: the shape of an exemplary vibrational mode of the considered plate structure made of aluminium (left) and the corresponding distribution of the induced electric charge induced on the theoretical point-sensors made of the considered piezoceramics (right).

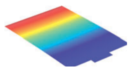
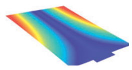
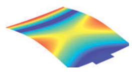
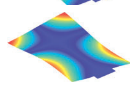
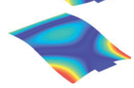
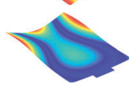
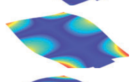
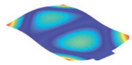
Figure 5: Thin aluminium plate with attached piezoelectric transducers in laboratory stands used in experimental investigations.

second case the transducers were assumed to be made of transversally isotropic piezoceramics, for which the material parameters were taken from the manufacturer's data catalog. The main reason for using two different models was to investigate the influence of the added mass and stiffness introduced by the transducers on the vibrational characteristics of the considered structures. The comparision between the obtained results indicates that – in the considered low-frequency range - including the comparatively small transducers in the simulations had no significant effect, neither on the shape functions of the eigenmodes, nor on the eigenfrequencies. The experimental investigations revealed that the mode shapes - determined using the laser vibrometer - were exactly as predicted, but the measured eigenfrequencies were not that consistent with the simulations. The results are presented in Table 2.

The modal sensitivity functions of the piezoelectric sensors attached to the considered structures were investigated numerically and experimentally. The

Table 2.

Resonant frequencies determined numerically (using the 2D plate model and the 3D structure model with piezoelements) and experimentally, and the corresponding mode shapes

Eigenrequency [Hz]			
2D plate model	3D model	Measured	Mode shape
8	8.1	7.1	
24.6	25	27.1	
50.3	50.5	51.7	
82.8	83.1	90.1	
127.2	129.2	123.5	
144.9	143.4	155.1	
160.2	163.5	172.4	
199	199	183	

normalized absolute values of the obtained results for the first several vibration modes are given in Tables 3 and 4 for the sandwich plate, and in Tables 5 and 6 for the aluminium plate. The value 1 in a cell of the Table indicates that the specific transducer is the most sensitive to the specific mode of all the piezo-elements (thus, the sensitivities are relative with respect to the result of the "best" sensor), while the value 0 indicates that it is not sensitive to this mode at all.

The locations of the piezo-elements were chosen based on the results of the numerical simulations, described in the previous section. The exemplary results are presented in figure 4. To determine the modal sensitivity or selectivity values the computed charge should be integrated over the desired surface corresponding to chosen transducer location. The purpose was to ensure negligible or high sensitivity to the selected structural modes. Once again, it can be seen that a relatively small, rectangle-shaped piezo-element can be placed in locations that ensure very high or, in other case, negligible sensitivity to one or two selected structural modes, but that the transducer will also respond to most of the other modes in the considered low-frequency range.

The comparision of the results given in Tables 5-6 reveals that the experimental and numerical results are in general similar, though some significant discrepancies between predicted and measured values are observed too. For example, in case of the sandwich structure the sensors 3 and 5 were in fact sources of the electric signal at the *all* considered resonant frequencies, although their locations were deliberately chosen in such way that the transducers should be – theoretically – insensitive or almost insensitive to some selected modes. As a matter of fact, none of the sensitivity values in Table 4 is close to zero. It seems obvious that one of the most important reasons for the observed discrepancies between the measured and simulated results is that damping was completely neglected in numerical simulations and thus only one vibrational mode was considered, whereas due to damping effects present in the real structure the amplitudes of non-resonant modes compared to the amplitude of the considered resonant mode usually had non-

Table 3.
Values of the normalized sensitivity function of piezoelectric sensors attached to the sandwich panel obtained from the numerical simulations

Mode number	Sensor 1	Sensor 2	Sensor 3	Sensor 4	Sensor 5
1	1	0.678	0.468	0.28	0.055
2	0.008	0.329	0.01	1	0
3	0.317	0.846	1	0.969	0.28
4	0.001	1	0.09	0.29	0
5	0.6	0.105	1	0.72	0.68

Table 4.
Values of the normalized sensitivity function of piezoelectric sensors attached to the sandwich panel obtained from the experimental investigations

Mode number	Sensor 1	Sensor 2	Sensor 3	Sensor 4	Sensor 5
1	1	0.5	0.634	0.2	0.062
2	0.089	0.523	0.178	1	0.103
3	0.662	0.625	1	0.6	0.185
4	0.465	0.922	1	0.52	0.367
5	0.052	0.091	0.973	1	0.445

Table 5.
Values of the normalized sensitivity function of piezoelectric sensors attached to the aluminium plate obtained from the numerical simulations

Mode number	Sensor 1	Sensor 2	Sensor 3	Sensor 4
1	1	0.34	0.9	0.03
2	0.29	0.6	0.07	1
3	0.24	1	0.68	0.22
4	1	0.3	0.02	0.88
5	0.25	1	0.22	0.37
6	0.56	0.65	1	0.34
7	0.52	1	0.02	0.09
8	0.56	0.34	1	0.16

Table 6.
Values of the normalized sensitivity function of piezoelectric sensors attached to the aluminium plate obtained from the experimental investigations

Mode number	Sensor 1	Sensor 2	Sensor 3	Sensor 4
1	1	0.42	0.91	0.08
2	0.17	0.99	0.55	1
3	0.4	1	0.68	0.33
4	1	0.19	0.05	0.75
5	0.1	1	0.4	0.27
6	0.69	0.14	1	0.42
7	0.34	1	0.1	0.05
8	0.56	0.34	1	0.2

negligible values and had to be taken into account.

The results are in general more consistent between the numerical predictions and the experiments for the aluminium plate. For some vibrational modes of this structure – see, for example the mode No. 8 in Tables 5 and 6 – the results are in fact almost exact. The main reason for this better agreement is obviously the exactness in modelling the material for the isotropic aluminium plate, and also a seemingly lower structural damping than in the case of the sandwich plate; a better adhesion of the transducers to the surfaces of the aluminium plate might also have its effect. The methods of mounting the piezoelements to the considered structures were different due to a need to ensure the electrical contact to the both electrodes of a transducer – including the 'bottom' electrode that is the one in contact with the plate. In case of electrically conductive aluminium the plate was used as the common ground so that the whole bottom side of a transducer could be thoroughly and fully glued to the plate with a conductive glue; in case when the piezoelements were attached to the sandwich plate additional electric wires had to be glued to the 'bottom' electrodes making the attachment no so complete.

4. CONCLUSIONS

The relation between the placements of piezoelectric transducers on the surfaces of beam, plate and panelled structures and the capability of sensing or exciting specific vibration modes were investigated using new, proposed form of description of the active vibroacoustic control system with multiple independent feedback loops. It has been shown that relatively small, rectangle-shaped piezo-elements will be sensitive enough to most of the forms of vibrations in the considered low-frequency range.

The radiation efficiency of various structural modes may differ significantly one from another, however, in the process of developing an active vibroacoustic control system, all of the forms of vibrations should be taken into account due to the complex, multimodal interaction in the closed feedback loop.

Due to the fact, that the amplitude of the excited mode is inversely proportional to the difference of the squared values of the corresponding eigenfrequency and excitation frequency, the number of considered modes can be significantly reduced in case of harmonic vibrations. This dependence can be used to limit the number of piezoelectric sensors necessary in adaptive control systems to determine the parameters of the external loading.

The best agreement between the numerical simulations and experimental investigations was obtained for the beam structures. Both, the eigenfrequencies and the structural mode shapes were predicted precisely and the behavior of the piezoelectric transducers attached to the surfaces of the considered structures was like expected. In case of the plate and the sandwich panel some discrepancies between the theoretical predictions and the results of measurements were observed – mainly due to some damping effects neglected in the modelling and some inaccuracy in estimating the effective material data for the sandwich plate.

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REFERENCES

[1] C. R. Fuller, S. J. Elliot, and P. A. Nelson. *Active Control of Vibration*. Academic Press, London, 1996.

[2] C.-K Lee.and F.C.Moon. Model sensor/actuators. Journal of Applied Mchanics, 57:434-441, 1990. doi: 10.1115/1.2892008

[3] C. K. Lee. Theory of laminated piezoelectric plates for the design of distributed sensors/actuators. Part I: Governing equations and reciprocal relationships. *Journal of the Acoustical Society of America*, 87(3): 1144–1158, 1990. doi: 10.1121/1.398788

[4] W. T. Baumann, W. R. Saunders, and H. H. Robertshaw. Active suppression of acustic radiation from impulsively excited structures. *Journals of the Acoustical Society of America*, 90(6):3202–3208, 1991.

[5] S. J. Elliott and M.E. Johnson. Radition modesand the active control of sound power. *Journal of the Acoustical Society of America*, 94(4):2194–2204, 1339

[6] T. G. Zielinski. Multiphysics modeling and experimental validation of the active reduction of structure-borne noise. *Journal of Vibration and Acoustics – Transactions of the ASME*, 132:(6), Art.No. 061008, 1–14, 2010. doi: 10.1115/1.4001844

[7] W. Wang and N. Atalla. A numerical algorithm for double surface integrals over quadrilaterals with a $1- R$ singularity. *Communications in Numerical Methods in Engineering*, 11:885–890, 1997. doi:10.1002/(SICI)1099-0887(199711)13:11<885::AID-CNM112>3.0.CO;2-D

[8] A. Alia, M. Souli, and F. Erchiqui. Variational boundary element acoustic modelling over mixed quadrilateral-triangular element meshes. *Communications in numerical methods in engineering*, 22:767–780, 2006. doi: 10.1002/cnm.848

[9] W. Desmet. Boundary element method in acoustics. *In: ISSAAC23-Course on Numerical and Applied Acoustics*, Leuven, Belgium, 2012.

[10] S. J. Elliott, P. Gardonio, T. C. Sors, and M. J. Brennan. Active vibroacoustic control with multiple local feedback loops. *Journal of the Acoustical Society of America*, 111(2):908–915, 2002. doi: 10.1121/1.1433810

[11] E. H. Anderson and N. W. Hagood. Simultaneous piezoelectric sensing/actuation: Analysis and application to controlled structures. *Journal of Sound and Vibration*, 174(5):617–639, 1994. doi: 10.1006/jsvi.1994.1298

[12] J. J. Dosch, D. J. Inman, and E. Garcia. A self-sensing piezoelectric actuator for collocated control. *Journal of Intelligent Material Systems and Structures*, 3:166–185, 1992. doi: 10.1177/1045389X9200300109

[13] J. S. Vipperman. *Adaptive piezoelectric sensoriactuators for active structural acoustic control*. PhD thesis, Duke University, 1997.

[14] J. S. Vipperman and R. L. Clark. Implementation of an adaptive piezoelectric sensoriactuator. *Journal of the Acoustical Society of America*, 34(10):2102–2109, 1996. doi: 10.2514/3.13358

[15] G. E. Simmers, J. R. Hodgkins, D. D. Mascarenas, G. Park, and H. Sohn. Improved piezoelectric self-sensing actuation. *Journal of Intelligent Material Systems and Structures*, 15(12):941–953, 2004. doi: 10.1177/1045389X04046308

[16] S. L. Padula and R. K. Kincaid. Optimization strategies for sensor and actuator placement. Technical report, National Aeronautics and Space Administration, 1999.

[17] M. I. Frecker. Recent advances in optimization of smart structures and actuators. *Journal of Intelligent Material Systems and Structures*, 14(4–5):207–216, 2003.

[18] V. Gupta, M. Sharma, and N. Thakur. Optimization criteria for optimal placement of piezoelectric sensors and actuators on a smart structure: A technical review. *Journal of Intelligent Material Systems and Structures*, 21(12):1227–1243, 2010. doi: 10.1177/1045389X10381659

[19] Z. Qiu, X. Zhang, H. Wu, and H. Zhang. Optimal placement and active vibration control for piezo¬electric smart flexible cantilever plate. *Journal of Sound and Vibration*, 301:521-543, 2007. doi: 10.1016/j.jsv.2006.10.018

[20] O. J. Aldraihem, T. Singh, and R. C. Wetherhold. Optimal size and location of piezoelectric actuator/sensors: practical considerations. *Journal of Guidance, Control, and Dynamics*, 23(3):509–515, 2000. doi: 10.2514/2.4557

[21] R. L. Clark and C. R. Fuller. Optimal placement of piezoelectric actuators and polyvinylidene fluoride error sensors in active structural acoustic control approaches. *Journal of the Acoustical Society of America*, 92(3):1521–1533, 1992.

[22] A. M. Sadri, J. R. Wright, and R. J. Wynne. Modelling and optimal placement of piezoelectric actuators in isotropic plates using genetic algorithms. *Smart Materials and Structures*, 8(4):490, 1999. doi: 10.1088/0964–1726/8/4/306

[23] S. T. Quek, S. Y. Wang, and K. K. Ang. Vibration control of composite plates via optimal placement of piezoelectric patches. *Journal of intelligent material systems and structures*, 14(4–5):29–245, 2003. doi: 10.1177/1045389X03034686

[24] S. Julai and M. O. Tokhi. Vibration suppression of flexible plate structures using swarm and genetic optimization techniques. *Journal of Low Frequency Noise, Vibration and Active Control*, 29(4):293–318, 2010. doi: 10.1260/0263-0923.29.4.293

[25] J.-H. Han and I. Lee. Optimal placement of piezoelectric sensors and actuators for vibration control of a composite plate using genetic algorithms. *Smart Materials and Structures*, 8(2):257, 1999. doi: 10.1088/0964–1726/8/2/012

[26] J. D. Sprofera, R. H. Cabell, G. P. Gibbs, and R. L. Clark. Structural acoustic control of plates with variable boundary conditions: Design methodology. *Journal of the Acoustical Society of America*, 122 (1):271–279, 2007. doi: 10.1121/1.2739404

[27] Ł. Nowak and T. G. Zielinski. Determining the optimal locations of piezoelectric transducers for vibroa- coustic control of structures with general boundary conditions. In: *Proceedings of ISMA2012: International Conference on Noise and Vibration Engineering*, pp. 369–383, 2012.

[28] A. W. Leissa and M. S. Qatu. *Vibrations of Continious Systems*. Mc Graw Hill, 2011.

[29] T. G. Zielinski, M. A. Galland, and M. N. Ichchou. Fully coupled finite-element modeling of active sandwich panels with poroelastic core. *Journal of Vibration and Acoustics – Transactions of the ASME*, 134(2), Art.No. 021007, 1–10, 2012. doi: 10.1115/1.4005026

[30] T. G. Zielinski. Fundamentals of multiphysics modelling of piezo-poro-elastic structures. *Archives of Mechanics*, 62(5):343-378, 2010.

[31] I. G. Masters and K. E. Evans. Models for the elastic deformation of honeycombs. *Composite Structures*, 35(4):403422, 1996. doi: 10.1016/S0263–8223(96)00054–2