Basics of Multiscale Modelling: Tutorial on Porous Media Flow

Introductory Course on Multiphysics Modelling

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Basics of multiscale modelling

Motivation:

- **Many complex phenomena** involve processes occurring at different scales (of space and/or time), or . . .
- . . . **multiple spatial and/or temporal scales can be distinguished** to differ between the process phases or to better/easier describe the process features.
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- Usually, **it is easier to deal with different scales individually**.

Multi-scale modelling

Mathematical solution techniques of dealing with problems that have important features at multiple scales of space and/or time.

Comment: For many problems, the processes (i.e., sub-problems) at various scales can be, in practice, solved (quasi) separately, which makes such multi-scale approach very efficient.

Basics of multiscale modelling

Multi-scale modelling

Mathematical solution techniques of dealing with problems that have important features at multiple scales of space and/or time.

Requirements:

- **Separation of scales** – allows to apply different approaches to treat problems at various scales. One can distinguish:
  - **different spatial scales –** when there are local and global phenomena, or there co-exist processes which are: essentially microscopic (i.e., occur at the micro-scale), mesoscopic (i.e., occur at the meso-scale), and macroscopic (i.e., occur at the macro-scale), etc.;
  - **different temporal scales –** when the involved processes are: relatively slow (static or quasi-static), dynamic, or relatively fast, etc.

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- **Representativeness** of the geometry or time-interval for the phenomenon considered on the scale related to this geometry or time-interval.
- Well defined way of **passing of the relevant information** (effective properties, behaviour, etc.) **between the scales**.

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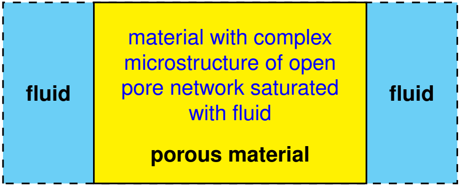
EXAMPLE: Flow in porous media

MACRO-SCALE

viscous flow through a porous material



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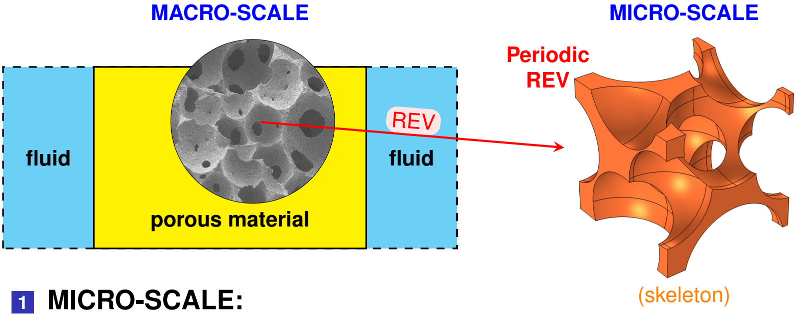

1. **MICRO-SCALE:**
   - Selection (construction) of a (periodic) **Representative Elementary Volume (REV)** of a porous medium.

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EXAMPLE: Flow in porous media

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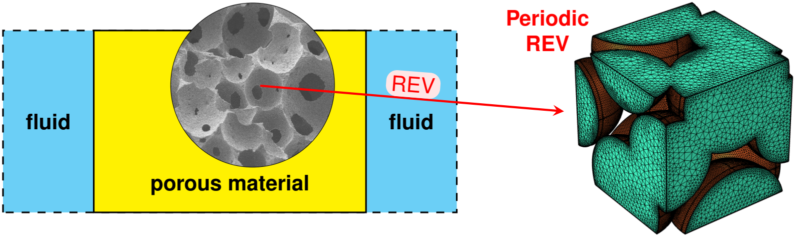

(fluid domain)

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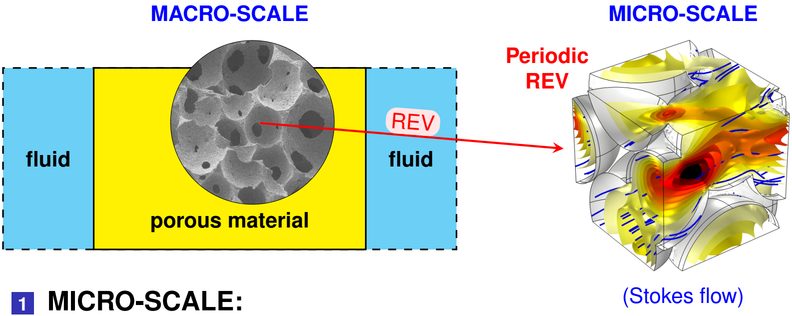

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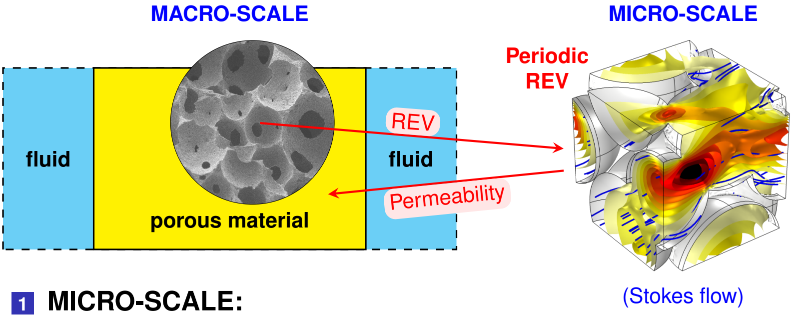

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- Selection (construction) of a (periodic) **Representative Elementary Volume (REV)** of a porous medium.
- **Stokes flow**, i.e., linear & steady, viscous, incompressible flow through the **periodic RVE**, driven by a uniform pressure gradient.
- Averaging of the computed velocity field to determine the **permeability of the porous medium**.

2 MACRO-SCALE:

- **Macroscopic flow through the porous material** characterised by its open porosity and permeability using the **Darcy's law**.

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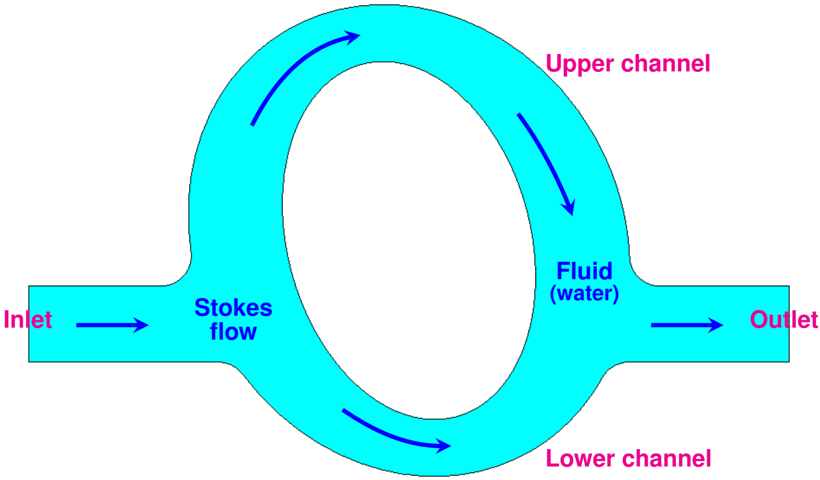
TUTORIAL: Steady viscous flow through channels clogged with a porous material



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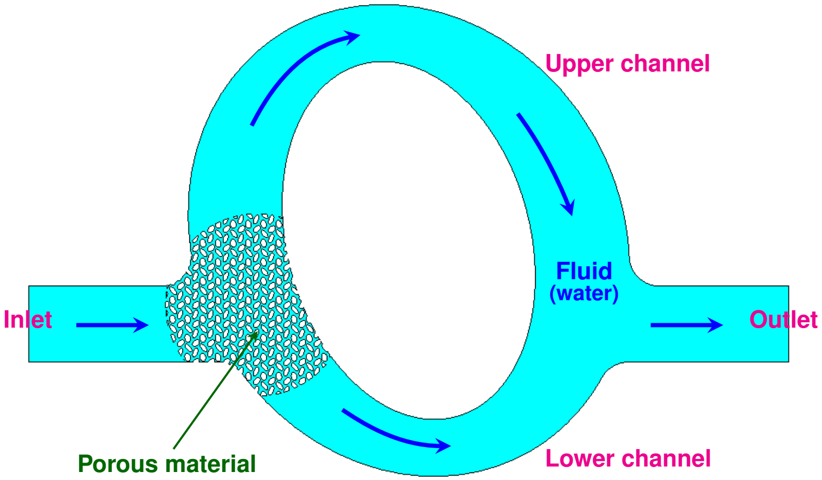

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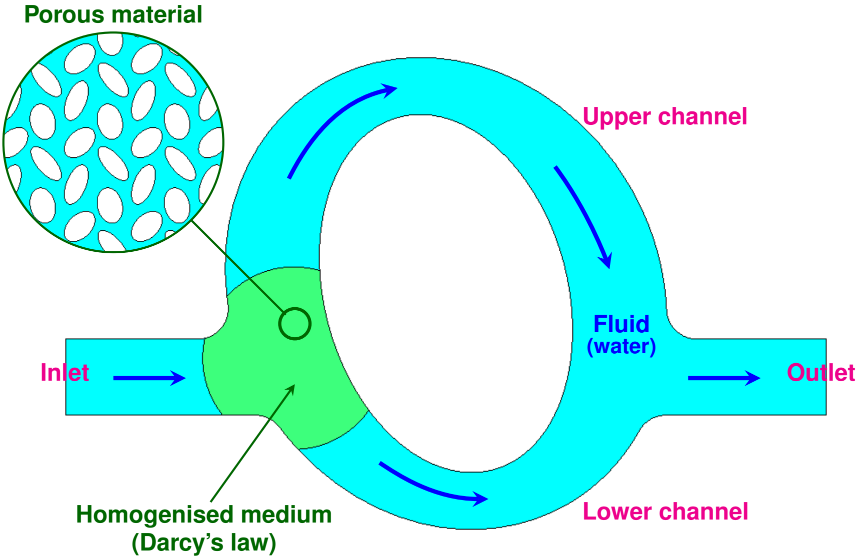

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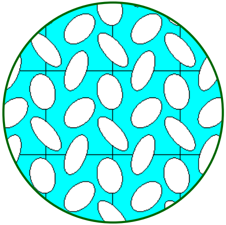

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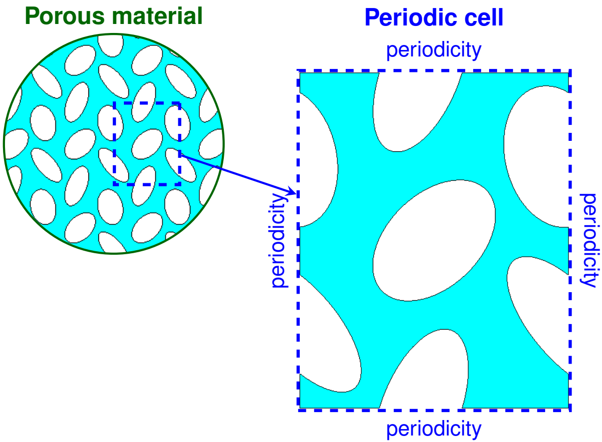

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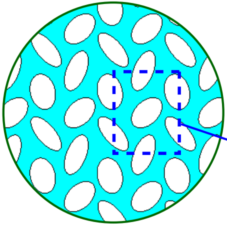
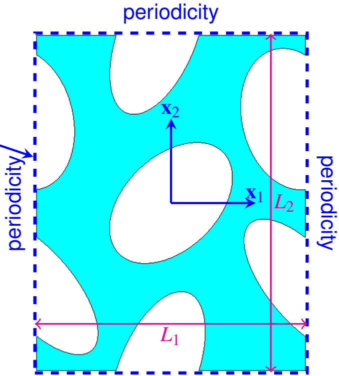


Periodic cell



Darcy's law

$$\mathbf{q} = -\frac{\mathbf{k}}{\mu}\nabla p$$

$\mathbf{q}$: flux [m/s] $\quad \mathbf{q} = \phi\langle\mathbf{v}\rangle_{\mathrm{f}}$

$\mathbf{v}$: velocity in the pores [m/s]

$\langle . \rangle_{\mathrm{f}}$: averaging over the pore fluid

ϕ : open porosity

∇p : pressure gradient [Pa/m]

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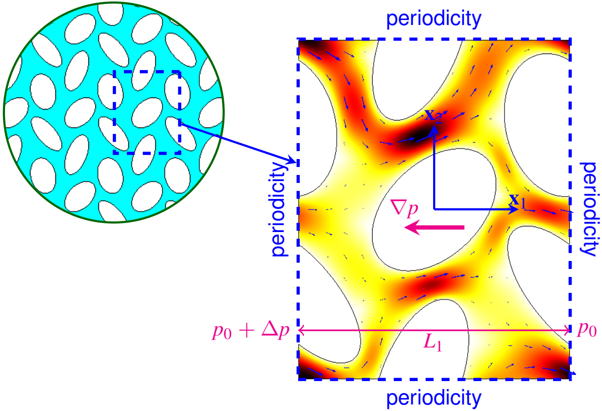

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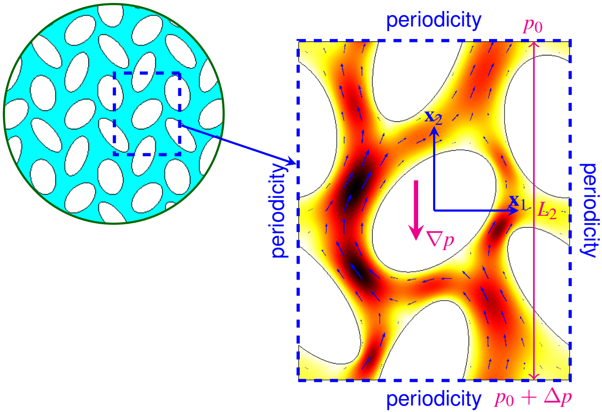

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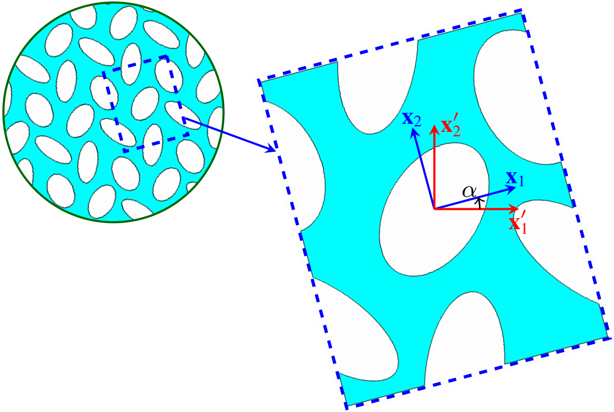

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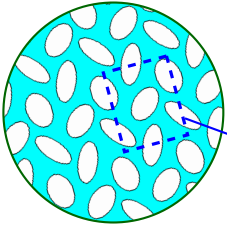

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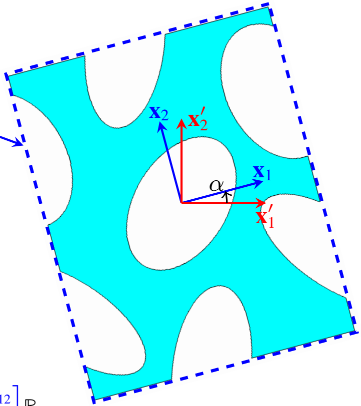

Axes (or material) rotation

α : rotation angle

$$\mathbb{R} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

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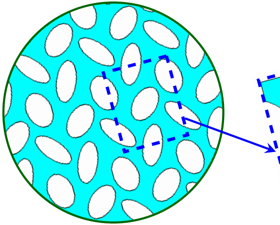

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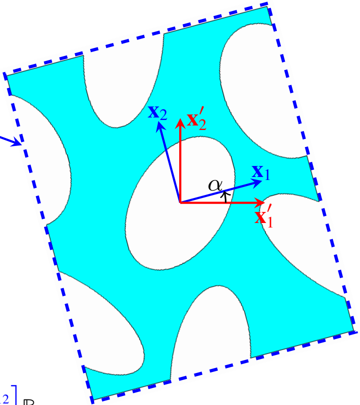

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Principal directions of anisotropy

$k_{12}' = k_{21}' = 0$ for

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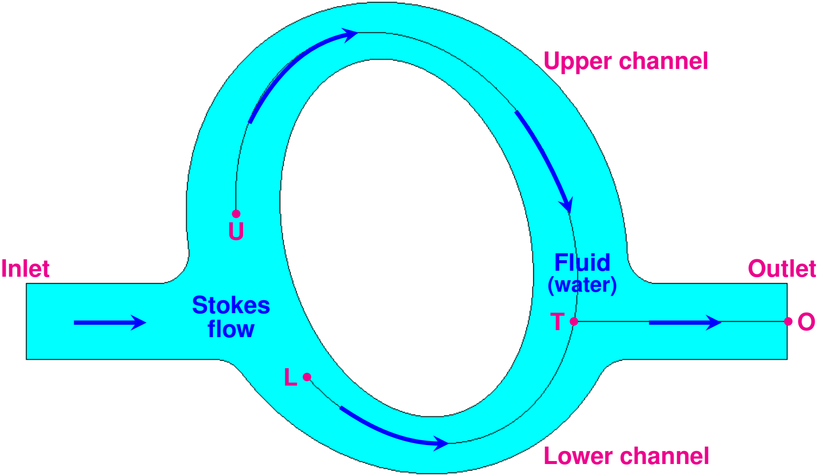
TUTORIAL: Steady viscous flow through channels clogged with a porous material



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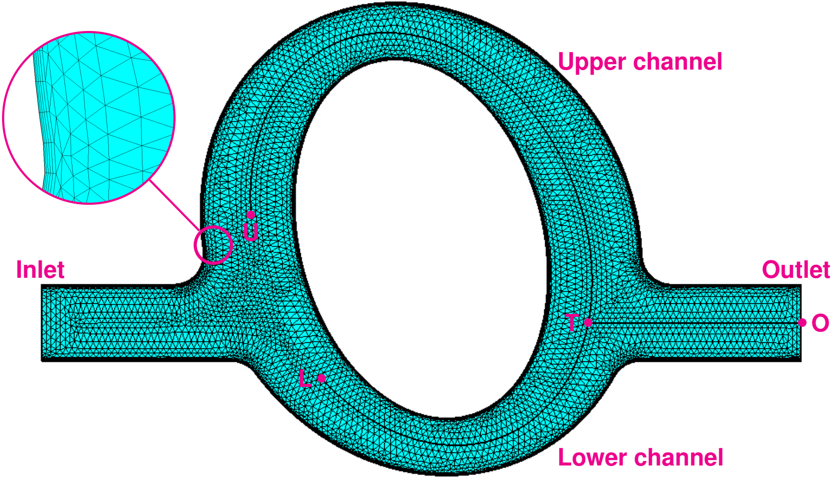

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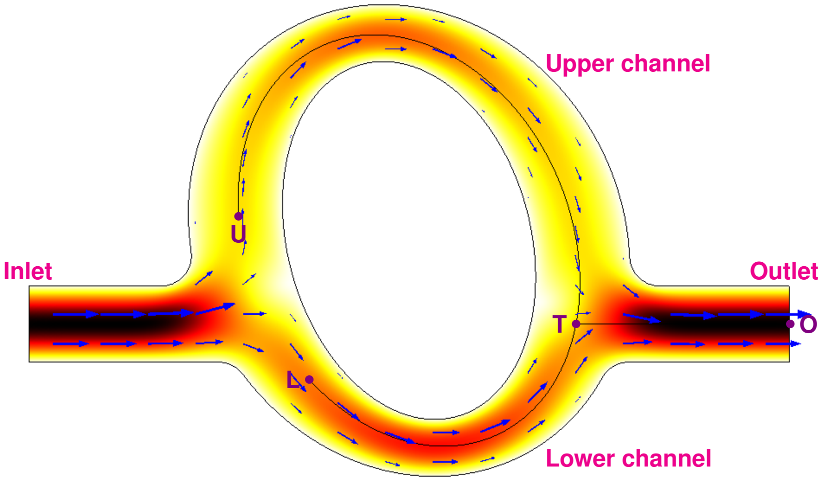

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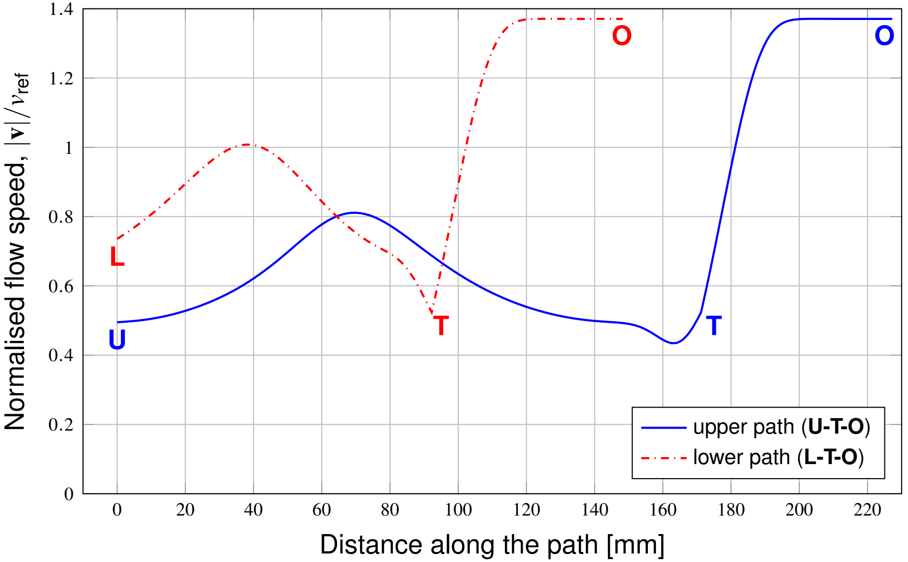

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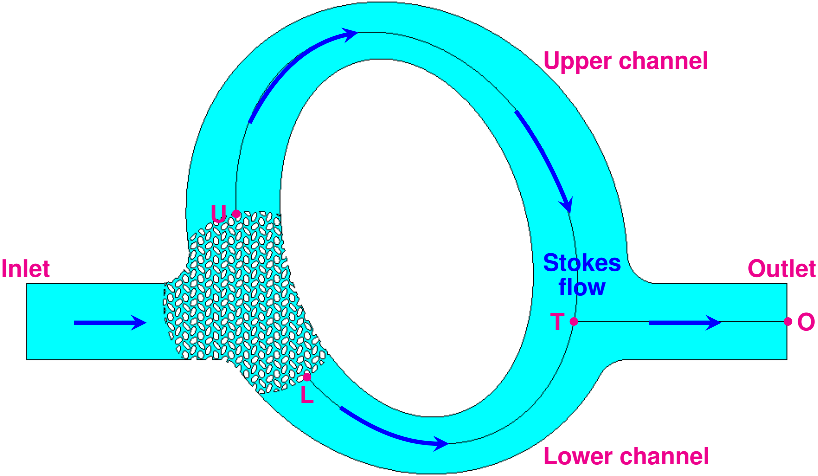

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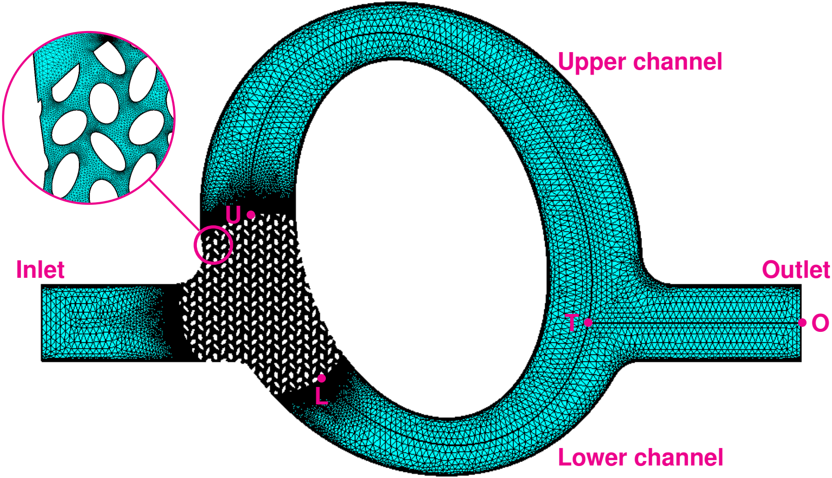

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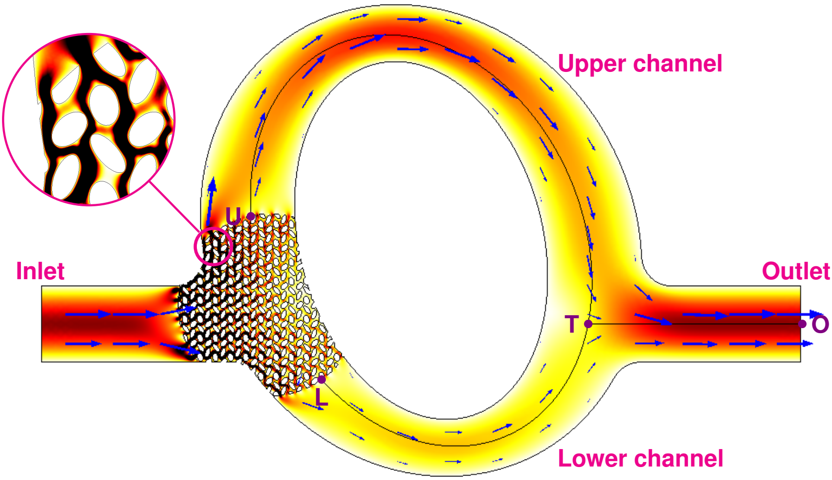

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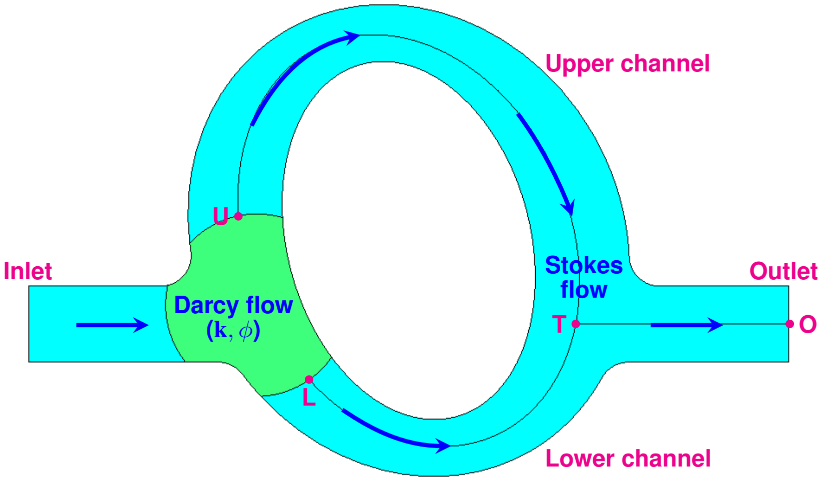

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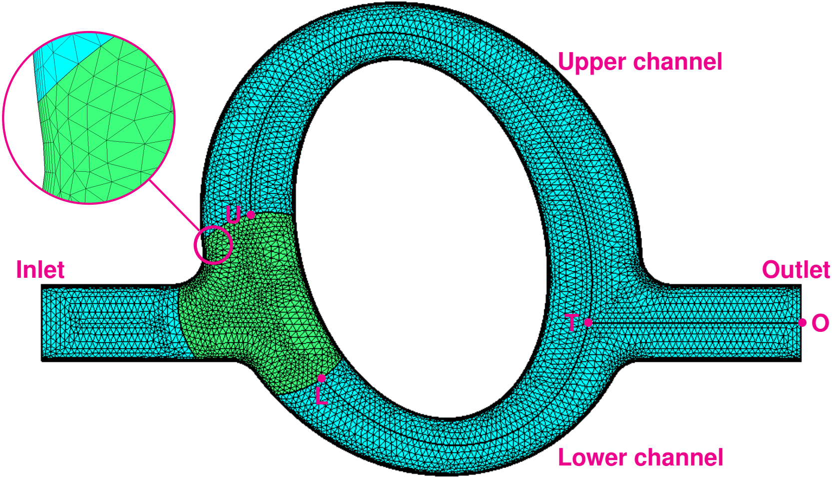

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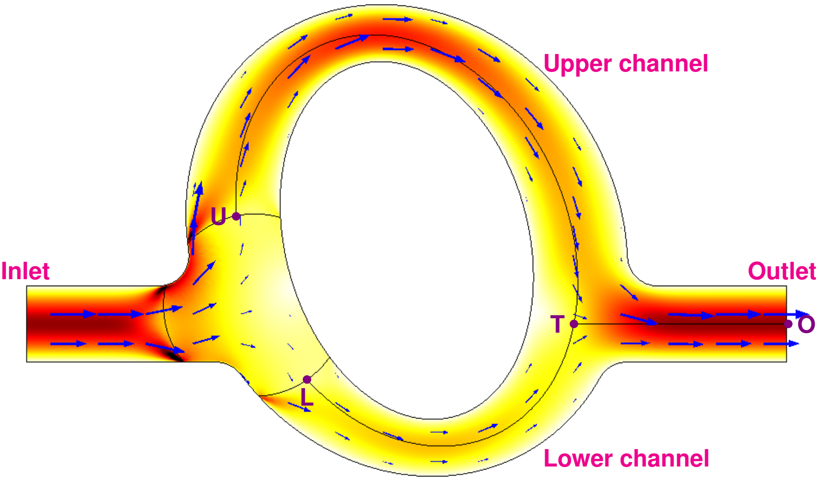

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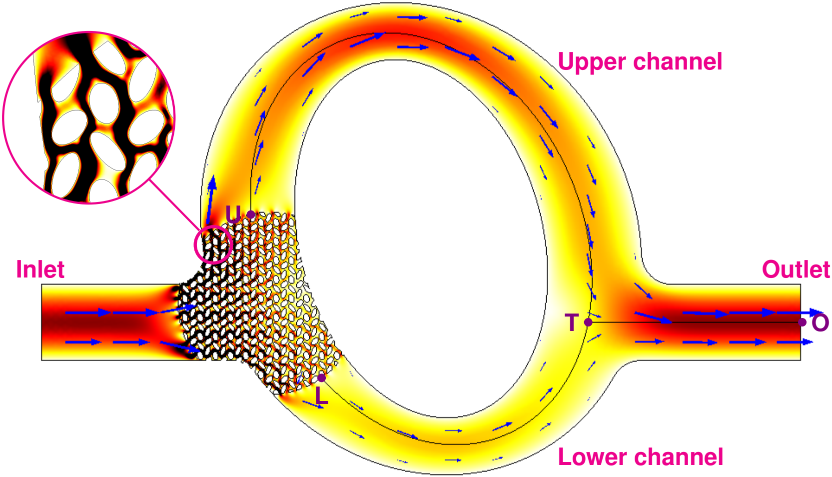

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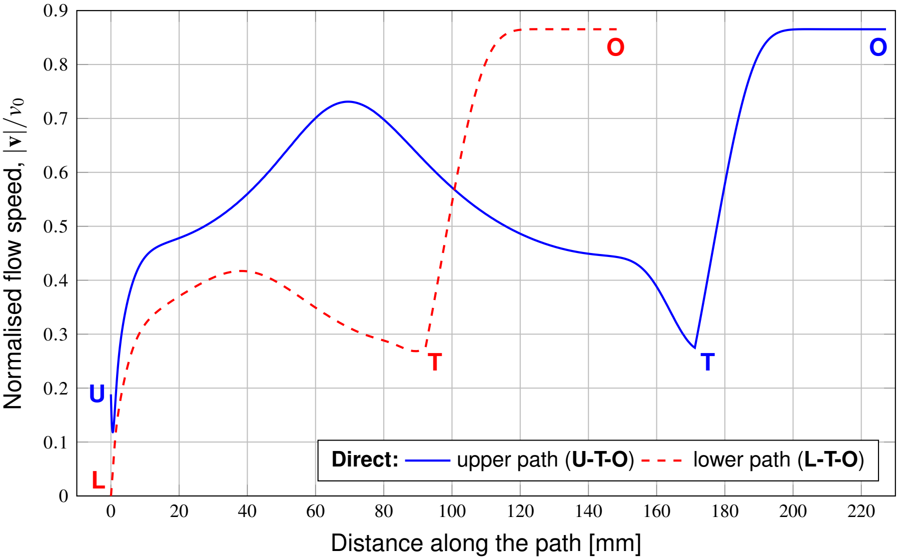

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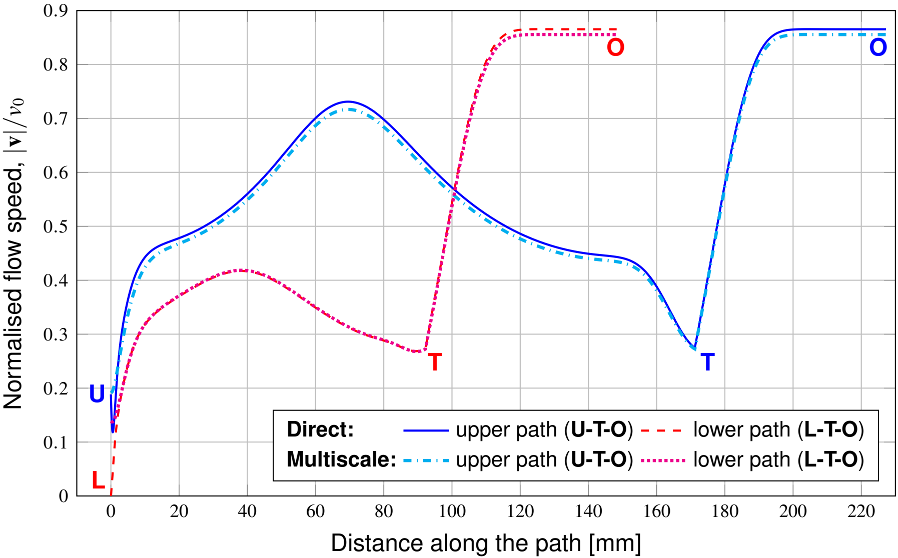

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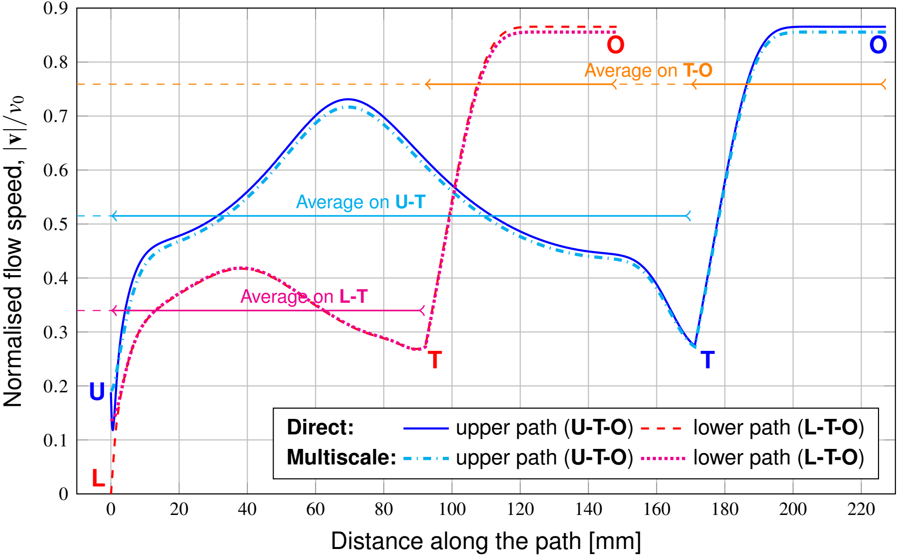

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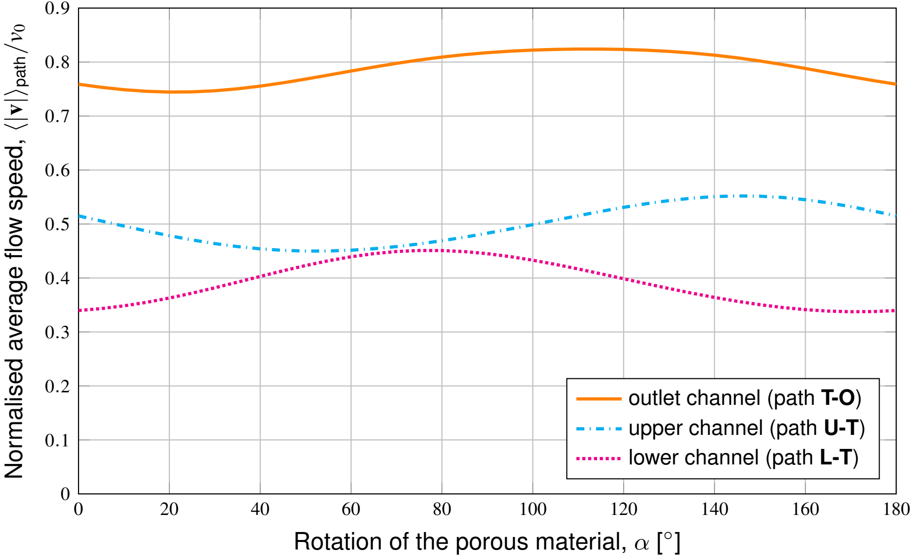

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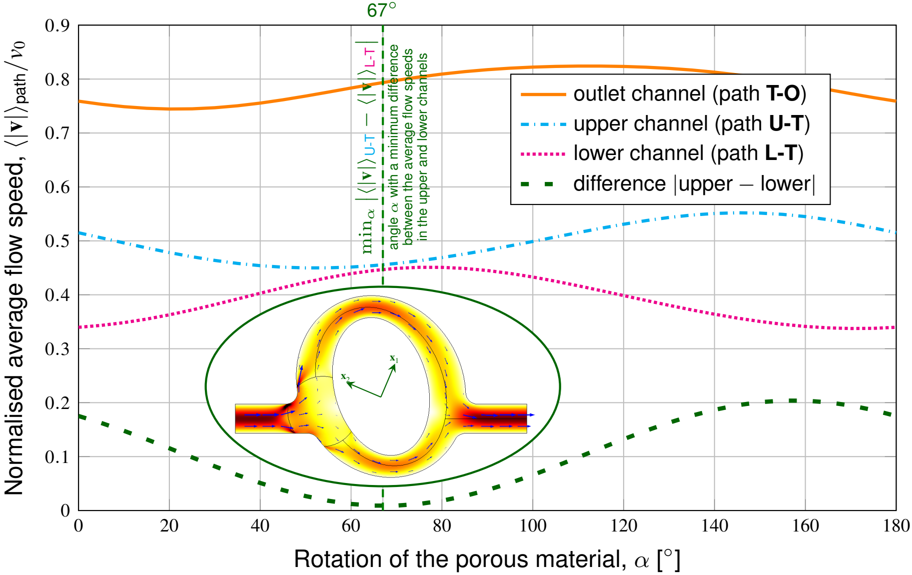

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