



# **STREAMING POTENTIAL AND STREAMING CURRENT OF A PARTICLE COVERED SURFACE**

**Part 1**

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# OVERVIEW

- Introduction
- Electrical Double Layer (EDL)
- *Gouy-Chapman-Stern* model
- Electrokinetics
  - Electro-osmosis
  - Electrophoresis
  - Streaming current/Streaming potential
- Measuring the Streaming Potential
- Particles adsorbed at the interface – what changes?



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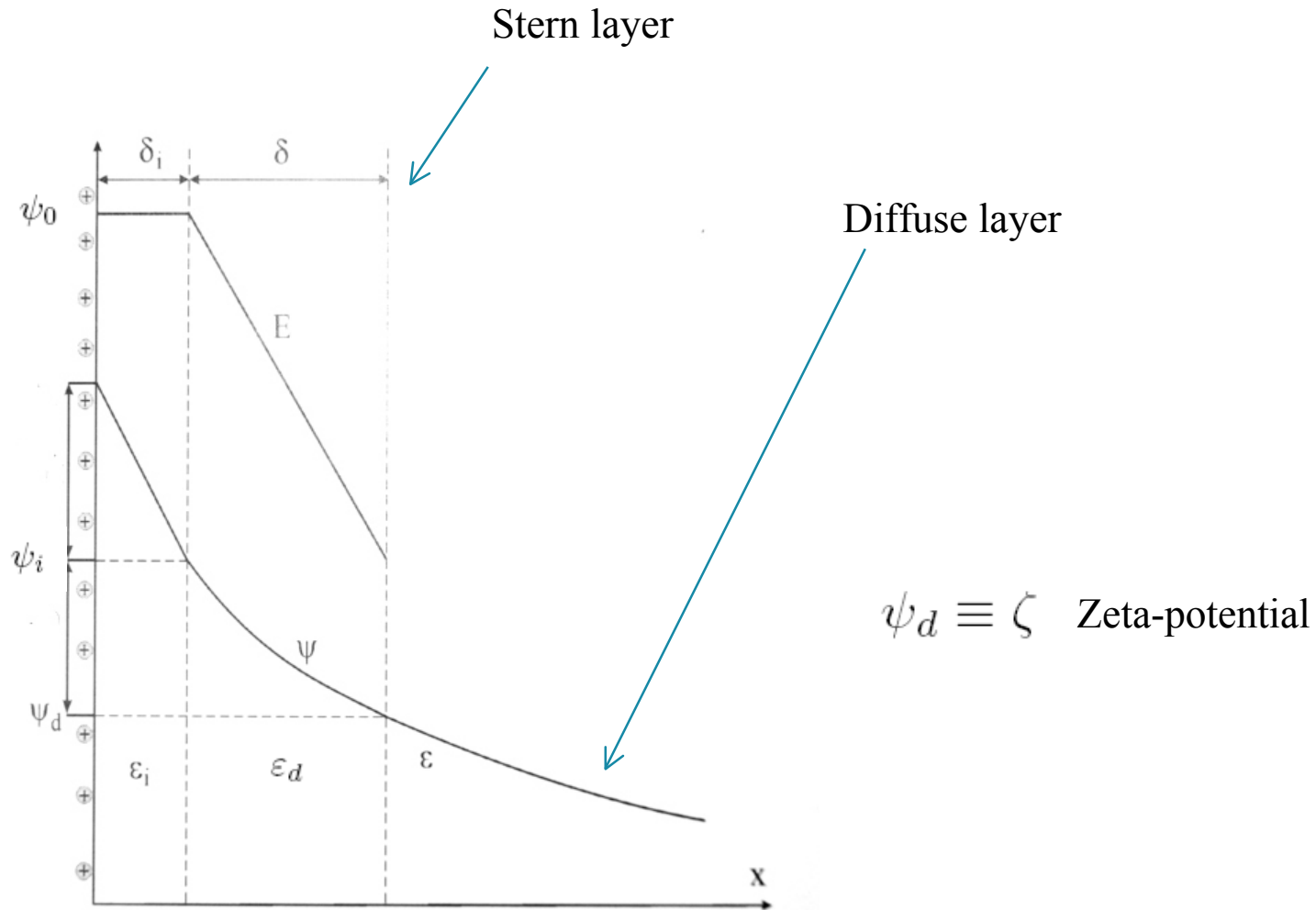
# INTRODUCTION

- Charge...

Surface charge [C m <sup>-2</sup> ]	a=10 <sup>-6</sup> m		a=10 <sup>-4</sup> m	
	ψ[V]	E[V m <sup>-1</sup> ]	ψ[V]	E[V m <sup>-1</sup> ]
0.16	1.8 x 10 <sup>4</sup>	1.8 x 10 <sup>10</sup>	1.8 x 10 <sup>6</sup>	1.8 x 10 <sup>10</sup>
	2.31 x 10 <sup>2</sup>	2.31 x 10 <sup>8</sup>	2.31 x 10 <sup>4</sup>	2.31 x 10 <sup>8</sup>
1.6 x 10 <sup>-3</sup>	1.8 x 10 <sup>2</sup>	1.8 x 10 <sup>8</sup>	1.8 x 10 <sup>4</sup>	1.8 x 10 <sup>8</sup>
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# ELECTRICAL DOUBLE LAYER (EDL)



# *GOUY-CHAPMAN-STERN* MODEL

$$\nabla \cdot (\varepsilon \nabla \psi) = -\rho$$

$$\rho = \sum_i n_i z_i e$$

Bulk concentration of ions

Concentration of ions

$$n_i = n_i^0 \exp(-w_i/kT)$$

$$w_i = z_i e \psi$$

Poisson-Boltzmann equation:


$$\nabla^2 \psi = -\frac{1}{\varepsilon} \sum_i n_i^0 z_i e \exp(-z_i e \psi / kT)$$




# GOUY-CHAPMAN-STERN MODEL

- Solution method of the Poisson-Boltzmann eq.

- Debye-Hückel approx. Works only for  $|z_i e \psi| \ll kT$
- Gouy-Chapman model:
  - Flat surface
  - Symmetric electrolyte  $z_+ = z_- = z$

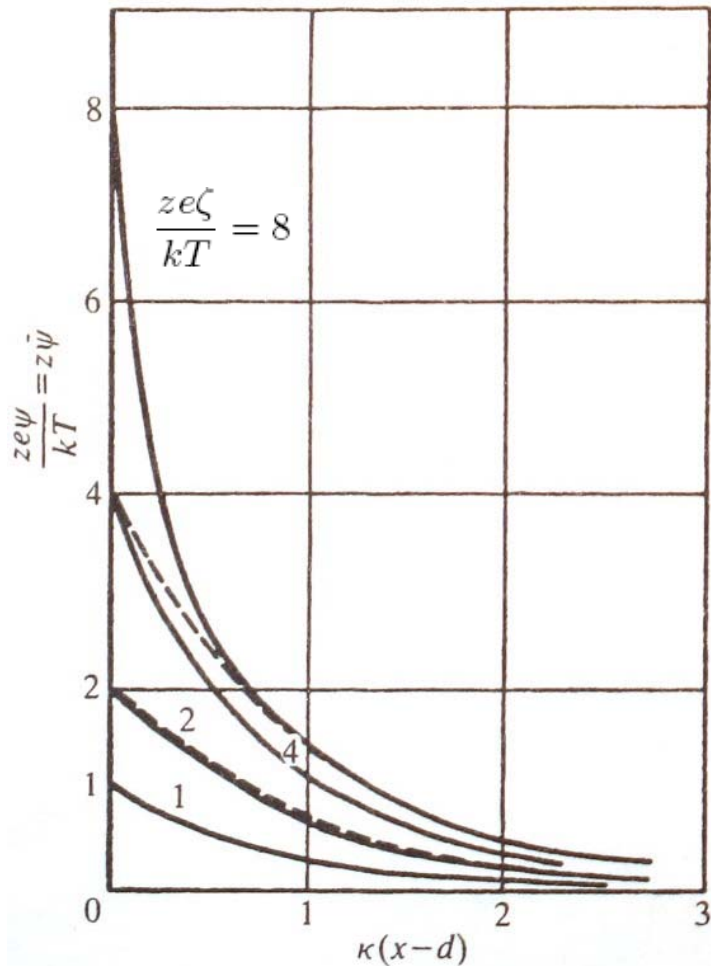

$$\frac{d^2 \psi}{dx^2} = \frac{2n^0 z e}{\epsilon} \sinh \frac{z e \psi}{kT}$$


$$\tanh(z e \psi / 4kT) = \tanh(z e \zeta / 4kT) \exp[-\kappa(x - d)]$$

Debye-Hückel parameter  $\kappa = \sqrt{\frac{2z^2 e^2 n^0}{\epsilon kT}}$



# GOUY-CHAPMAN-STERN MODEL



$$\frac{ze\psi}{kT} \ll 1$$

$$\tanh(ze\psi/4kT) \simeq ze\psi/4kT$$



$$\psi = \zeta \exp[-\kappa(x - d)]$$





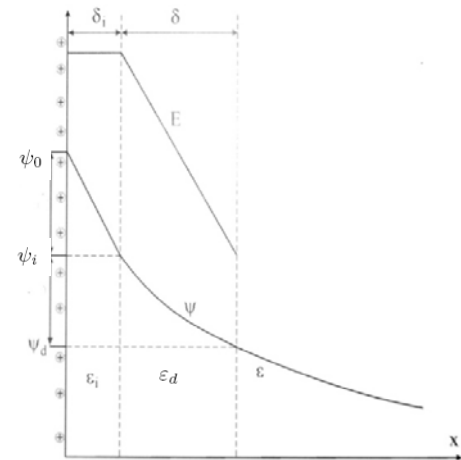
# GOUY-CHAPMAN-STERN MODEL

- Diffuse layer charge

$$\sigma_d = \int_d^{\infty} \rho dx \quad \longrightarrow \quad \sigma_d = -\frac{4n^0 ze}{\kappa} \sinh \frac{ze\zeta}{2kT}$$

- Adsorbed charge in the inner region (Stern layer, Stern 1924, Graham 1947)
  - No charge within the layer of thickness  $\delta_i$

$$\nabla \cdot (\epsilon_i \nabla \psi) = 0 \quad \Delta \psi^0 = \frac{\sigma_0 \delta_i}{\epsilon_i}$$



# GOUY-CHAPMAN-STERN MODEL

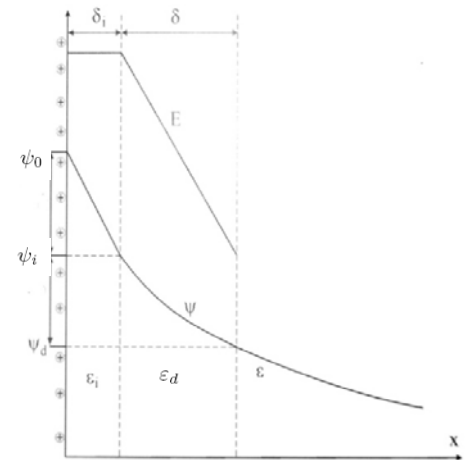
- What is the charge distribution in the layer  $\delta$ ?
  - Uniform space charge distribution, distant-dependent permittivity

Or

- All ions are assumed to be confined to a layer and are treated as point charges.

$$\psi_0 - \psi_i = \sigma_0 \delta_i / \epsilon_i$$

$$\psi_i - \psi_d = -\sigma_d \delta / \epsilon_d$$



# ELECTRO-OSMOSIS

- Motion of liquid induced by an applied electric field

$$\begin{aligned}\eta \nabla^2 \mathbf{v} - \nabla p &= -\rho_e \mathbf{E} \\ \nabla \cdot \mathbf{v} &= 0\end{aligned}$$

Stokes equations



$$\mathbf{F} = \rho_e \mathbf{E} = -\varepsilon_0 \varepsilon_r \nabla^2 \psi \mathbf{E}$$



Slip velocity

$$v = \varepsilon_0 \varepsilon_r (\psi - \zeta) E / \eta$$

Smoluchowski 1903



# ELECTROPHORESIS

- Motion of suspended particles in an applied electric field

$$\mathbf{v} = \mu_E \mathbf{E}$$



Electrophoretic mobility

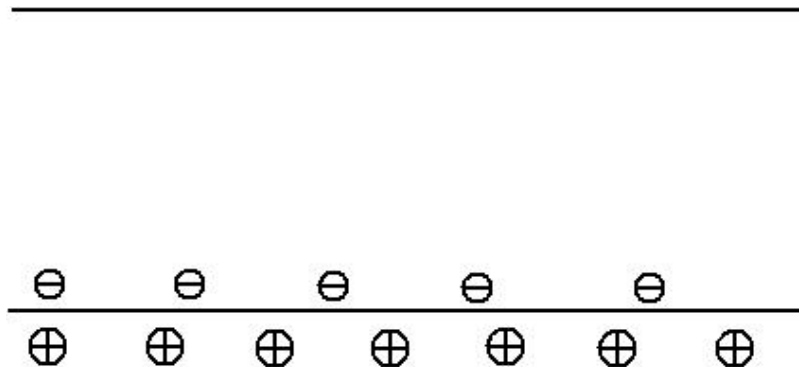
$$\mu_E = \varepsilon_0 \varepsilon_r \zeta / \eta$$

Smoluchowski 1921



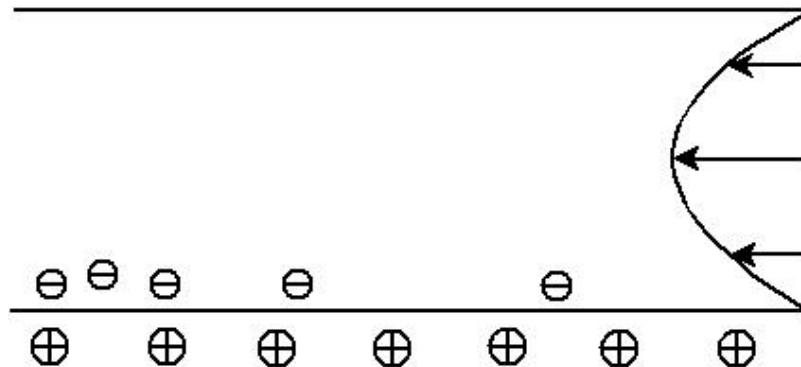
# STREAMING CURRENT AND STREAMING POTENTIAL

- Current appearing due to double-layer charge movement with the fluid
- Transfer of charge downstream (due to pressure gradient) is balanced by current due to electric field
- Potential drop associated with this field: **STREAMING POTENTIAL**



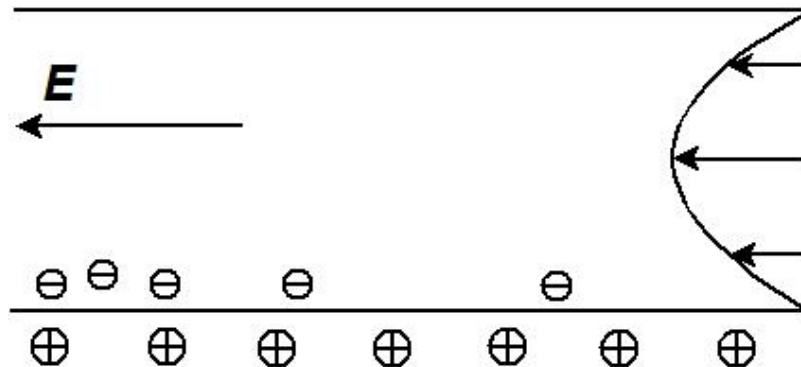
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# STREAMING CURRENT AND STREAMING POTENTIAL

Poiseuille flow in a tube:

$$v = \frac{\Delta p}{4\eta L} (a^2 - r^2)$$

Pressure difference

Tube length

Electric current due to convection:

$$I_1 = \int_0^a 2\pi r \rho_e(r) v(r) dr$$

Dominant contribution from the double layer:

$$v \simeq [\Delta p a / 2\eta L] (a - r) \quad \rho_e(y) = -\varepsilon \frac{d^2 \psi}{dy^2}$$

$$y = a - r$$





# STREAMING CURRENT AND STREAMING POTENTIAL

$$I_1 = -\frac{\Delta p a^2 \pi}{\eta L} \int_0^a \rho_e(y) y dy$$

$$\rho_e(y) = -\varepsilon \frac{d^2 \psi}{dy^2} \quad y = a - r$$



Balance of currents:

$$I_1 = -\frac{e\zeta \pi a^2}{\eta L} \Delta p$$

$$I_2 = K \pi a^2 E$$

Electrolite conductivity



$$E = \frac{\varepsilon \zeta}{\eta K} \frac{\Delta p}{L}$$

$$\Delta \psi = \frac{\varepsilon \zeta}{\eta K} \Delta p$$

Streaming potential



# MEASURING THE STREAMING POTENTIAL

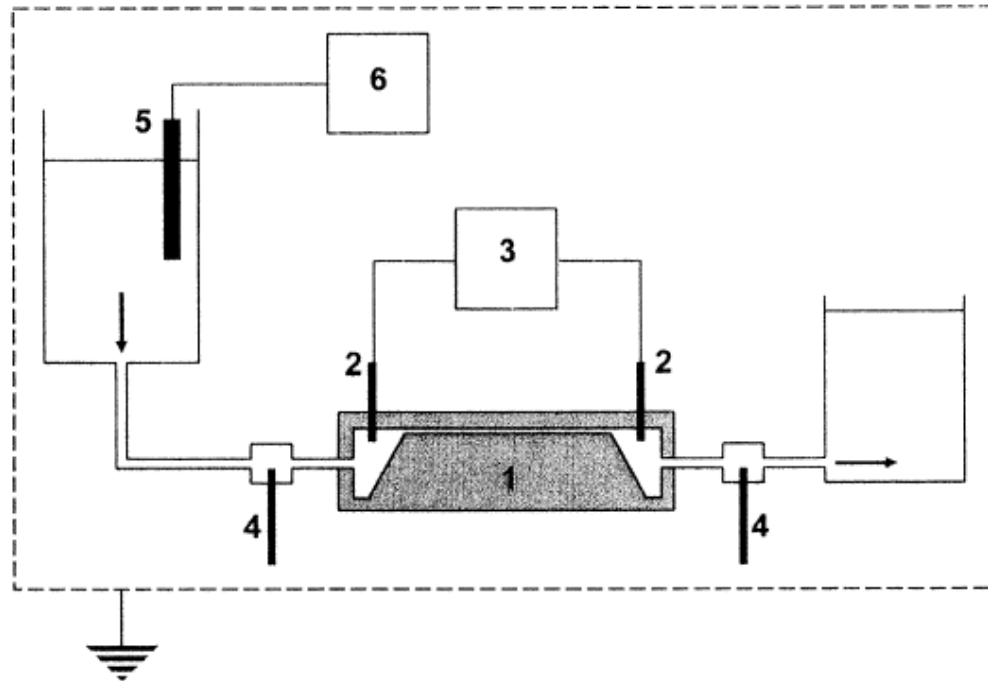
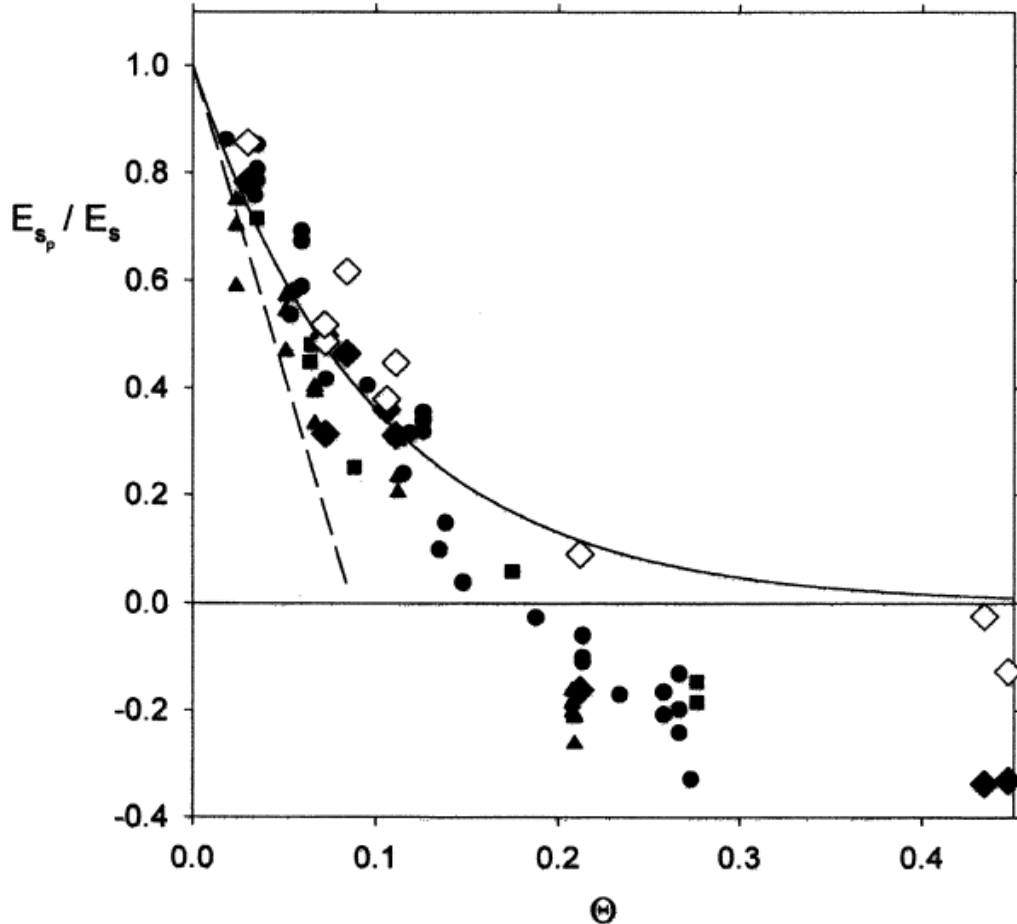


Fig. 4. A schematic view of the set up used for the streaming potential measurement: (1) the cell; (2) electrodes for streaming potential measurements; (3) electrometer; (4) electrodes for cell resistance measurements; (5) conductivity cell; (6) conductometer.



# PARTICLES ADSORBED AT THE INTERFACE – WHAT CHANGES?



Rapid decrease of the streaming current/potential with particle concentration  $\theta$



# TO BE CONTINUED...

- Can this dependence be explained theoretically??
  - Hydrodynamics
  - Statistical Physics



- Virial expansion, Simulations

