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On the Inclusion of the Interfacial Area Between Fluids in Models of Multiphase Porous Media Flows

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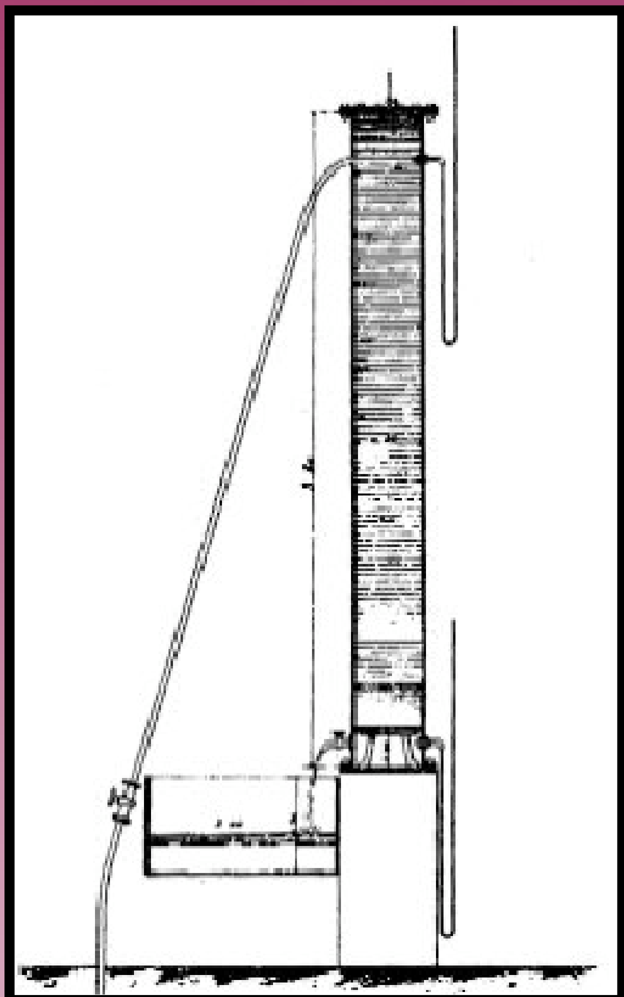
Outline of Presentation

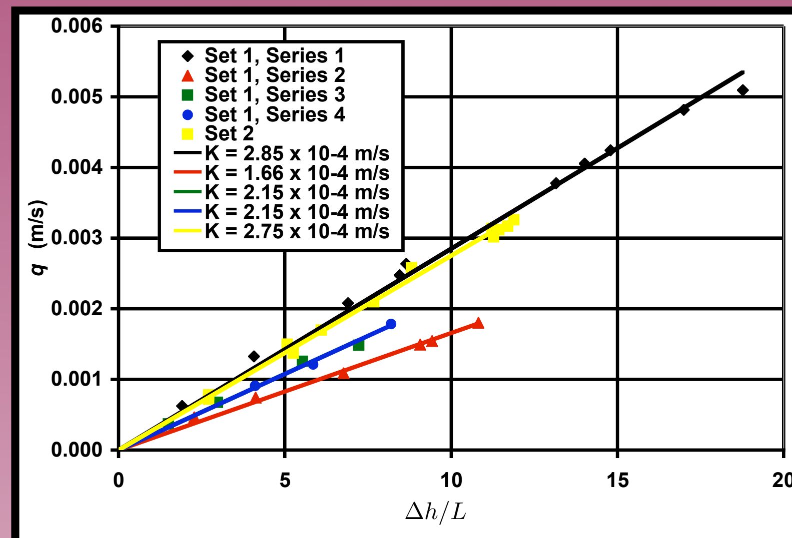
- Review of Darcy's Law
- Pressure
- Elements of TCAT Approach
- Considerations for Unsaturated Flow
- Formulation of Capillary Pressure
- Stress Tensor for an Elastic Solid
- Conclusion

Review of Darcy's Law

Darcy’s Experiment, 1855

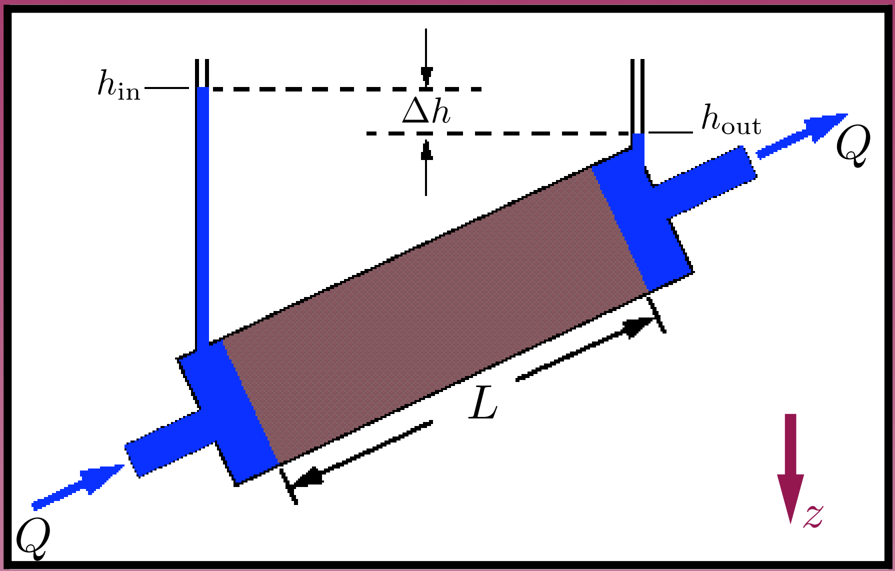


- 2.5m column; 35cm diameter
- 35 experiments with uniform sand
- water inflow from top
- mercury manometers
- length between 0.58 and 1.71m



$$q = K\left|\frac{\Delta h}{L}\right|$$

Darcy's Law



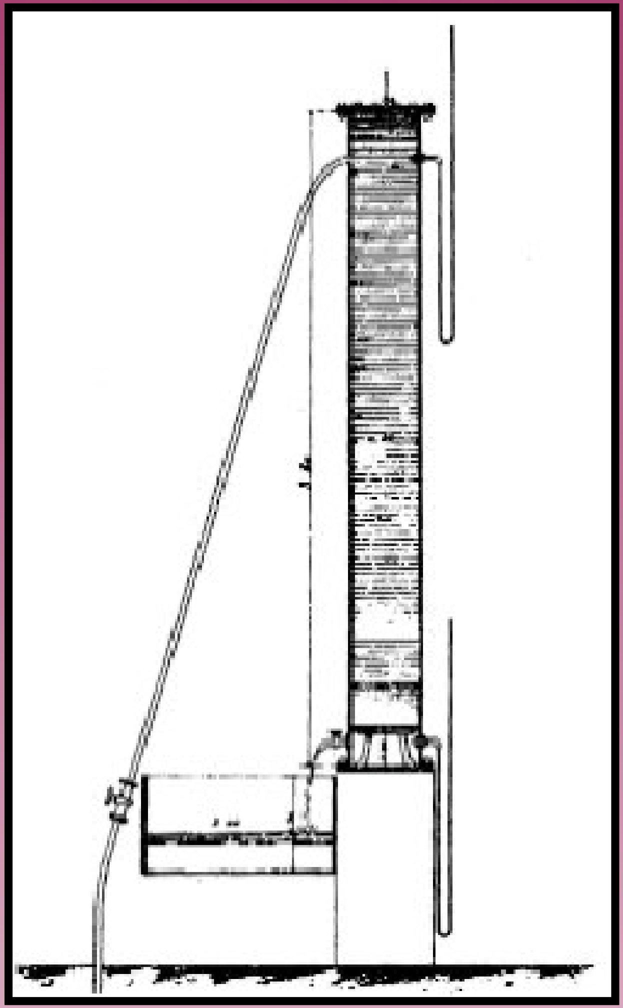
Algebraic Form

- $$q = \frac{Q}{A} = -K\left(\frac{h_{out} - h_{in}}{L}\right)$$
- $$h = \frac{p}{\rho g} + (z_{ref} - z)$$
- $$q = -\frac{K}{\rho g}\left[\frac{p_{out} - p_{in}}{L} - \rho g \frac{z_{out} - z_{in}}{L}\right]$$

Differential Form

- $$q = -\frac{K}{\rho g}\left[\frac{\mathrm{d}p}{\mathrm{d}l} - \rho g \frac{\mathrm{d}z}{\mathrm{d}l}\right]$$

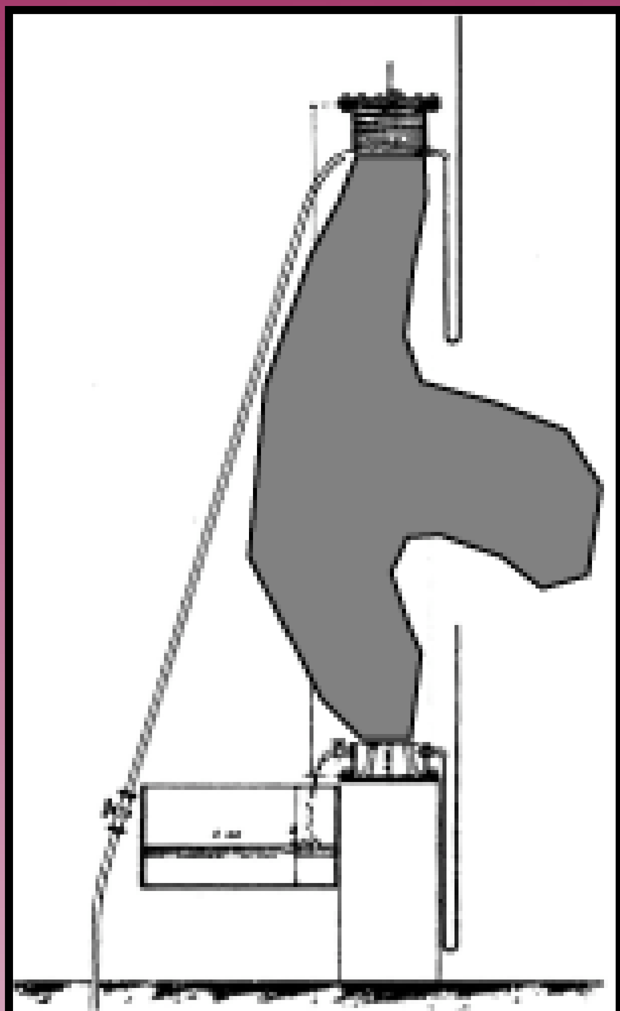
Observations on Darcy's Experiment



$$Q = KA\frac{h_2 - h_1}{L}$$

- Equation does not depend on sand uniformity
- A is a property of the column, not the porous material
- L is a measure of the distance between sampling points
- h_2 and h_1 are indicators of pressure in the reservoirs exterior to the medium
- Q does not provide an indication of the actual fluid velocity within the medium

Alternative Darcy Experiment



- shell houses some arbitrary packed, saturated-flow system
- head is measured using manometers in reservoirs external to the shell region
- Darcian correlation is
- $$Q = K_{\text{eff}} \left(h_2 - h_1 \right)$$
- or divide by some selected cross-sectional area to obtain

$$q = K_{\text{eff}}^{*} \left(h_2 - h_1 \right)$$

- no suggestion of a differential form

Nevertheless... "Darcy's Law"

$$\mathbf{q} = -\frac{K}{\rho_{\text{avg}} g}(\nabla p_{\text{avg}} - \rho_{\text{avg}}\mathbf{g})$$

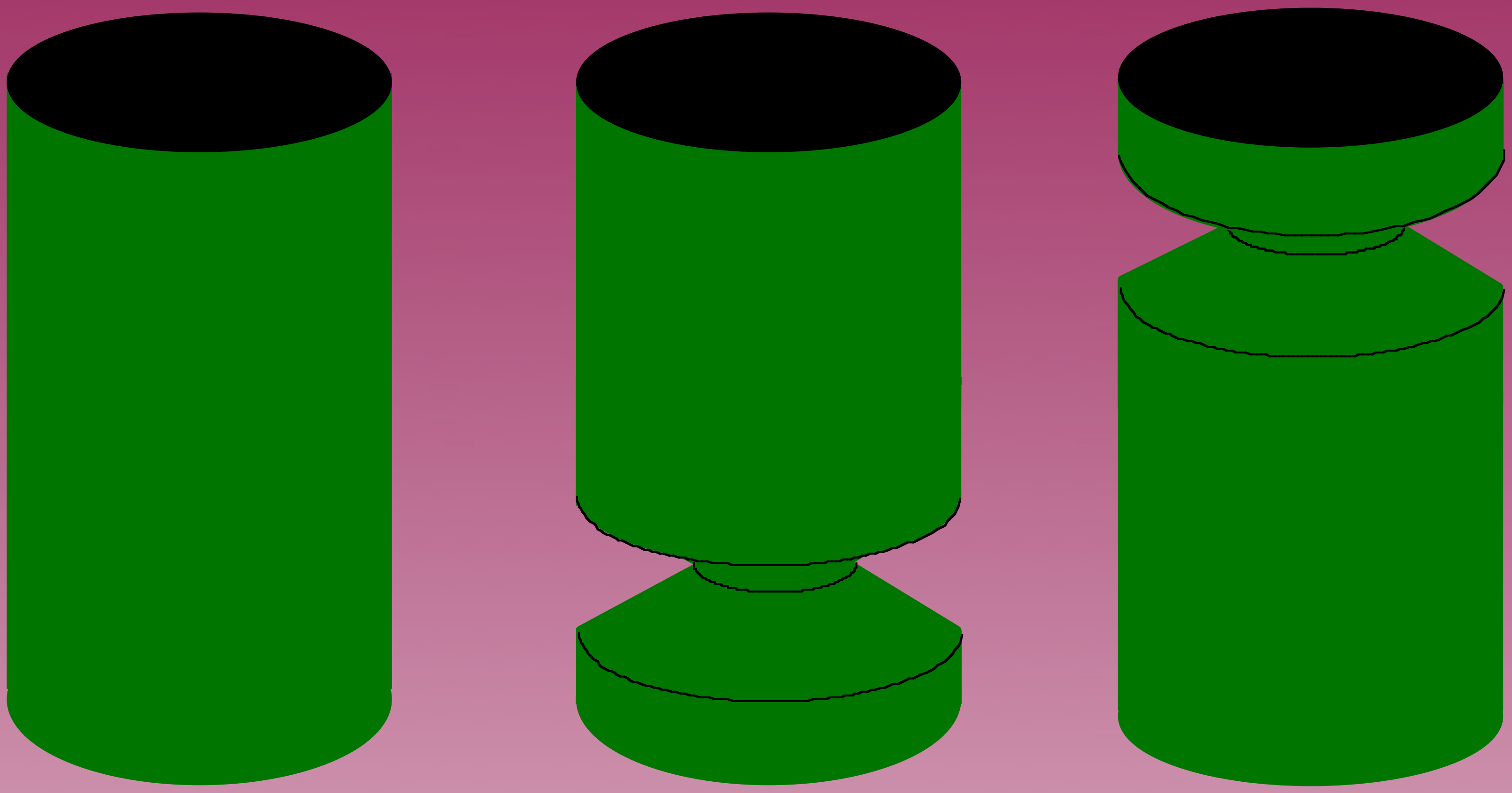
- differential vector equation in terms of "macroscale" variables
-
- $\mathbf{q} = \epsilon \mathbf{v}$ is the Darcy velocity
-
- p_{avg} is the macroscale pressure

Observations on Darcy Experiments

- Darcian experiments provide infomation at the ends of the domain but do not identify the scaling or variation within the domain.
- The suggestion of a differential form of Darcy's Law written in terms of variables within the porous medium is an unjustifiable artifact of the serendipitous selection of a straight tube with constant cross-sectional area and of the placement of manometers for the classic experiments.
- The pervasive use of differential forms of Darcy's Law to describe single-phase, unsaturated, and multiphase flow cannot be justified on the basis of the classic experiments.
- Because variables that appear in the differential forms of Darcy's Law are undefined and are not related to measured quantities in a rigorous fashion, information transfer among scales is precluded.

Pressure

Average Pressure

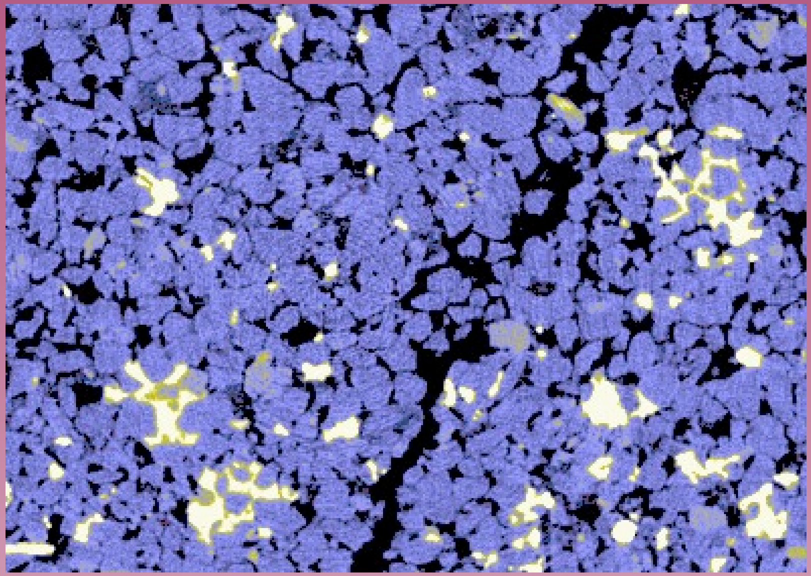


$p_{\mathrm{vol}} = p_{\mathrm{surf}}$ $p_{\mathrm{vol}} < p_{\mathrm{surf}}$ $p_{\mathrm{vol}} > p_{\mathrm{surf}}$

Porous Medium

How should the macroscale pressure be defined?

$$\mathbf{q} = -\frac{K}{\rho_{\mathrm{avg}} g}(\nabla p_{\mathrm{avg}} - \rho_{\mathrm{avg}}\mathbf{g})$$



Berea sandstone at resolution of 9μ

- What should be measured?
- What scale?
- Pressure needed for Darcy's Law?
- Pressure used in Darcy's Law?
- Capillary pressure?
- Thermodynamic description?
- Definitions of other variables?

Elements of Thermodynamically Constrained Averaging Theory

TCAT Approach to Modeling at Macroscale

- Formulate conservation equations and thermodynamic relations at microscale
- Develop microscale equilibrium conditions using a variational analysis
- Employ theorems that allow for a rigorous change in scale of universal relations
- Constrain entropy inequality with conservation equations and averaged thermodynamic relations
- Close equations by requiring that irreversible processes occurring within the system produce entropy and through application of additional constraints (e.g., geometric evolution)

Distinguishing Attributes of TCAT Approach

- Conservation equations are employed for phases, interfaces, common lines, and common points
- Rigorous change of scale ensures that relations between microscale and macroscale variables is preserved
- Averaging of thermodynamic expressions ensures that macroscale pressure, temperature, and chemical potentials are defined in terms of microscale counterparts.
- Averaging of microscale equilibrium conditions to macroscale ensures proper exploitation of the constrained entropy inequality
- Geometric evolution relations are employed for saturations, interfacial areas, and common line lengths

Microscale Conservation Equations and Thermodynamic Relations

- Species Conservation: $\mathcal{M}_{i\alpha} = 0$
- Momentum Conservation: $\boldsymbol{\mathcal{P}}_{i\alpha} = 0$
- Energy Conservation: $\mathcal{E}_{i\alpha} = 0$
- Entropy Equation: $\sum_i \mathcal{S}_{i\alpha} = \Lambda_\alpha \geqslant 0$
- Thermodynamic Dependence: $E_{i\alpha} = E_{i\alpha}(\mathbf{X}_{i\alpha})$

Approaches to Thermodynamics

Formulation	Functional Form
Equilibrium Thermo	$\mathbb{E} = \mathbb{E}(\mathbb{S}, V, M), \text{ for } \Omega$
Classical Irreversible Thermo	$E(\mathbf{x}, t) = E[\eta(\mathbf{x}, t), \rho(\mathbf{x}, t)]$
Rational Thermo	$E(\mathbf{x}, t) = E[\eta(\mathbf{x}, t), \rho(\mathbf{x}, t), \ldots]$
Extended Irreversible Thermo	$E(\mathbf{x}, t) = E[\eta(\mathbf{x}, t), \rho(\mathbf{x}, t), \mathbf{J}]$
Rational Extended Thermo	$E(\mathbf{x}, t) = E[\eta(\mathbf{x}, t), \rho(\mathbf{x}, t), \mathbf{J}, \ldots]$
Theory of Internal Variables	$E(\mathbf{x}, t) = E[\eta(\mathbf{x}, t), \rho(\mathbf{x}, t), \mathbf{I}]$

Employ Theorems for Scale Change

- Entities now occupy portion of same space
- Constrain Entropy Equation with additional relations
- Lagrange multipliers are selected to reflect system

$$\sum_{\alpha}\sum_{i}\mathcal{S}^{i\alpha}+\sum_{\alpha}\sum_{i}\lambda_{\mathcal{E}}^{i\alpha}\mathcal{E}^{i\alpha}+\sum_{\alpha}\sum_{i}\boldsymbol{\lambda}_{\boldsymbol{\mathcal{P}}}^{i\alpha}\cdot\boldsymbol{\mathcal{P}}^{i\alpha}+\sum_{\alpha}\sum_{i}\lambda_{\mathcal{M}}^{i\alpha}\mathcal{M}^{i\alpha}$$

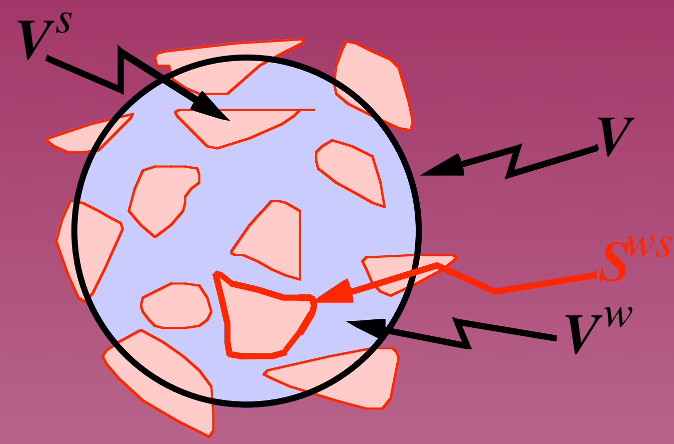
$$+\sum_{\alpha}\sum_{i}\lambda_{t}^{i\alpha}\left(\frac{\partial E^{i\overline{\overline{\alpha}}}}{\partial t}-\frac{\partial E^{i\overline{\overline{\alpha}}}}{\partial \mathbf{X}^{i\alpha}}\cdot\frac{\partial \mathbf{X}^{i\alpha}}{\partial t}\right)$$

$$+\sum_{\alpha}\sum_{i}\boldsymbol{\lambda}_{\mathbf{x}}^{i\alpha}\cdot\left(\nabla E^{i\overline{\overline{\alpha}}}-\nabla \mathbf{X}^{i\alpha}\cdot\frac{\partial E^{i\overline{\overline{\alpha}}}}{\partial \mathbf{X}^{i\alpha}}\right)=\Lambda\geqslant 0$$

Close Equations

- Require that irreversible processes produce entropy
- Examine “near” equilibrium situation
- Linearized constitutive theory
- Make use of approximations relating to geometric changes

Porous Media Flow Equations:



Darcy's Law

- $$\mathbf{q} = -\frac{K}{\rho_{\mathrm{avg}} g}(\nabla p_{\mathrm{avg}} - \rho_{\mathrm{avg}}\mathbf{g})$$

TCAT

- $$\mathbf{q} = -\frac{K}{\rho^w g}(\nabla p^w - \rho^w \mathbf{g}) + \frac{K}{\rho^w g}\left[\frac{1}{V^w}\int_{S^{ws}} (p^w - p)\mathbf{n}^w \mathrm{d}S\right]$$

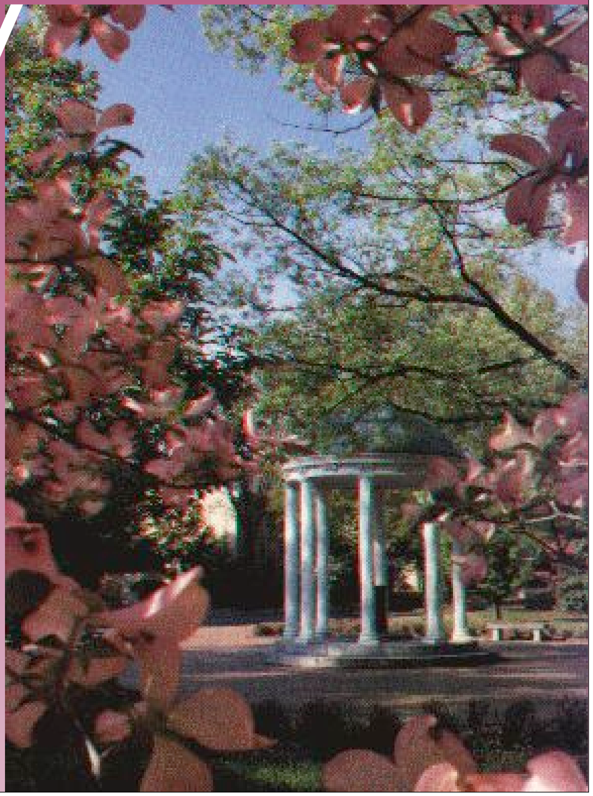
with $$p^w = \frac{1}{V^w}\int_{V^w} p\mathrm{d}V$$

and $$\rho^w = \frac{1}{V^w}\int_{V^w} \rho\mathrm{d}V$$

Summary

- Darcy's Law applies for the measurements and systems that led to this correlation.
- Extension of Darcy's Law to differential form and for general systems ignores both processes and scale.
- Despite the widespread, essentially universal, application of the differential form of Darcy's Law, the quantities appearing in the equation are poorly defined.
- Definitions of other quantities at the macroscale are also ill-defined (e.g., temperature).

Unsaturated Flow



Standard Form for Air-Water Flow:

Water Phase

- $\mathbf{q}^w = -\frac{K k^r(s^w)}{\rho^w_{\text{avg}} g}\left(\nabla p^w_{\text{avg}} - \rho^w_{\text{avg}}\mathbf{g}\right)$

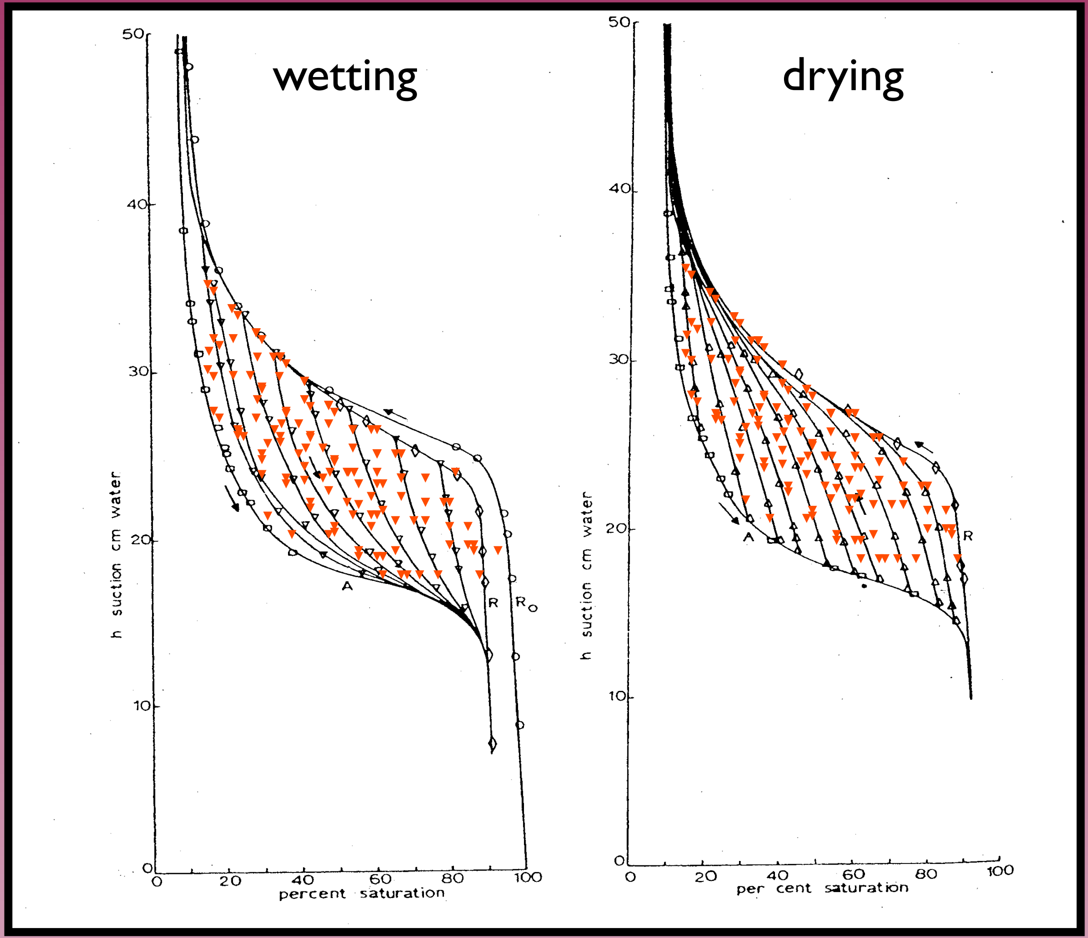
Air Phase Pressure

- $p^a_{\text{avg}} = \text{constant}$

Capillary Pressure

- $p^a_{\text{avg}} - p^w_{\text{avg}} = p^c_{\text{avg}}(s^w)$

Capillary Pressure vs. Saturation



Microscale Surface Momentum Equation

$$\frac{\mathrm{D}(\rho\mathbf{v})}{\mathrm{D}t} - \nabla^s \cdot \mathbf{t}^s + \mathbf{t}_w \cdot \mathbf{n}^w + \mathbf{t}_n \cdot \mathbf{n}^n = 0 - \rho\mathbf{g}$$

$$- [\rho_w \mathbf{v}_w(\mathbf{v}_w - \mathbf{v})] \cdot \mathbf{n}^w - [\rho_n \mathbf{v}_n(\mathbf{v}_n - \mathbf{v})] \cdot \mathbf{n}^n$$

Surface Stress Tensor

$$-\nabla^s \cdot \mathbf{t}^s + \mathbf{t}_w \cdot \mathbf{n}^w + \mathbf{t}_n \cdot \mathbf{n}^n = 0$$

$$\mathbf{t}^s = \gamma_{wn}\mathbf{I}^s$$

Phase Stress Tensors

$$\mathbf{t}_\alpha = -p_\alpha \mathbf{I} + \boldsymbol{\tau}_\alpha$$

Tangential Stress Balance

$$-\nabla^s \gamma_{wn} + \mathbf{I}^s \cdot \boldsymbol{\tau}_w \cdot \mathbf{n}^w + \mathbf{I}^s \cdot \boldsymbol{\tau}_n \cdot \mathbf{n}^n = 0$$

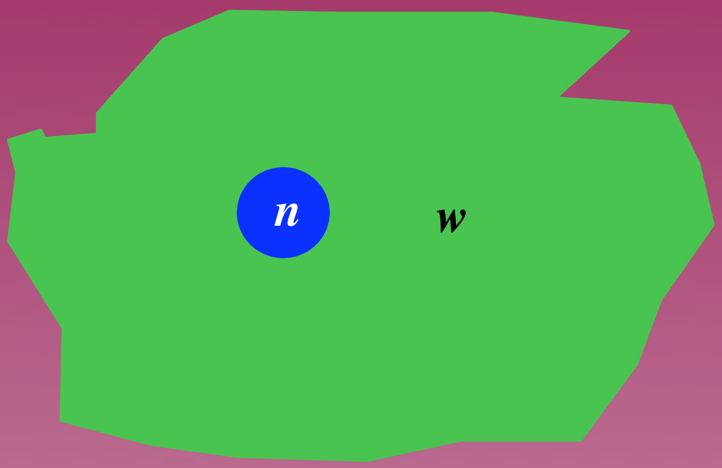
Normal Stress Balance

$$\gamma_{wn} \nabla^s \cdot \mathbf{n}^w - p_w + p_n + \mathbf{n}^w \cdot \boldsymbol{\tau}_w \cdot \mathbf{n}^w + \mathbf{n}^n \cdot \boldsymbol{\tau}_n \cdot \mathbf{n}^n = 0$$

Microscale Example: Expanding Bubble

Consider

- $\mu_w \gg \mu_n$
- $\nabla \cdot \mathbf{v}_w = 0$
- $p_c = -\gamma_{wn} \nabla^s \cdot \mathbf{n}^w$
- $\mathbf{n}^w \cdot \boldsymbol{\tau}_w \cdot \mathbf{n}^w = -\frac{2}{r} \mu_w v_r$



Normal Stress Balance

- $\gamma_{wn} \nabla^s \cdot \mathbf{n}^w - p_w + p_n = -\mathbf{n}^w \cdot \boldsymbol{\tau}_w \cdot \mathbf{n}^w - \mathbf{n}^n \cdot \boldsymbol{\tau}_n \cdot \mathbf{n}^n$
- $-p_c - p_w + p_n = \frac{2}{r} \mu_w v_r$

Observations from Microscale

- Interfacial curvature is defined by $J^w = \nabla^s \cdot \mathbf{n}^w$

- Capillary pressure is a unique function of curvature:

$$p_c = -\gamma_{wn} J^w$$

- Capillary dynamics involves quantities at the interface

- For the case of the bubble with constant properties at the interface, we can integrate over the interface to obtain:

$$-p_c - p_w + p_n = \frac{\mu_w}{2\pi r^3} \frac{\mathrm{d}V}{\mathrm{d}t}$$

TCAT Form for Air-Water Flow:

Water Phase

- $$\mathbf{q}^w = -\frac{K k^r(s^w, a^{wa})}{\rho^w g}(\nabla p^w - \rho^w \mathbf{g}) + \frac{K\, k^r(s^w, a^{wa})}{\rho^w g}\left[\frac{1}{V^w}\int_{S^w} (p^w - p)\mathbf{n}^w \mathrm{d}S\right]$$

Capillary Pressure

- $$A\frac{\partial s^w}{\partial t} = p^{\overline{\overline{w}}} - p^{\overline{\overline{a}}} + p^{\overline{\overline{c}}}(s^w, a^{wa}, ...)$$

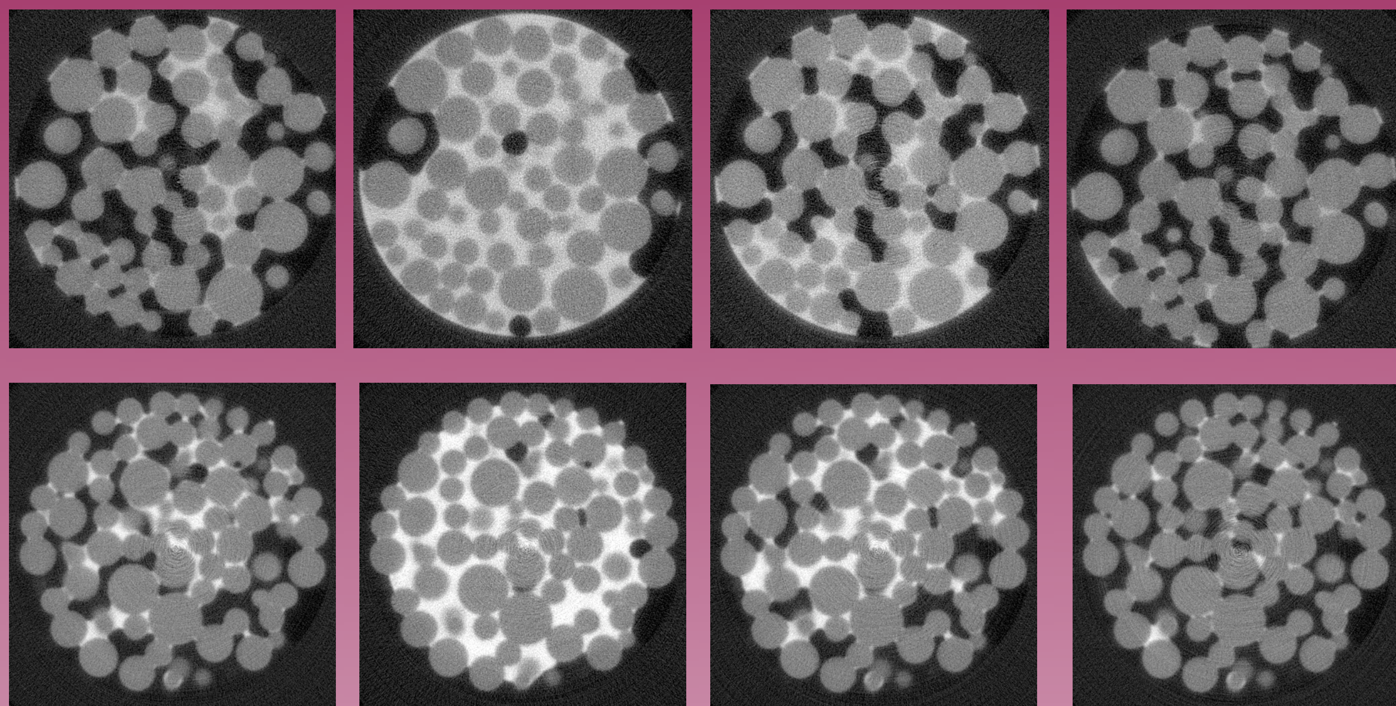
Interface Evolution

- $$\gamma^{wa}\frac{\partial a^{wa}}{\partial t} + p^{\overline{\overline{\mathbf{c}}}}\epsilon\frac{\partial s^w}{\partial t} = -\nabla \cdot (a^{wa}\, \mathbf{G}^{wa} \cdot \mathbf{q}^w)$$

Formulation of Capillary Pressure

Microscale Experiments: 7mm Column

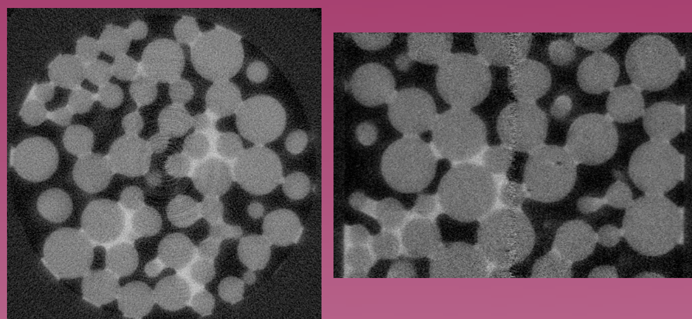
$s^w = 0.24$ $s^w = 0.75$ $s^w = 0.34$ $s^w = 0.10$



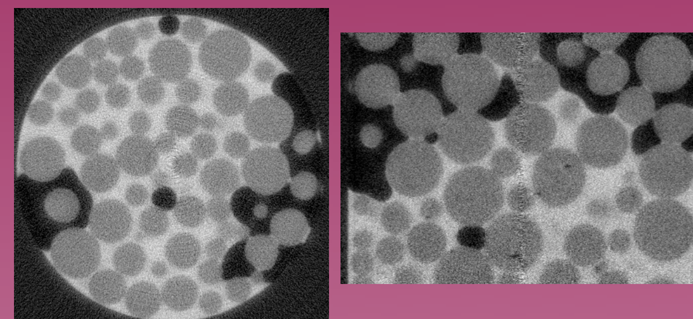
$s^w = 0.24$ $s^w = 0.72$ $s^w = 0.37$ $s^w = 0.10$

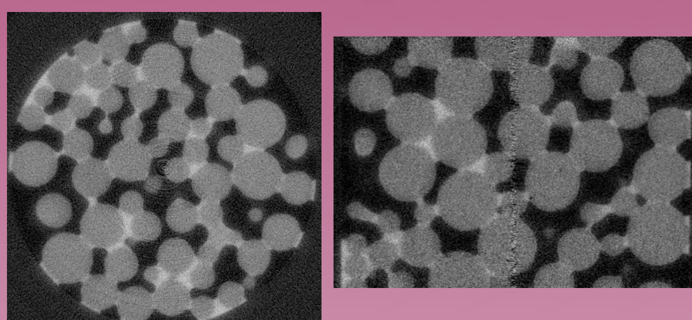
Imbibition

$s^w = 0.13$

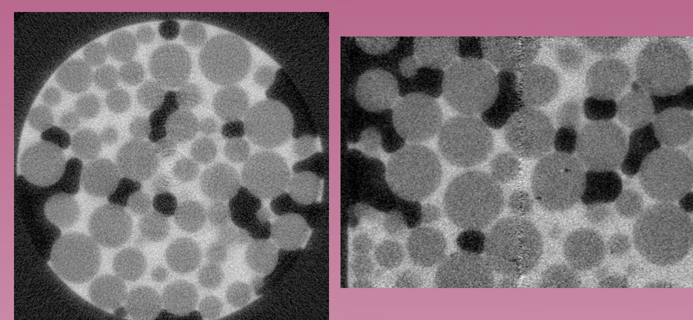


$s^w = 0.59$





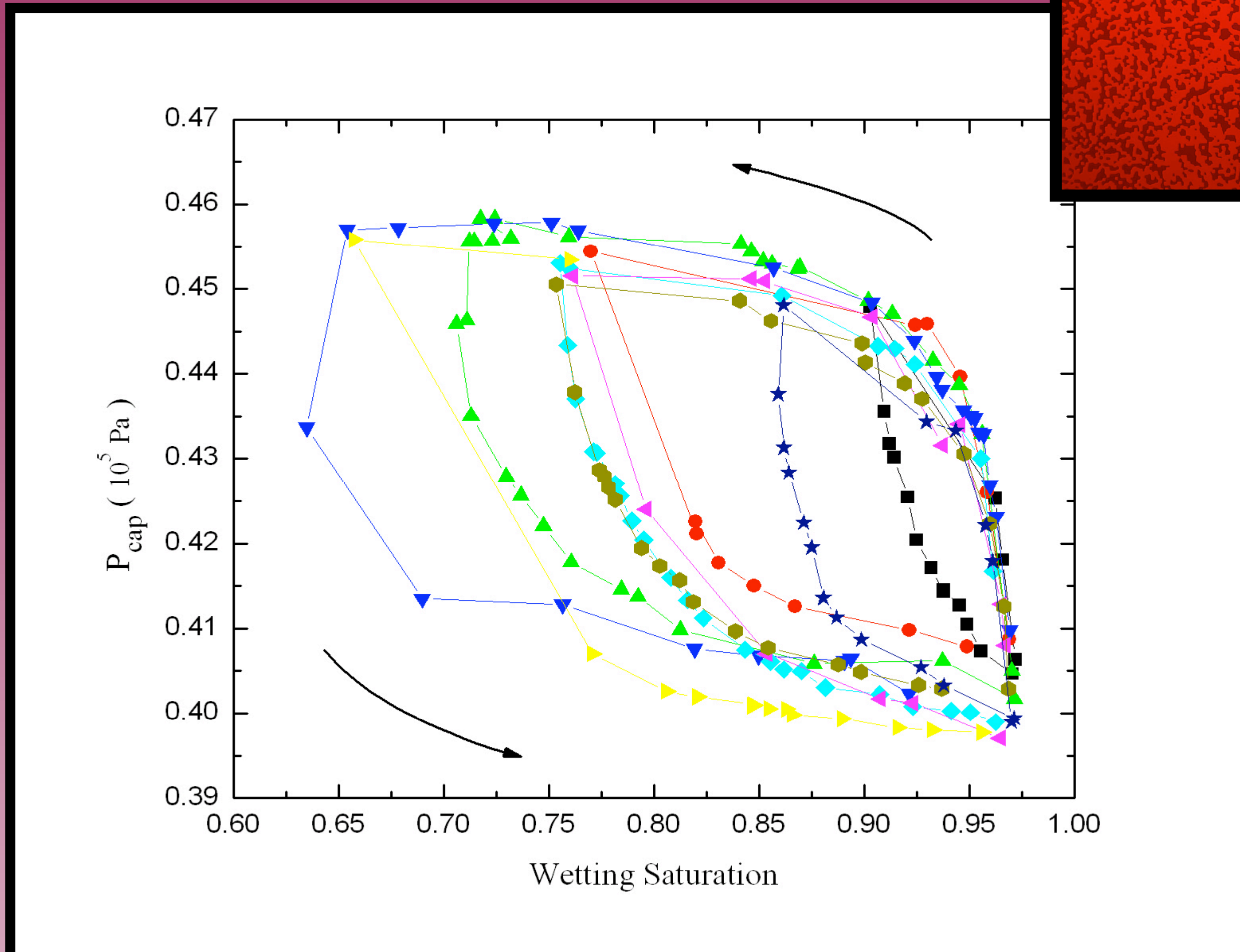
$s^w = 0.15$

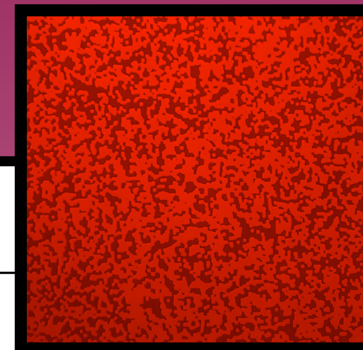


$s^w = 0.61$

Drainage

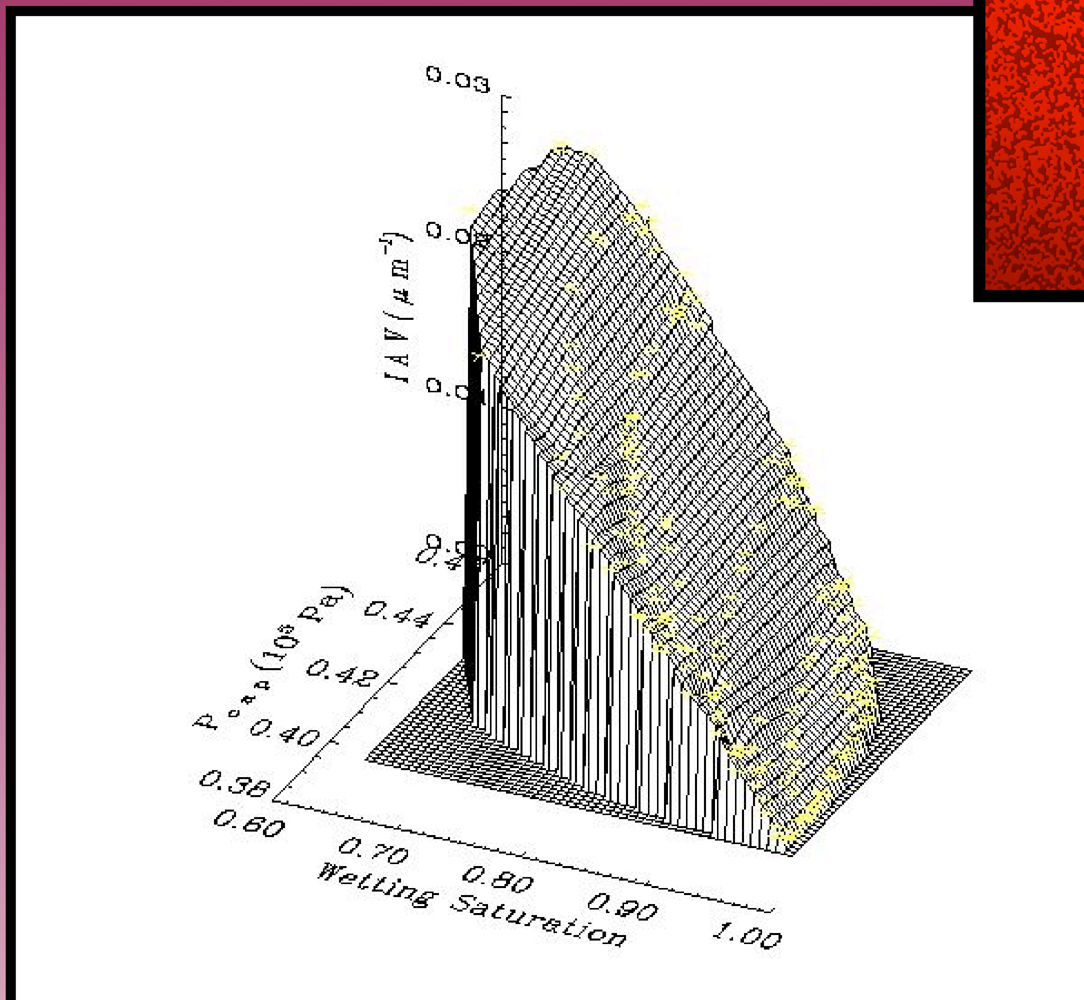
Capillary Pressure - Saturation

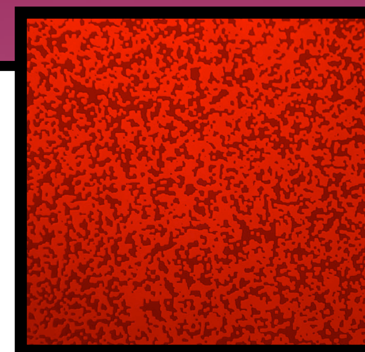




Cheng, Pyrak-Nolte, Giordano (2002)

Capillary Pressure - Area - Saturation





Cheng, Pyrak-Nolte, Giordano (2002)

Derived Form for Solid Stress

Solid Phase Stress Tensor

- $\mathbf{t}^s = \boldsymbol{\tau}^s - p^s \mathbf{I}$

In Terms of Biot Coefficient

- $\mathbf{t}^s = \boldsymbol{\tau}^s + \alpha \left(\mathbf{n}_s \cdot \mathbf{t}_s \cdot \mathbf{n}_s\right)^{\text{surf}} \mathbf{I}$

Normal Surface Stress

- $\left(\mathbf{n}_s \cdot \mathbf{t}_s \cdot \mathbf{n}_s\right)^{\text{surf}} = -x_s^{ws} \left(p_w^{ws} + \gamma^{ws} J_s^{ws}\right) - \left(1 - x_s^{ws}\right) \left(p_n^{ns} + \gamma^{ns} J_s^{ns}\right)$

Conclusion

For Study of Multiphase Flow

- Application of "laws" beyond the conditions for which they were developed must be approached with caution.
- Theory requires ability to transfer experimental information between scales.
- Identification of symbols in a theory as "pressure" or "temperature" does not necessarily indicate consistency.
- "Hysteresis" at the larger scale is due, probably in large part, to loss of information.
- Careful manipulation of equations can provide scientific studies that faithfully reproduce reality.

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