Seminar of Institute of Fundamental Technological Research PAS, June 06, 2007

REGULAR AND SINGULAR COMPONENTS OF ENVIRONMENTAL FLOWS AND THEIR IMPACT ON TRANSPORT OF SUBSTANCES

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- Introduction;
- Visualization of environmental flows;
- Exact solutions of fundamental governing equations (without external forcing, linearized and slightly nonlinear models);
- Periodic internal waves: calculations and observations;
- Flow past obstacles and redistribution of a dye;
- Conclusion and speculations.

Крым и Азов с орбиты (ручная съемка)

Ship wave wakes and filaments in Sun glitter (view from S-A orbit)

Lava ropes in Mexico Botanical Garden

Evolution of experimental fluid mechanics

1756-1757 J. Borda – rotational machine (drag forces on prisms, cylinders
1775 D'Alembert, Bossue, Condorce (pool 20x16x2 m, mast 23 m
20 models, wave drag, shapes of stern (total drag)
Du Buat (Pitot tubes), "Principles of hydraulics" (1779, 1786, 1816): «deficit of pressure», (drag on body: effect of the body shape and friction force);
M. Bofua 10000 experiments in the dock 122x20x3.55 м опытов (plates on floats)
"Marine and hydraulic experiments" (time and distances registration)
1838 – Hagen's, 1841 Poiseuillelle's discharge from capillary tubes
W. Rankine 1853 ("added wet surface")
W. Froude (1859-Torquay, 1870 tank 120x6.7x3.0 м; forces and moments measurements)
Replicated in St. Petersburg in 1894 г. (120x6.7x3.0 м, enlarged in 1935 г.)
1894, 1896, 1902 – aerodynamics Wind tunnels in Denmark, England, France, Russia
1896 O. Lilliental searches a wing with a "negative" drag profile
1906 V.Ekman model of the "Fram"
1907 – L. Prandtl, Goettingen "Boundary layer calculations of drag and lifts on profiles"
XX Century(20th - 50th) – modern Water tanks and Wind tunnels test facilities
XX Century(60th - now) – stratified tanks

•Excursion to the history of scientific fluid mechanics

- Intuitive-observational period: definition of quantities:
  $-\infty$ - XVIII century
- Mathematical formulation of foundations of fluid mechanics (D'Alembert, J.&D.Bernoulli, L.Euler, J.Lagrange, J.Fourier (1822))
- Formulation of governing equations for a homogeneous fluid

  (Navier (1822). Cauchy, Poisson, Sain Venant, G.G.Stokes (1845), H.Helmhotz, H.Lamb, I.Gromeko...)
- Identification of flow components and their mathematical images (vortices, different waves, wakes, jets, shock waves..). Model description (Saint Venant, Boussinesq, Kortewed-de Vries, Kelvin, Mach, Reynolds, Prandtl...,
- Adequate Laboratory modeling of flows (1942 – now)
- New classification (Regular and Singular flow components)

Mathematical definition of properties of fluids and flow parameters

Momentum, Descartes, $$\mathbf{p} = \rho\,\mathbf{v}$$

Density, acceleration: Galileo, Mass, force: Newton,

Pressure: Stevin, Pascal: hydrostatic; D.Bernoulli (1739), Lagrange: dynamical

$$\frac{p}{\rho} + \frac{\mathrm{v}^2}{2} + gz + w = \mathrm{const} \qquad \frac{\partial \varphi}{\partial t} + \frac{p}{\rho} + \frac{\mathrm{v}^2}{2} + gz = \mathrm{const}$$

Acceleration of fluid particles:
J.Bernoully (1738-1741), L.Euler (1744)

$$\frac{d(\rho\mathbf{v})}{dt} = \frac{\partial(\rho\mathbf{v})}{\partial t} + (\mathbf{v}\nabla)(\rho\mathbf{v})$$

Tensor of velocity shear, two viscosities:
A. Cauchi (1818)

$$e_{ij} = \frac{\partial \mathrm{v}_j}{\partial x_i} + \frac{\partial \mathrm{v}_i}{\partial x_j}$$

Vorticity, Circulation: G.G.Stokes (1845), Kelvin, Helmholtz; Helicity: K.Moffat

$$\mathrm{rot}\,\mathbf{v} = [\nabla\,\mathbf{v}] \quad \Gamma = \int \mathbf{v}\cdot d\mathbf{l}$$

$$S = (\mathbf{v}\cdot\mathrm{rot}\mathbf{v})$$

Scalar (Euler) and vector potentials, Stream function,toroidal-poloidal decomposition

$$\mathbf{v} = \mathrm{grad}\,\varphi \quad \mathbf{v} = \mathrm{rot}\,\mathbf{A}$$

J. D’Alembert published his
***Traité de l'équilibre et du mouvement des fluides* in 1744 ,**
***Theorie générale des vents*, in 1747:**

He arrived at a partial differential equation for description of string motions and sounds and derived continuity equation both for incompressible and compressible (2D case) fluids .

$$\operatorname{div}\mathbf{v} = 0 \quad \text{or} \quad \frac{\partial\rho(\mathbf{x},t)}{\partial t} + \operatorname{div}(\rho\cdot\mathbf{v}) = 0 \quad dp = c_s^2\, d\rho$$

$$\rho(p)\left(\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\mathbf{v}\right)=-\nabla p+\rho\,\mathbf{g};$$

$$\frac{\partial \rho}{\partial t}+\operatorname{div}(\rho\cdot\mathbf{v})=0 \qquad dp=c_s^2\,d\rho \qquad \textbf{or} \qquad \operatorname{div}\mathbf{v}=0$$

Euler L. Principes généraux du mouvement des fluids // Mémoires de l'Academié royalle des sciences et belles letters. Berlin. 1757. V. 11 (papers of 1755 year). P. 274-315. = Opera omnia. Ser.II. V.12. P. 54-91.**. (+ 1750, 1751, 1752**)

"Nevertheless everything what contains theory of fluids is contained in the two above mentioned equations (§ 34), so for continuation of the study we need not in principles of mechanics but only instruments of analysis which is still not enough developed for this purpose".

Euler-D'Alembert paradox

First fundamental classification of a flows is based on of potential and stream function,
later vorticity, complex potential, complex velocity were introduced
velocity fields were separated on potential and vortex parts

$$\mathbf{v} = \operatorname{grad}\varphi \qquad v_x = \partial\Psi/\partial y \quad v_y = -\partial\Psi/\partial x$$

vorticity $$\boldsymbol{\omega} = \operatorname{rot}\mathbf{v} \qquad w = \phi + i\psi \qquad z = x + iy$$

$$\mathbf{v} = \mathbf{v}_p + \mathbf{v}_s = \nabla\varphi + \operatorname{rot}\mathbf{A}$$

Components of flows are waves, jets, wakes, vortices

Advantages:

Clear mathematical definitions of basic flow components;
A lot of solved particular problems;

Disadvantages:

Lost of physically proved boundary conditions
Existence of discontinuities on some characteristics and specific lines of surfaces

Toroidal-poloidal decomposition $$\mathbf{v} = \nabla\times\mathbf{e}_z\Psi + \nabla\times\left(\nabla\times\mathbf{e}_z\Phi\right)$$

Jean Baptiste Joseph Fourier (1768 - 1830) published his

Théorie analytique de la chaleur (1806-1814-1822)

Dimension and Dimension homogeneity;

Functional connection between heat flux and temperature gradient:

$$\mathbf{q}_T = -\kappa_q \Delta T$$

$$\frac{\partial T}{\partial t} + (\mathbf{v}\nabla)T = \chi \Delta T, \quad \kappa_q = \rho c_p \chi$$

Developed expansions of functions in Fourier series and integrals

$$F(\mathbf{k},\omega) = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} f(\mathbf{x},t) \exp\left(i(\mathbf{kx} - \omega t)\right) d\mathbf{x}dt$$

Fick’s law: $\mathbf{q}_S = -\kappa_s \Delta S$

$$\frac{\partial(\rho S)}{\partial t} + (\mathbf{v}\nabla)(\rho S) = \kappa_s \Delta(\rho S),$$

Claude-Louis-Marie-Henri (M.) Navier.

Mémoire sur les Lois du Mouvement des Fluids // Mém. d l'Acad. des Sciences. 1822. V. 6. P. 389.

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho(\mathbf{v}\nabla)\mathbf{v} = -\nabla p + \rho \mathbf{g} + \mu \Delta \mathbf{v} \qquad \frac{\partial \rho}{\partial t} + \mathrm{div}\, \rho \mathbf{v} = 0$$

Interpretation of P.-S. Girard's (1816) negligent Experiments leads M.Navier to posing wrong boundary conditions

$$E\,\mathbf{v} + \mu \frac{\partial \mathbf{v}_{\parallel}}{\partial \mathbf{n}_{\perp}} = 0$$

Mr. Navier could present his basic principles in form of hypotheses which must be proved experimentally. However, if common equations of fluid mechanics are so difficult for analysis what can we receive from this new ones even more complicated equations? Antoine Cournot, 1828.

"Useless equations"

Augustin-Louis Cauchy

Recherches sur l'équilibre et le mouvement des corps solides ou fluides, élastiques ou non élastiques // Bulletin des sciences de la Société philomatique de Paris. 1823. P. 9-13. (Extract of a memoir read on September 30, 1822)

Found connections between tehsors of shear of velocity and rate of deformation (wrote tensor of stresses in canonical diagonal form)

$$e_{ij} = \frac{\partial \mathrm{v}_j}{\partial x_i} + \frac{\partial \mathrm{v}_i}{\partial x_j}$$

He analysed the case of absolutely inelastic body, rederived the Navier equation and did not mentioned Navier's original paper in references; introduced two characteristic viscosities of fluid (the first and second viscosities)

$$e_{ij} = K_1 \left(\frac{\partial \mathrm{v}_j}{\partial x_i} + \frac{\partial \mathrm{v}_i}{\partial x_j} \right) + K_2 \delta_{ij} \frac{\partial \mathrm{v}_k}{\partial x_k}$$

Simon-Denis Poisson.

Mémoire sur les Équations générales de l’ Équilibre et du Mouvement des Corps Solides elastiques et des Fluides // Journ. De l’Ecole Polytechn. 1829. V. 13. P. 1-174 (V.20, 1831).

$$\tau_{ij} = N\left[\lambda\delta_{ij} + (\lambda' - \lambda)\delta_{ij}\frac{\partial \mathrm{v}_k}{\partial x_k} + (\lambda' + \lambda)\left(\frac{\partial \mathrm{v}_j}{\partial x_i} + \frac{\partial \mathrm{v}_i}{\partial x_j}\right)\right]$$

Saint-Venant, Adhémar Barré de .

Note á joindre au memoire sur la dynamique des fluids.

Présenté le 14 avril 1834) (extrait) // Comptes Rendus de l'Académie des Sciences. 1843. V.17. P. 1240-1243. (Nouveaux Commissaires, MM. Cauchy, Poncele, Lame, en rempacement de MM Ampére, Navier, Felix Savary)

$$e_{ij} = \varepsilon \left(\frac{\partial \mathrm{v}_j}{\partial x_i} + \frac{\partial \mathrm{v}_i}{\partial x_j} \right)_i + \varpi \, \delta_{ij}$$

Saint-Venant discussed:
Problems of resolvability of governing equations, forms of boundary conditions,
Introduction of variable ("turbulent") viscosity введение переменой вязкости,
unification of hydrodynamics and hydraulics by universal theory

Stokes G. G. On On the theories of the internal friction of fluids in motion, and of the equilibrium and motion of elastic bodies // Transaction of the Cambridge Philosophical Society. 1845. V. 8. P. 287 (Mathematical and Physical Papers. 1880. V. I. Cambridge: at the University Press. 1880. V. I. P. 75-129).

Separated shear of velocity tensor on symmetric and antysymmetric parts

$$\frac{\partial \mathrm{v}_j}{\partial x_i} dx_i = \frac{1}{2}\left[\left(\frac{\partial \mathrm{v}_j}{\partial x_i} + \frac{\partial \mathrm{v}_i}{\partial x_j}\right) dx_i + \left(\frac{\partial \mathrm{v}_j}{\partial x_i} - \frac{\partial \mathrm{v}_i}{\partial x_j}\right) dx_i\right]$$

Used two characteristic viscosities, calculated a sound attenuation; indicated solution of the Hagen-Poiselle problem

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Stokes G.G. On the effect of the internal friction on the motion of pendulums // Trans. Cambridge Phil. Soc. 1851. V. 9. P. 8. (Mathematical and Physical Papers. V. III. Cambridge: at the University Press. 1901. P. 1-141).

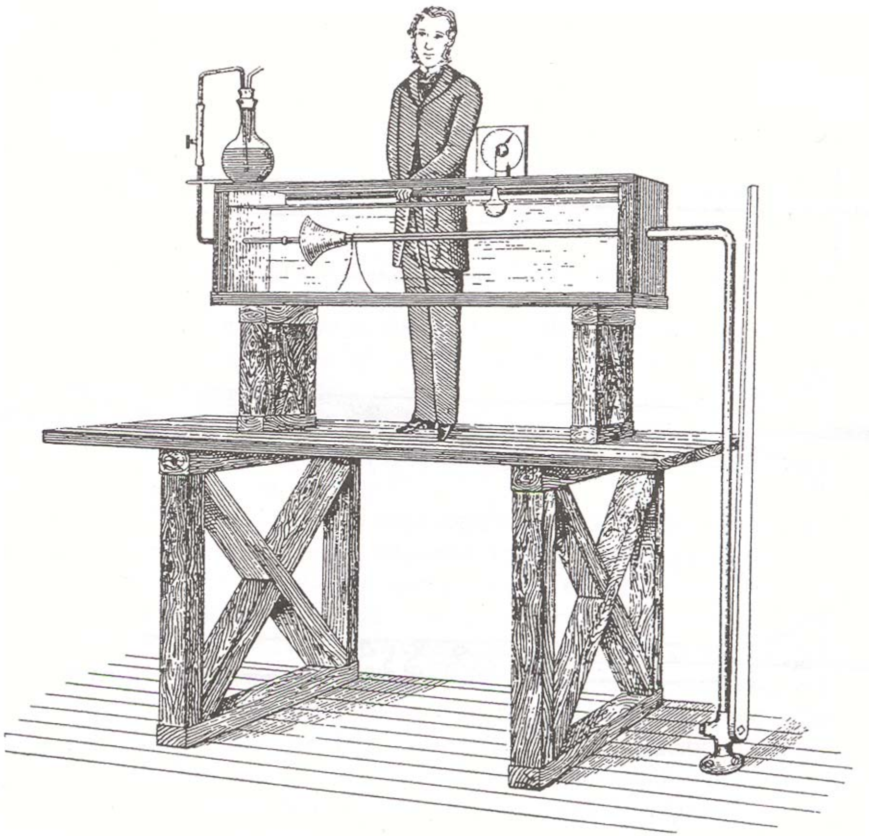
$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \nabla) \mathbf{v} = -\nabla p + \mu \Delta \mathbf{v} + \rho \mathbf{g}$$

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \rho \mathbf{v} = 0 \qquad \textbf{or} \qquad \operatorname{div} \mathbf{v} = 0$$

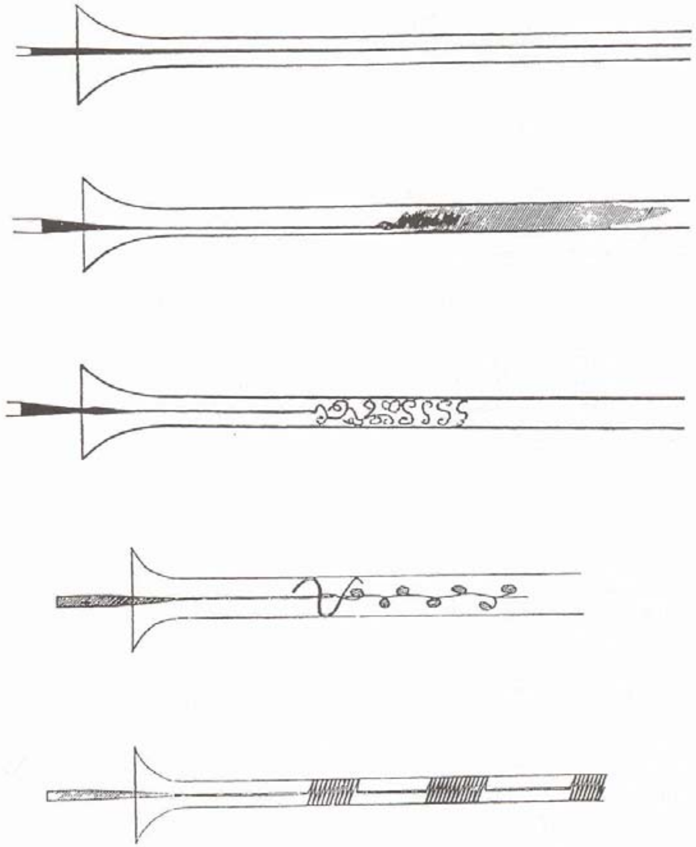
$$\rho = \text{const}, \ \rho = \rho(p)$$

Collection of specific exact solutions: linear oscillations of a plane (discovery of periodic boundary layer), rotational oscillation of a disc, oscillations and uniform motion of a sphere, oscillations of a cylinder

General idea is to use symmetry of the problem for reducing the order of the governing equation set: solved only 1D or 2D problems



Reynolds' apparatus for studying the turbulent transition of the flow of water in a tube. Water from the tank enters the horizontal glass tube through the conical funnel. The valve with the long handle on the right controls the flux, whose value is inferred from the lowering of the floater. Ink from the flask is injected continuously in the middle of the entrance of the tube. *Source:* Reynolds (ref. 100), 71.



Reynolds' drawings of kinds of flow in a tube, as indicated by the ink jet method. "Direct" flow (a), "sinuous" flow (b), the same observed with a flash of light (c), disturbance of direct flow by a wire (d), intermittent "flashes" of eddying (e). *Source:* Reynolds (ref. 100), 59, 60, 76, 77.

Mathematical schemes in “theories of turbulence”

Reynolds O. An experimental investigation of the circumstances, which determine whether the motion of water shall be direct or sinuous, and the law of resistance in parallel channels // Phil. Trans. Roy. Soc., London. 1883. V. 174. P. 935-982.)

Reynolds O. On the dynamic theory of incompressible viscous fluids and the determination of the criterion // Phil. Trans. Roy. Soc., London.1895. V. 186. P. 123-161.

Classification of flows, rules of averaging, new spectral characteristic of flows, energy of mechanical motions, rate of its dissipation.

Ludwig Prandtl Lecture „Über Flüssigkeitsbewegung bei sehr kleiner Reibung“ // III International Congress of Mathematicians’. Heidelberg. Germany. s. 484-481. on August 12, 1904. (Gesammelte Abhandlungen zur Mechanik, Hydro- und Aerodynamik, 3 vols., Göttingen, 1961).

$$\frac{\partial u}{\partial x} + \frac{\partial \mathrm{v}}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + \mathrm{v}\frac{\partial u}{\partial y} - \nu\frac{\partial^2 u}{\partial y^2} = -\frac{1}{\rho_0}\frac{\partial P}{\partial x} = U\frac{dU}{dx}$$

„When complex mathematical problem looks like hopeless, it is useful to analyze what is happened when one of important parameters reach its limited zero value “ (Prandtl L. Mein Weg zu hydrodynamischen Theorien // Physikalische Blätter, 1948).

Lord Kelvin (J.J.Thomson)

Surface long and short ship waves:
Capillary waves;
Theory of vortex motions (vorticity, “Stokes theorem”)

O.Reynolds, Rayleigh (W.Strutt): celerity of waves, group and phase velocities

S. Russel, Boussinesq, Korteweg & De Vries…
Non-linear waves (solitons)

N.E. Joukovskiy Lift and circulation, calculations of lift and drag

G.I.Taylor, hypothesis "passive contaminants" and "permanent entrainment velocity"

Different kinds of a flow instabilities:

Hydrostatic instability
- Rayleigh-Taylor instability (in a stationary gravity field)
- Richtmayer-Meshkov instability (in growing gravity field)

Hydrodynamic instability (shear, tangential jump)
- in a homogeneous fluid (Kelvin-Helmholz)
- in a non homogeneous fluid (Miles-Huppert)

and appropriate critical conditions which are really non-universal

Constructive resuts of N.E.Joukovskiy, S.A.Chaplygin, A.M.Lyapounov, A.A.Fridman, N.E.Kochin, S.A.Khristianovich, A.M. Oboukhov, A.S.Monin, Sir James Lighthill...

Admittedly, as useful a matter as the motion of fluid and related sciences has always been an object of thought. Yet until this day neither our knowledge nor our command of the mathematical principles of nature have permitted a successesful treatment.

(Daniel Bernoulli, Letter to Shoepflin, Sept. 1734).

Now I think hydrodynamics is to be root of all physical science, and is at present second to none in the beauty of its mathematics.

(W.Thomson, Letter to Stokes 20 Dec. 1857)

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In order to celebrate mathematics in the new millennium, The Clay Mathematics Institute of Cambridge, Massachusetts (CMI) has named seven Prize Problems. The Scientific Advisory Board of CMI selected these problems, focusing on important classic questions that have resisted solution over the years. The Board of Directors of CMI designated a $7 million prize fund for the solution to these problems, with $1 million allocated to each.

EXISTENCE & SMOOTHNESS OF THE NAVIER–STOKES EQUATION
Charles L. Fefferman

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho(\mathbf{v}\nabla)\mathbf{v} = -\nabla p + \mu\Delta\mathbf{v}$$

$$\operatorname{div}\mathbf{v} = 0$$

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \rho\mathbf{v} = 0$$

$$\rho = \text{const}, \quad \rho = \rho(p)$$

- Birch and Swinnerton-Dyer Conjecture
- Hodge Conjecture
- P vs NP
- Poincaré Conjecture
- Riemann Hypothesis
- Yang-Mills Theory

Stratified flows $\rho_o = \rho_{oo}(z)$

Benjamin Franklin Behavior of oil on water. Letter to J. Pringle. Dec. 1, 1762.
Experiments and observation on electricity. 1769

G.G.Stokes, On the theory of Oscillatory waves. Camb. Trans., 1847

Jevons W.S. (1857)
Rayleigh, Lord (1883)
Väisälä V. (1925) - Brunt D. (1927)

$$N^2 = \frac{g}{\rho}\left|\frac{d\rho}{dz}\right|$$

F.Nansen (Fram, 1893-1896)-V.Ekman (1906)

Brever, Barus (1880)
Mendenholm and Mason (1924)
H. Stommel (1964) - M. Stern (1968) Thorpe et al. (1968) C.F. Chen (1970)

Prandtl-Lyra-Görrtler (1942-1244)
Lighthill et al. + ….(1964-1967)

We study a fine structure and dynamics of continuously stratified fluid in a gravity field with acceleration **g**

Density is $\rho(z) = \rho_o \exp(-z/\Lambda)$, $\Lambda = d \ln \rho / dz$ is buoyancy scale ,

$N = \sqrt{g/\Lambda}$ $T_b = 2\pi / N$ are buoyancy frequency and period.

…"And I think it (oscillation of a stratified fluid) is worth considering; for a new appearance, and if it cannot be explained by our old principles, may afford us new ones, of use perhaps in explaining some other obscure parts of natural knowledge."

Benjamin Franklin (1706-1790) Behavior of oil on water. Letter to John Pringle. Philadelphia, Dec. 1, 1762. Experiments and observation on electricity. 1769.

Euler equations

$$\mathbf{v}=\nabla\times\mathbf{e}_z\Psi+\nabla\times(\nabla\times\mathbf{e}_z\Phi)$$

$$\rho(\mathbf{x},t)\left(\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\mathbf{v}\right)=-\nabla p+\rho g;\qquad \operatorname{div}\mathbf{v}=0,\ \rho=\mathrm{const}$$

Second order; four or five variables

Classical Navier-Stokes equations

$$\rho=\rho(p),\ c_s^2=\left(\frac{\partial p}{\partial\rho}\right)_s$$

$$\rho\left(\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\mathbf{v}\right)=-\nabla p+\rho\nu\Delta\mathbf{v}+\rho\mathbf{g};\quad \frac{\partial\rho}{\partial t}+\operatorname{div}(\rho\mathbf{v})=0$$

Sixth order; four or five variables

Additional equation or state and conservation of stratified component

$$\rho=\rho(p,S(\mathbf{x},t));\quad \frac{\partial(\rho S)}{\partial t}+\operatorname{div}((\rho(\mathbf{x},t)S)\mathbf{v})=\frac{\partial}{\partial\mathbf{x}}\left(\kappa_S\frac{\partial(\rho S)}{\partial\mathbf{x}}\right)$$

Doppler effect $\omega=\omega_o+\mathbf{k}\cdot\mathbf{U},$

What else we can extract from the First Principles taking into account:

Definitions of the fundamental flow parameters;

Compatibility and non-degeneracy conditions for the governing equations set

And for solutions:

Conditions of uniqueness and exact satisfaction to the right boundary conditions? Definitions of the fundamental flow parameters, Compatibility and non-degeneracy condition for the governing equations set

Solutions must be constructed in the form, permitting direct comparison with data of Laboratory Experiments,
Visualization and Measurements of the Environmental status and flows

Results

Periodic flows in a baroclinic stratified fluid with a real positive frequency

$$\omega = \mathrm{Re}\,\omega = \omega_0 + \mathbf{kU}; \quad \omega_0 > 0$$

$$\mathbf{v},\, p,\, \rho \sim A \exp i\Theta \qquad \omega = \omega(\mathbf{k}),\ \omega = \omega(\mathbf{k}, A\mathbf{k}) \qquad \mathbf{k} = \mathbf{k}_1 + i\mathbf{k}_2$$

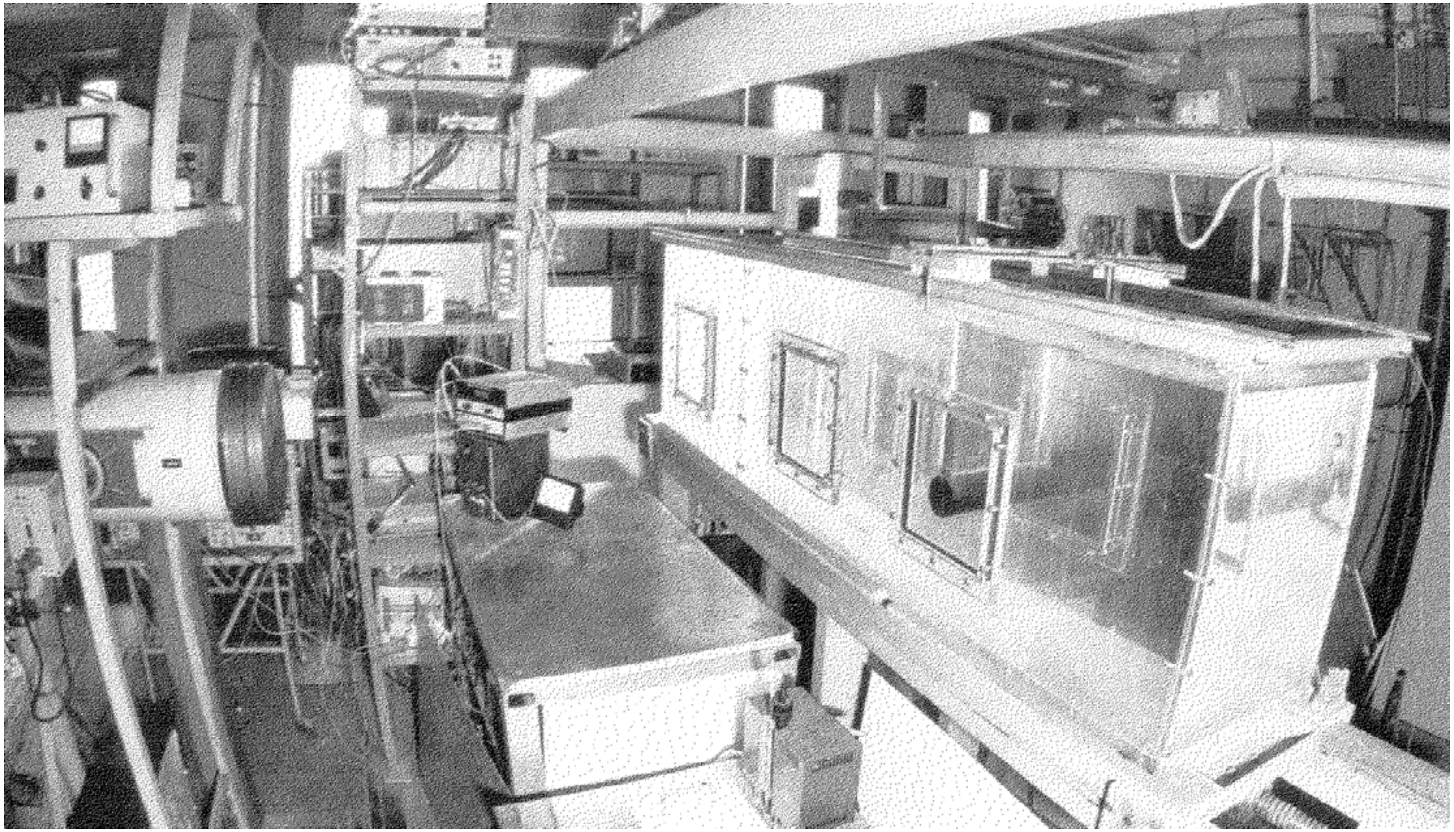
consist of regular components (waves and vortices)

$$\mathbf{v}, p, \rho \sim A \exp i\Theta, \qquad \mathrm{Re}\,\Theta \gg \mathrm{Im}\,\Theta, \qquad \mathrm{Im}\,\Theta \sim \nu^{\alpha},\ \alpha > 0$$

and singular components: thin elongated features, forming sets of boundary layers on contact surfaces and their analogues in a fluid interior (fine structure of stratification, envelopes of vortices, jets, wakes).

$$\mathbf{v}, p, \rho \sim A \exp i\Theta, \qquad \Theta \sim \sqrt{i}, \qquad \mathrm{Re}\,\Theta \sim \mathrm{Im}\,\Theta, \quad \mathrm{Im}\,\Theta \sim \nu^{-\alpha}, \alpha > 0$$

There are **two** distinguished singular on viscosity components, one is Stokes' type and second one is before unknown internal type



Лабораторный бассейн, T_b = 5-20 s.

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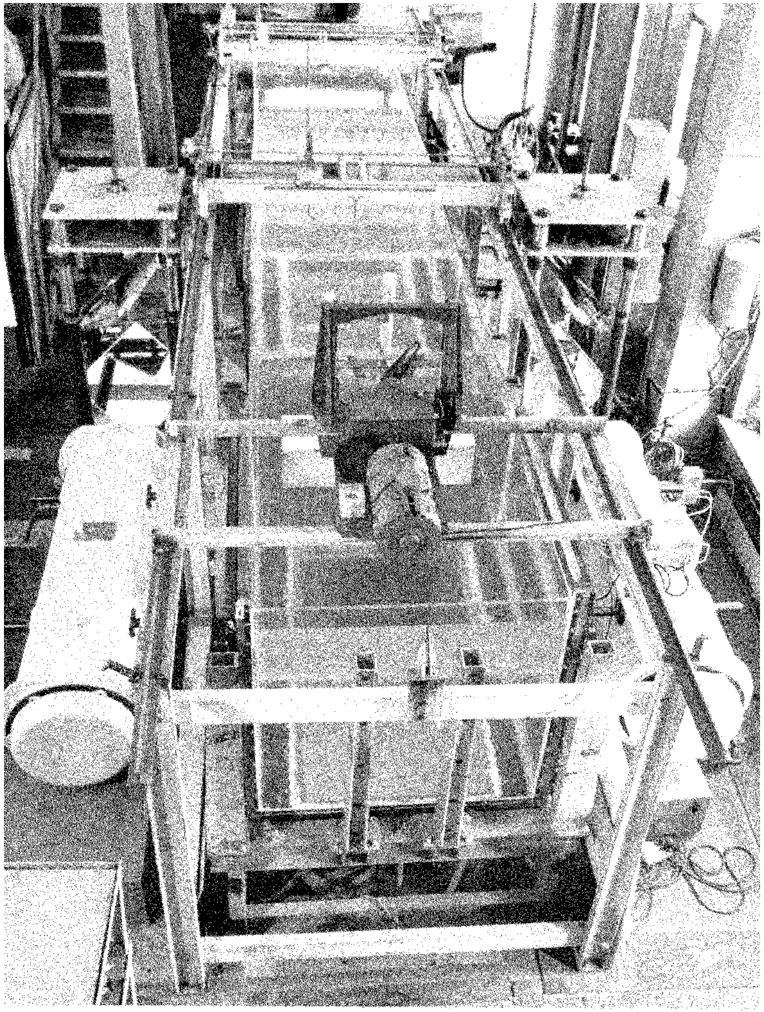

Large stratified tank $L \times W \times H = 7 \times 1.2 \times 1.2$ **cub. m.**

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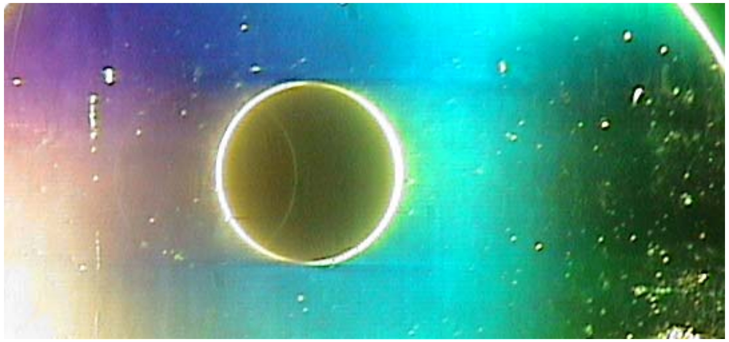

Diffusion induced flow formation on a motionless sphere submerged in exponentially stratified fluid at rest

$D = 5$ cm $\tau = 10\,000$ $T_b = 7.5$ s,

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SLOW BUOYANCY INDUCED FLOWS:
3D DIFFUSION INDUCED BOUNDARY CURRENTS

Baydulov V.G. et al. Doklady Physics. 2005. V. 50. No. 4. P. 195.bb

Classical governing equations including the Navier-Stokes Equations for a non-homogeneous fluid

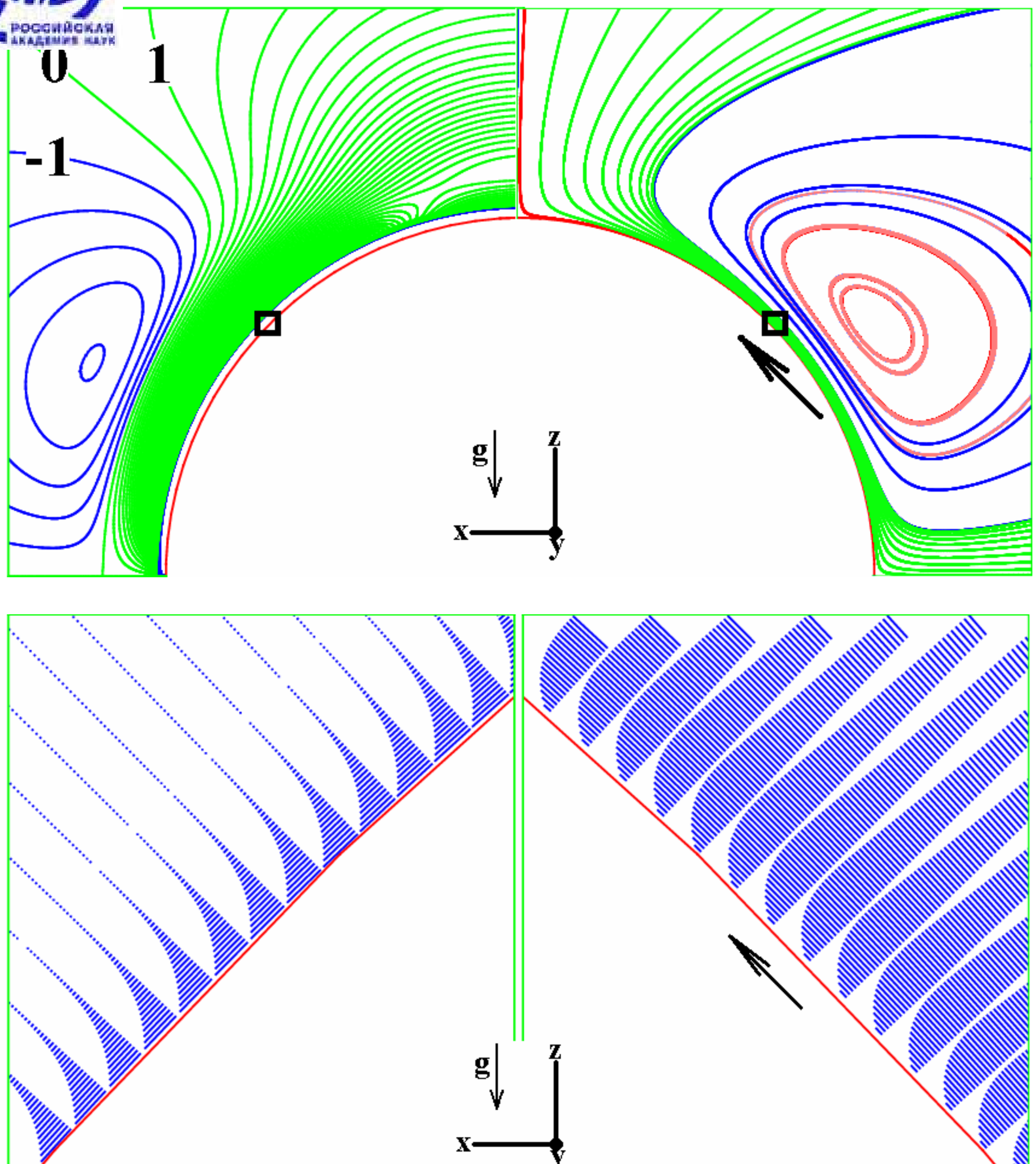
$$\rho(S)\left(\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}\nabla)\mathbf{u}\right)=-\nabla P+\nabla(\mu(S)\nabla\mathbf{u})+\rho(S)(\mathbf{g}-2\Omega\times\mathbf{u})$$

$$\frac{\partial S}{\partial t}+(\mathbf{u}\nabla)S=\nabla(\kappa_S(S)\nabla S)$$

$$\operatorname{div}\mathbf{u}=0,\quad \rho=\rho(S)$$

$$\rho=\rho_0(z)+\rho'(x,\ y,\ z,\ t)\qquad \Lambda=\left|\frac{1}{\rho_0(z)}\frac{d\rho_o(z)}{dz}\right|^{-1}\qquad N=\frac{2\pi}{T_b}=\sqrt{\frac{g}{\Lambda}}$$

\+ no-slip and no-flux boundary conditions



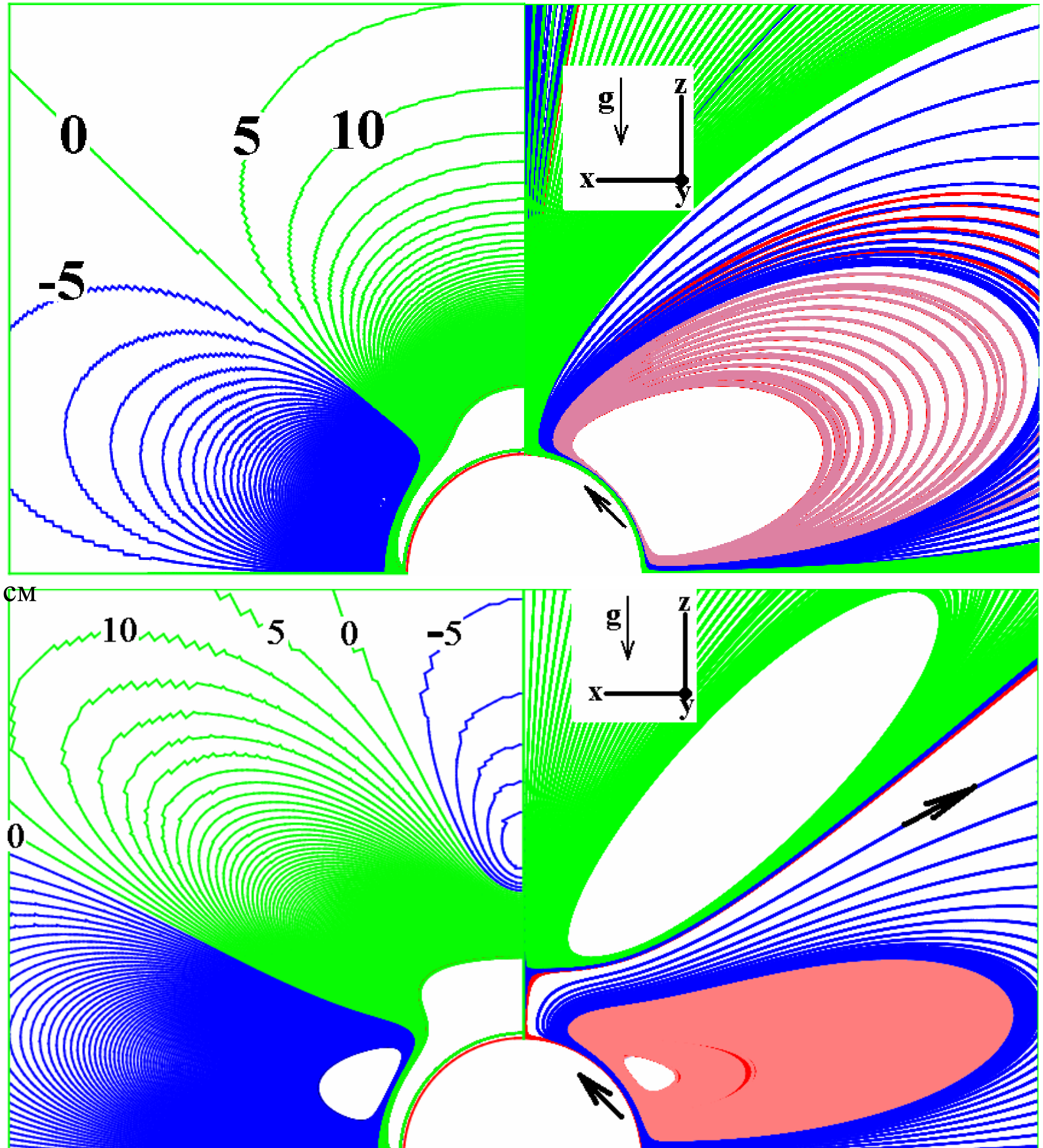
d = 2 см,
T_b = 6.34

$t = 0.5 \cdot T_b$,

42º< < 48º

mesh 1000×360×1, B_S = 5,
$h_{min} = 0.75 \cdot 10^{-3}$ см

б)

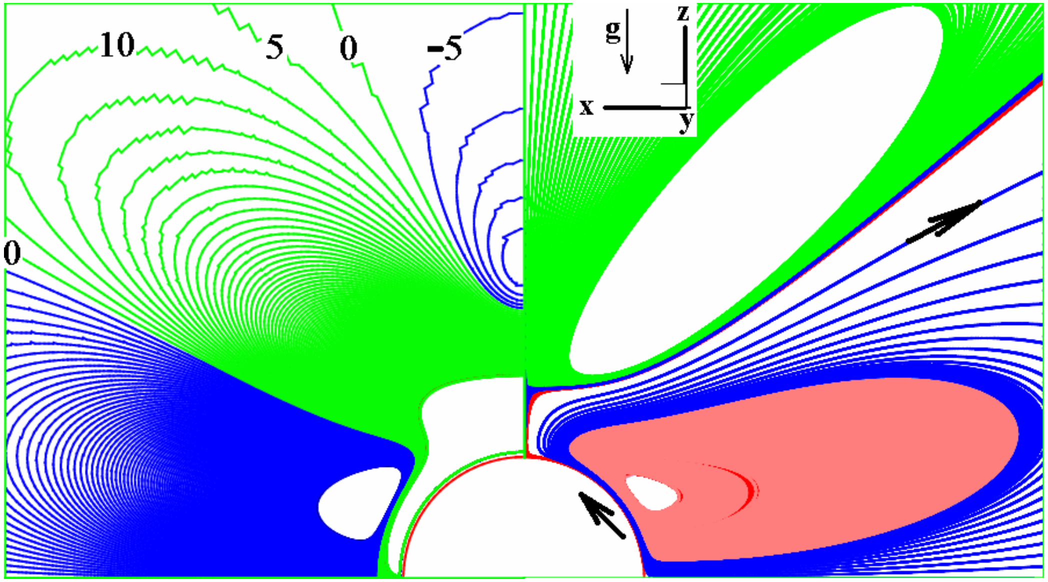


d = 2 см

d = 2 см, T_b = 6.34

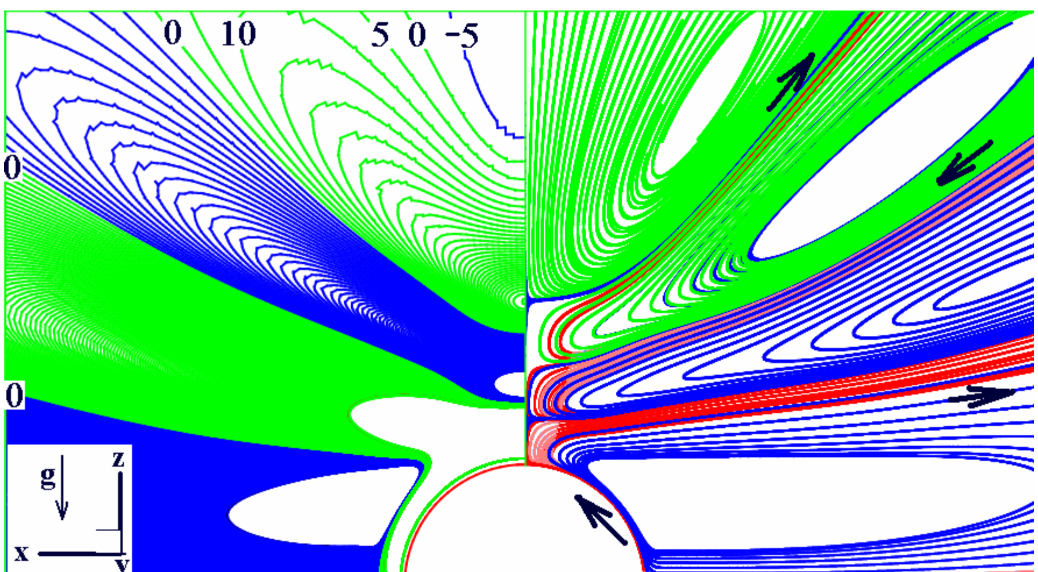
$t = 0.5 \cdot T_b$,

$t = T_b$



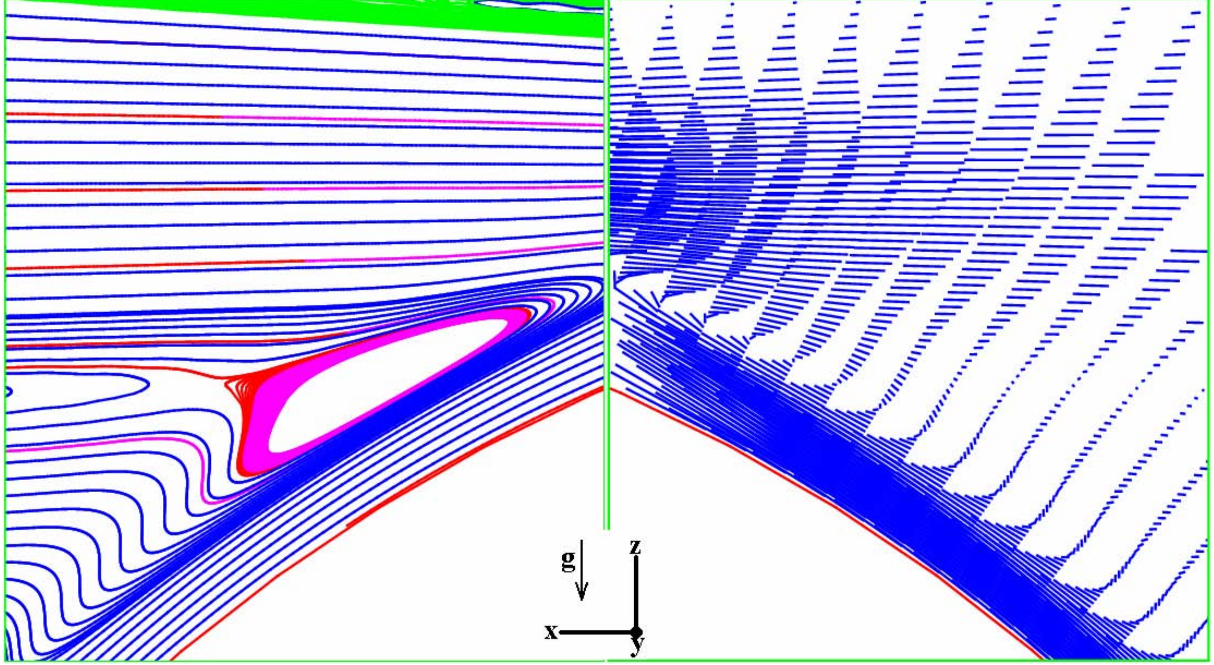
$\tau = t / T_b = 1,$

Transient internal waves and diffusion induced flow on a motionless sphere in an exponentially stratified fluid



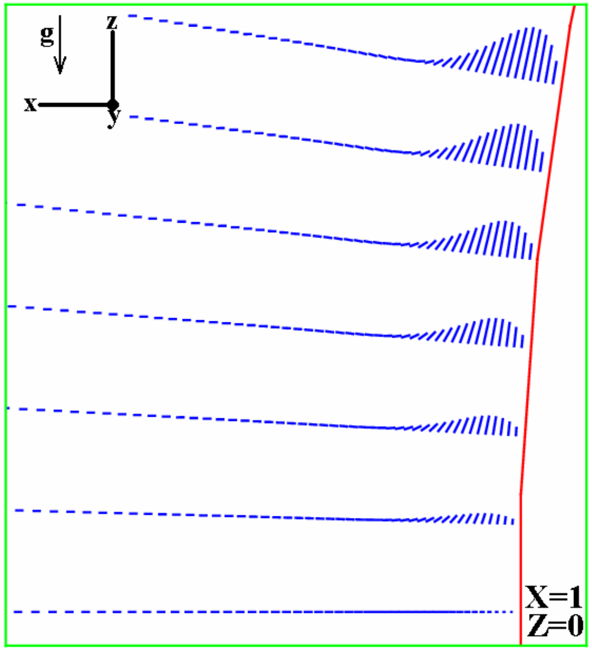
$D = 2$ ñì, $T_b = 6.2$ c,

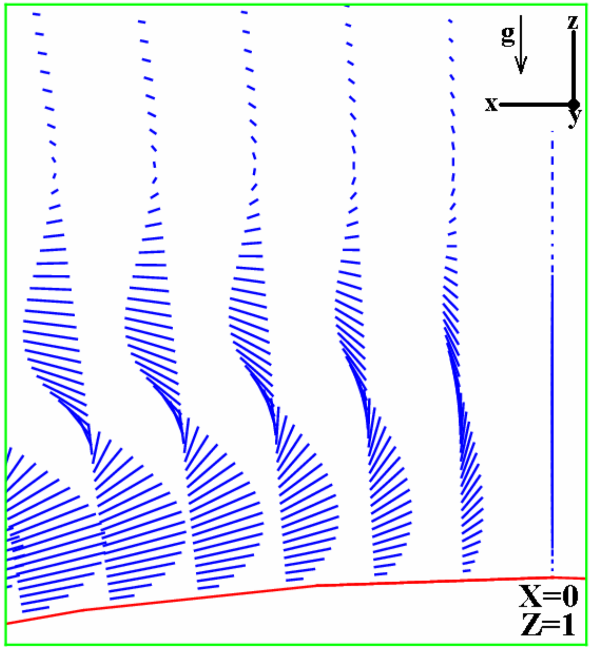
$\tau = 2$



Streamlines and velocity vectors near a vortex center

d = 4 см,
T_b = 6.34 с,
t = 1075 T

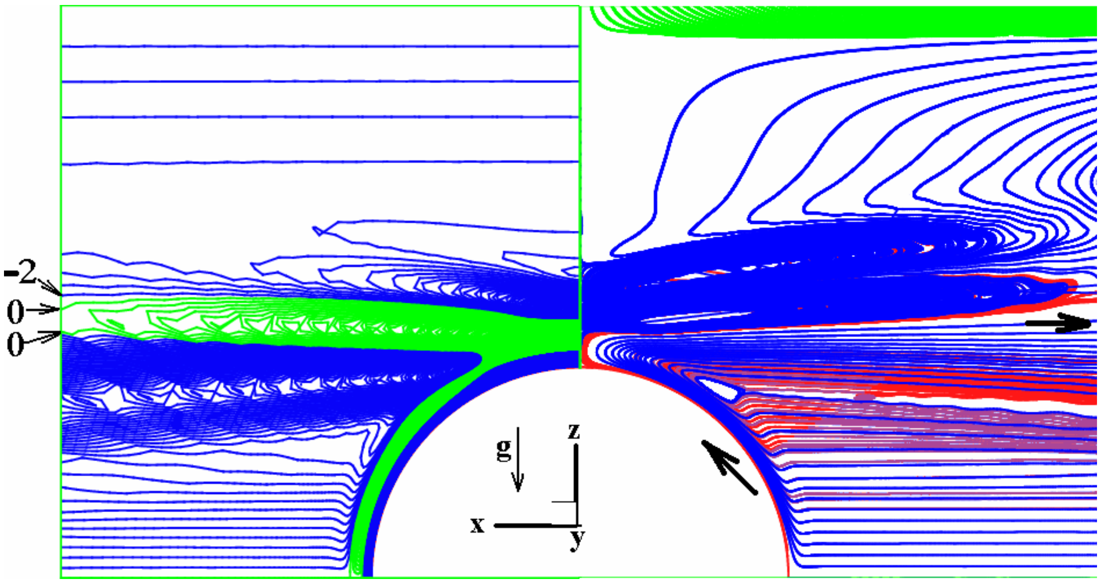




320×120×1,
B = 100,
$h_{min} = 4.48 \cdot 10^{-3}$ см.

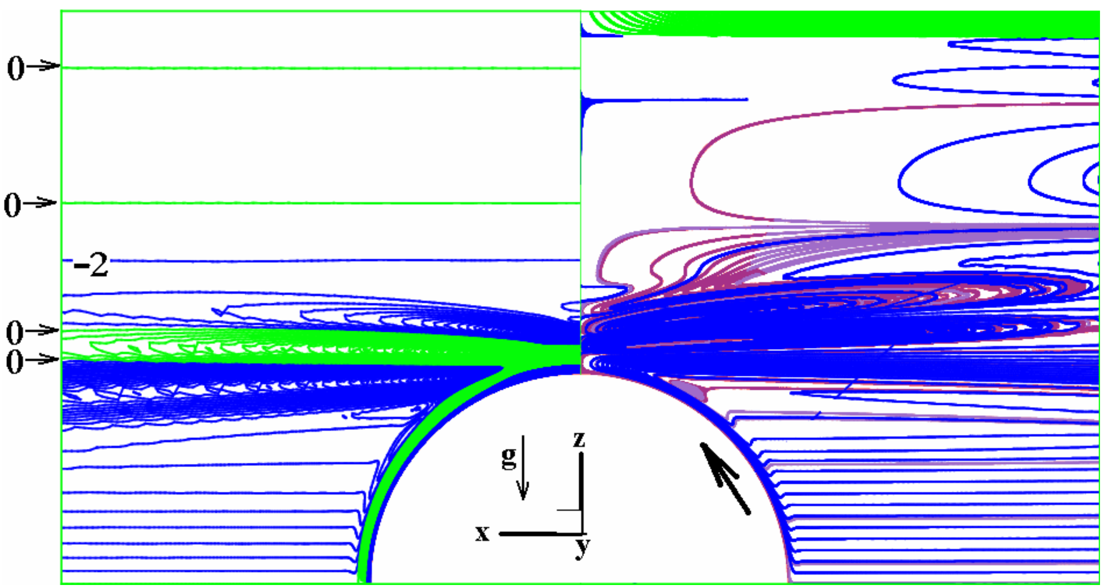
velocity vectors

near equator and upper pole



$D = 2$ ñì, $\tau = 1662$

Almost steady flow induced by a diffusion on a motionless sphere placed inside exponentially stratified fluid at rest

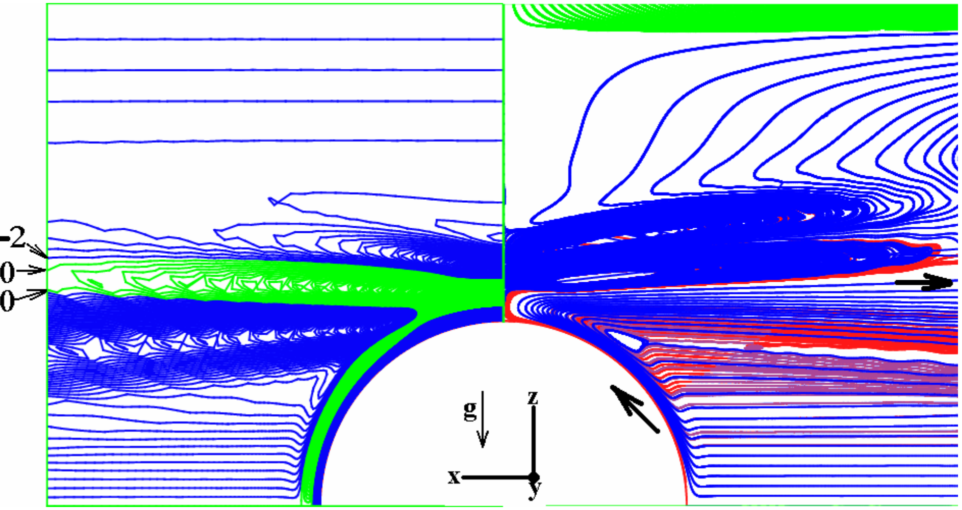


$T_b = 6.2$ c,

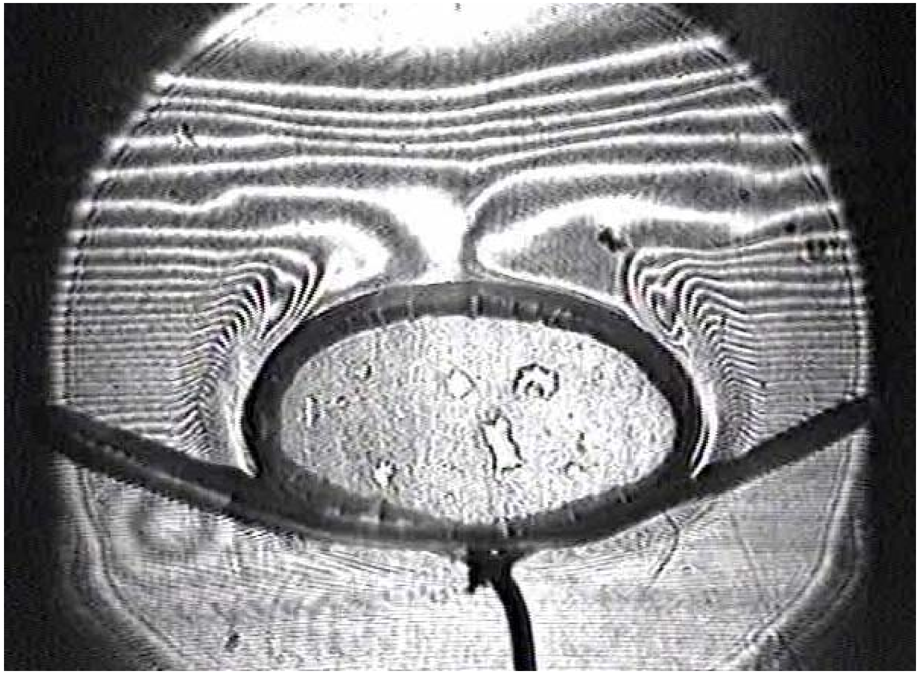
$D = 4$ ñì, $\tau = 1075$

Diffusion induced flow formation on a motionless sphere introduced in exponentially stratified fluid at rest

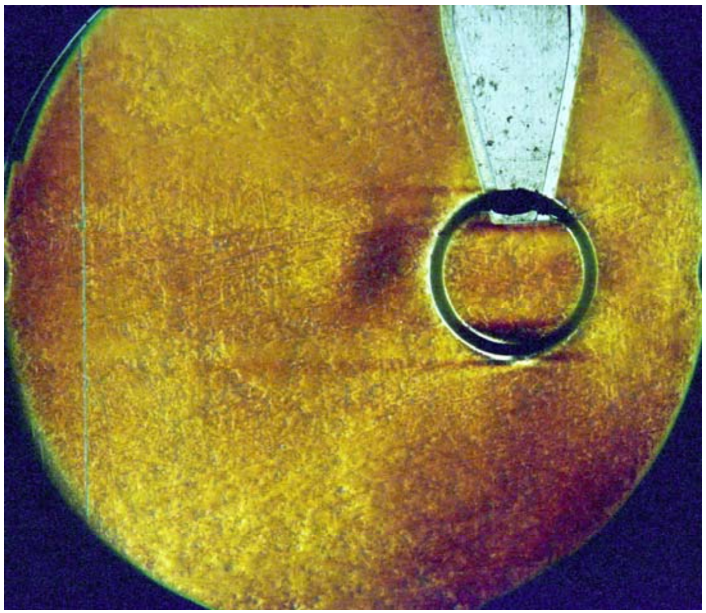
Numerical solution



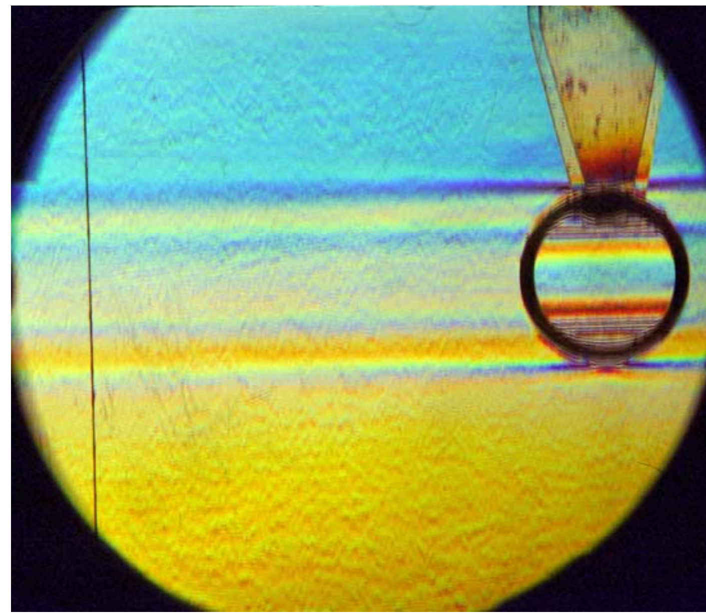
Inteferometric image of diffusion induced flow on a gas bubble in a stratified solution of acetic acid



$\Delta C = 10\%$, $D = 4.0$ mm
Kostyrev et al., Perm



$$T_b = 10.5 \text{ s}$$



$$D = 5 \text{ cm}$$

Common and 'natural rainbow' Schlieren images of diffusion induced boundary currents on the motionless cylinder in a stratified fluid at rest,

t = 48 hours.

Set of classical governing equations for a non-homogeneous fluid including the Navier-Stokes Equations is singular disturbed problems

$$\rho(z)\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\mathbf{u}\right) = -\nabla P + \nabla(\mu(S)\nabla \mathbf{u}) + \rho(z)\mathbf{g}$$

$$\frac{\partial S}{\partial t} + (\mathbf{u}\nabla)S = \nabla(\kappa_S(S)\nabla S)$$

$$\text{div}\ \mathbf{u} = 0, \quad \rho = \rho(S)$$

$$\rho = \rho_0(z) + \rho'(x, y, z, t) \qquad \Lambda = \left|\frac{1}{\rho_0(z)}\frac{d\rho_o(z)}{dz}\right|^{-1} \qquad N = \frac{2\pi}{T_b} = \sqrt{\frac{g}{\Lambda}}$$

+ no-slip and no-flux boundary conditions

Set of its solutions contains regular $\mathrm{Re}\,F, \mathrm{Im}\,F \sim (\nu, \kappa_s)^{\alpha}, \ \alpha > 0$ and singular $\mathrm{Re}\,F, \mathrm{Im}\,F \sim (\nu, \kappa_s)^{-\beta}, \ \beta > 0$ on dissipative factors terms

Doppler effect $\omega = \omega_o + \mathbf{k} \cdot \mathbf{U},$

Intrinsic scales

Density $\rho = \rho_0 \exp(-z/\Lambda)$

Regular: scale of stratification $\Lambda = (d \ln \rho / dz)^{-1}$,

internal wave length $\lambda = UT_b$

Size of an obstacle d,

Singular: $\delta_N = \sqrt{2\nu / N}$ $\delta_\rho = \sqrt{2\kappa_s / N}$ $\delta_u = \nu/U$

$$\Lambda >> d >> \delta_\nu, \quad \Lambda >> L_\nu >> \delta_\nu \qquad \delta_\nu >> \delta_\rho$$

Dimensionless parameters – ratios of intrinsic scales: Re, Fr, C, Ar, St, (Pe, Sc)

$$\mathrm{Re} = d/\delta_\nu = U_0 d/\nu, \quad Fr = \lambda/2\pi d = U_0/Nd$$

$$C_\Lambda = \Lambda/d$$

Infinitesimal periodic motions $\omega = const$

For Euler's equations

$$\omega^2 k^2 - N^2 (k_x^2 + k_y^2) = 0$$

- 2-nd order dispersion equation

For Navier-Stokes stratified fluid equations

$$\left[(\omega - i\kappa_s \Delta)(\omega - i\nu\Delta)\Delta - N^2 \Delta_\perp\right]\Phi = 0, \quad (\omega - i\nu\Delta)\Psi = 0,$$

$$\left[\left(\omega + i\nu k^2\right)k^2 - N^2 (k_x^2 + k_y^2)\right]\left(\omega + i\nu k^2\right) = 0$$

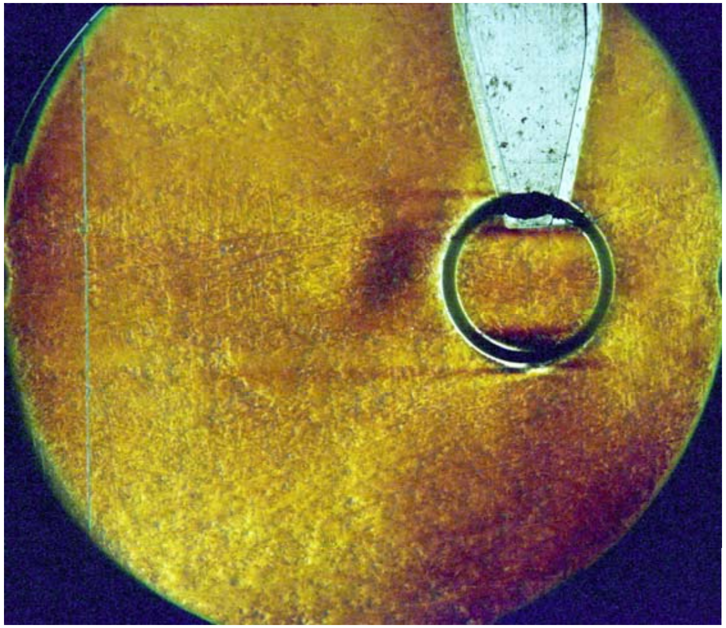
dispersion equation of 6-th order

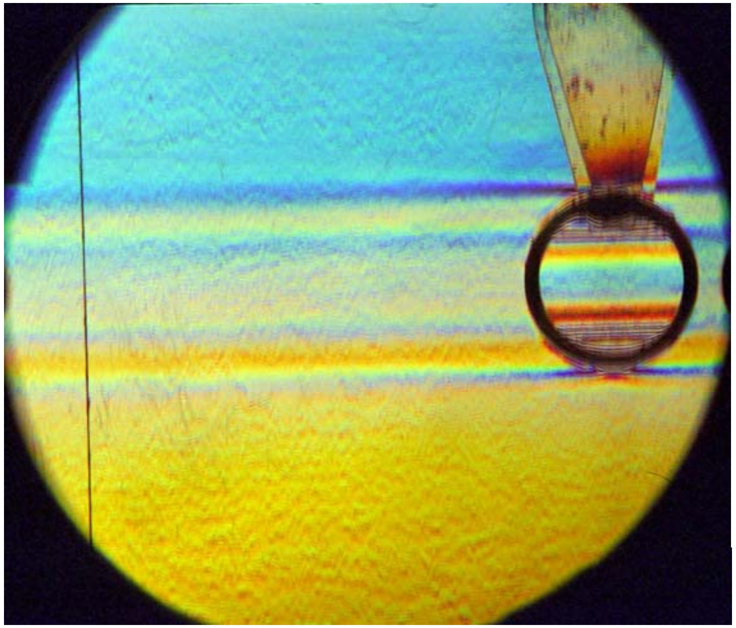
NSE + diffusion effects: dispersion equation of 8-th order

$$\left[(\omega - i\kappa_s \Delta)(\omega - i\nu\Delta)\Delta - N^2 \Delta_\perp\right]\Phi = 0, \quad (\omega - i\nu\Delta)\Psi = 0,$$

$$\left[\left(\omega + i\kappa_s k^2\right)\left(\omega + i\nu k^2\right)k^2 - N^2 (k_x^2 + k_y^2)\right]\left(\omega + i\nu k^2\right) = 0$$

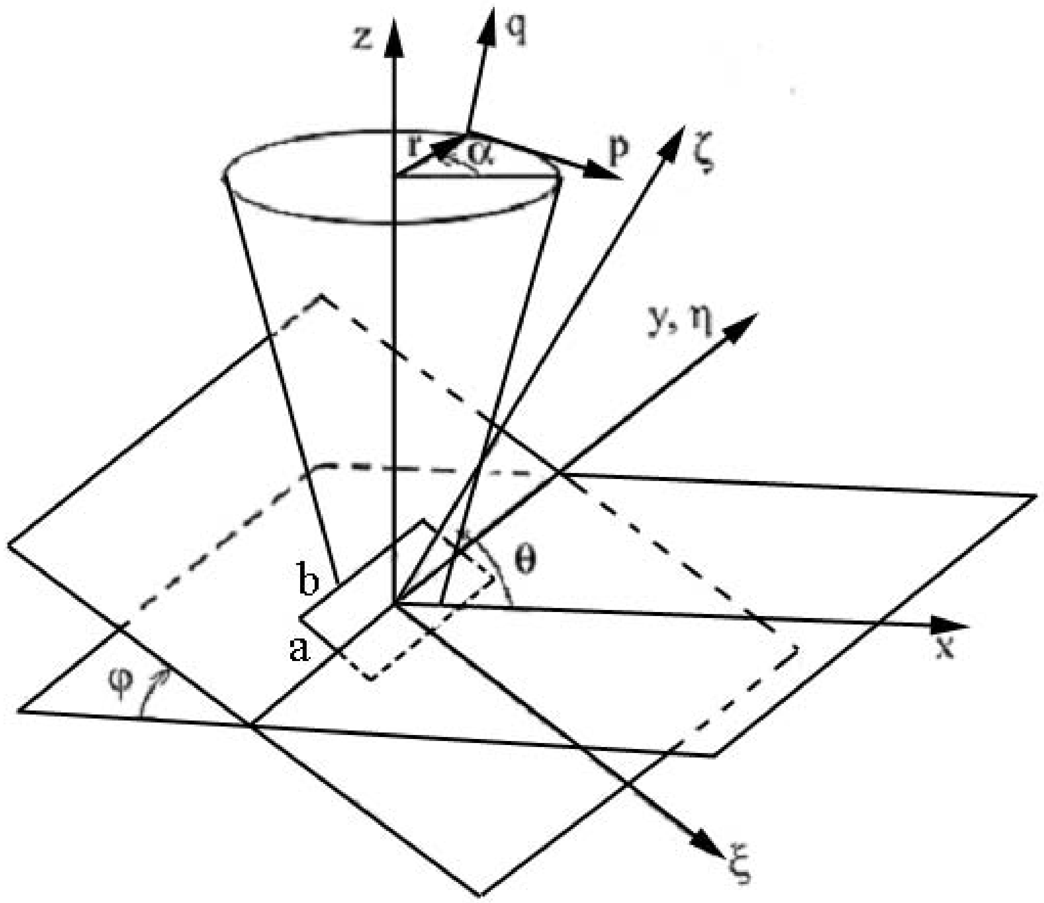
Vasiliev A., Chashechkin Yu. J. applied mechanics and technical physics. 2006 **47**(3), 314.





$D = 5$ cm $\quad \tau = 10\,000 \quad T_b = 7.5\,\text{s},$

Diffusion induced flow formation on a motionless cylinder introduced in exponentially stratified fluid at rest



Coordinate frame

Expansions for Φ, Ψ, S

$$\Phi = e^{-i\omega t} \sum_{j=1}^{3} \int_{-\infty}^{+\infty} A_j(k_\xi, k_\eta) \exp\left(ik_j\zeta + ik_\xi\xi + ik_\eta\eta\right) dk_\xi dk_\eta$$

$$\Psi = e^{-i\omega t} \int_{-\infty}^{+\infty} B(k_\xi, k_\eta) \exp\left(ik_4\zeta + ik_\xi\xi + ik_\eta\eta\right) dk_\xi dk_\eta$$

$$S = -\frac{\rho_0}{\Lambda} e^{-i\omega t} \sum_{j=1}^{3} \int_{-\infty}^{+\infty} \frac{\left(k_\xi \cos\varphi - k_j \sin\varphi\right)^2 + k_\eta^{\ 2}}{i\omega - D\left(k^2 + k_j^2\right)} A_j(k_\xi, k_\eta) \times$$

$$\times \exp\left(ik_j\zeta + ik_\xi\xi + ik_\eta\eta\right) dk_\xi dk_\eta$$

Coefficients are defined from boundary conditions

Wave numbers are solutions of the dispersion equation

Dispersion relation of the 8-th order

$$\omega = \text{const}$$

$$\left[\nu D\left(k^2 + k_j{}^2\right)^3 - i\omega(\nu + D)\left(k^2 + k_j{}^2\right)^2 - \omega^2\left(k^2 + k_j{}^2\right)\right.$$
$$\left. + N^2\left[\left(k_\xi \cos\varphi - k_j \sin\varphi\right)^2 + k_\eta{}^2\right]\right]\left(k_j{}^2 + \frac{\omega}{i\nu} + k^2\right) = 0$$

$$\left(k_j{}^2 + \frac{\omega}{i\nu} + k^2\right) = 0, \quad k_j = \pm\sqrt{\frac{\omega}{i\nu} + k^2}$$

Stokes solution

Vasiliev A. Yu, Chashechkin Yu.D. Fluid Dynamics, 2007 (In press)

Internal boundary layer

$$k_2 = \frac{i + \operatorname{sign}\mu}{\delta_\varphi} - \frac{k_\xi \sin 2\varphi}{2\mu}$$

thickness

$$\delta_\varphi = \delta_N \sqrt{2\sin\theta / |\mu|} = f_{in}(\theta, \varphi)\,\delta_N$$

Stokes' boundary layer

$$k_3 = \frac{i+1}{\delta_\nu}$$

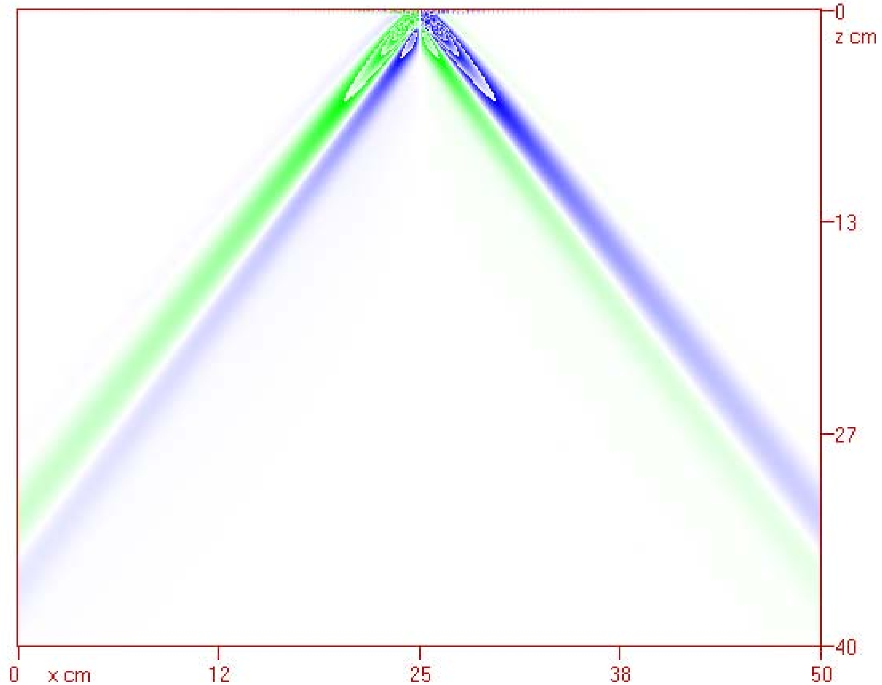
thickness

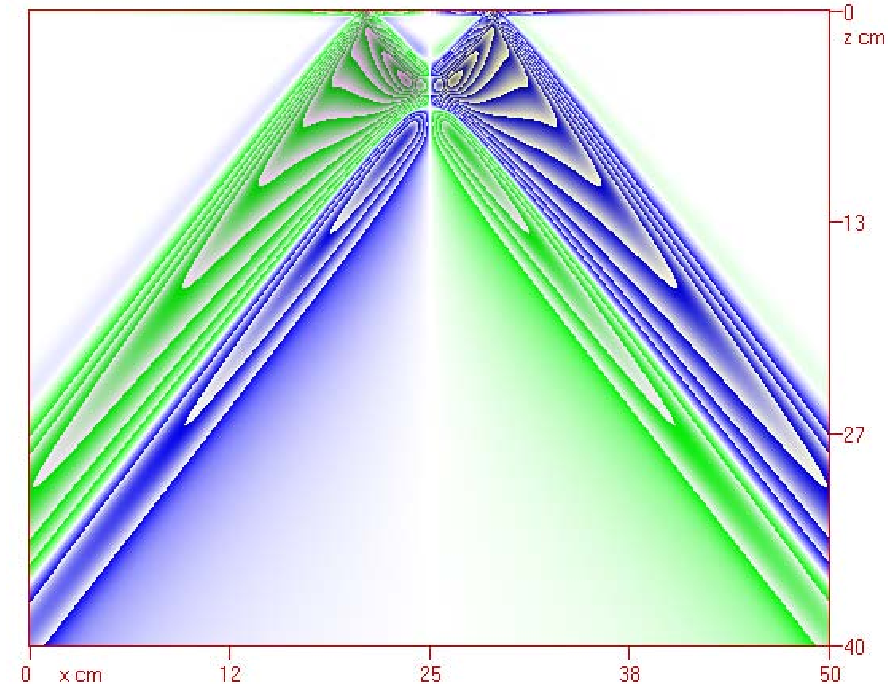
$$\delta_\nu = \sqrt{2\nu/\omega} = \delta_N \sqrt{2/\sin\theta} = f_{st}(\theta)\,\delta_N$$

$$\mu = \sin^2\varphi - \sin^2\theta \qquad \delta_N = \sqrt{\nu / N}$$

1. Chashechkin Yu.D., Vasiliev A.Yu.. Doklady Physics. 2004. **49(** 2).. P. 122..

Horizontal component of velocity
Sources are horizontal disks of different radius



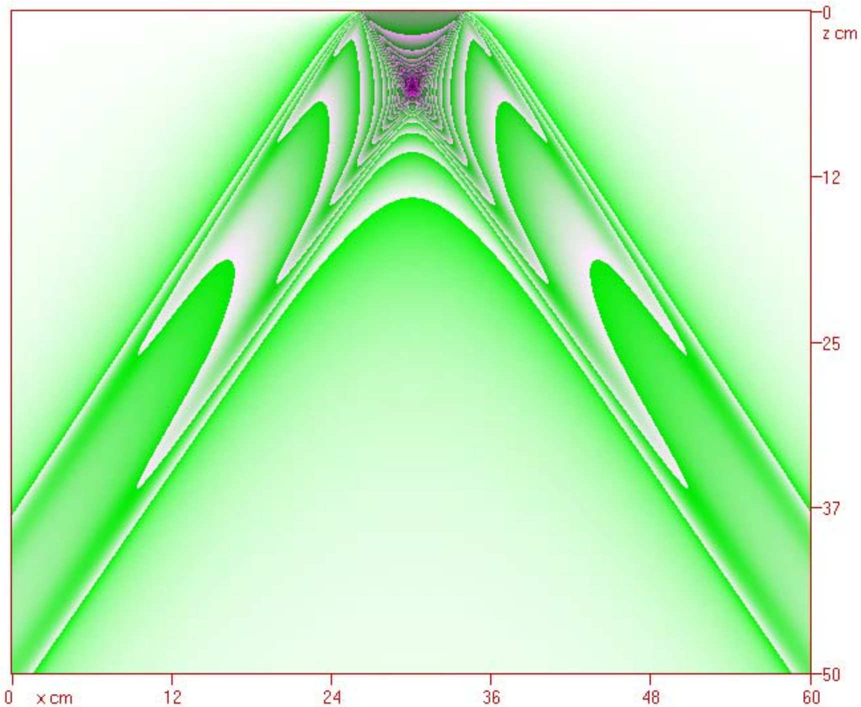


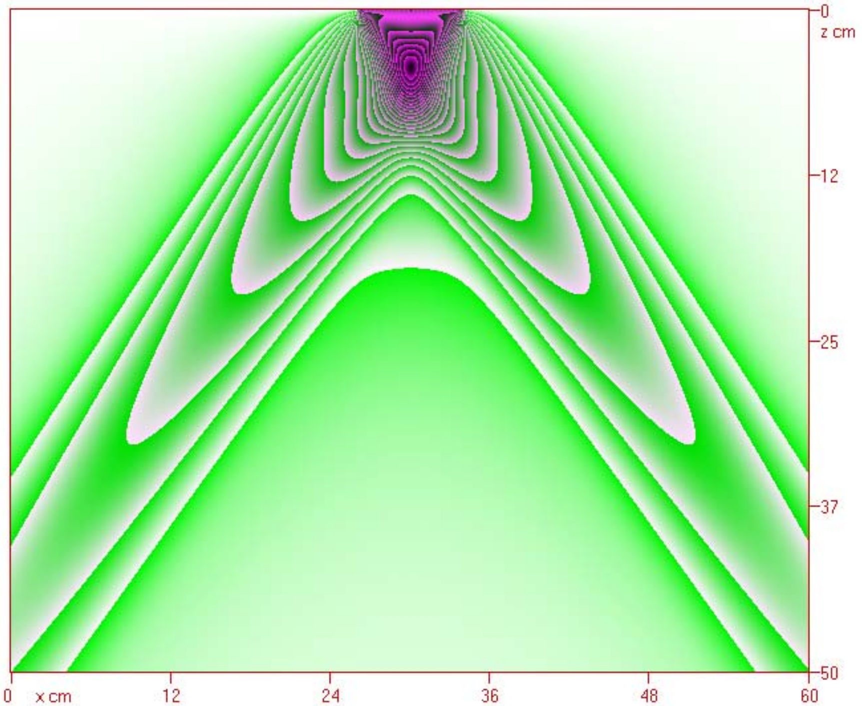
R = 0.1 cm < L_ν

R = 4 cm > L_ν

$L_\nu = \sqrt[3]{g\nu}/N$ N=1.2 s^{-1} $\omega = 1.0\ s^{-1}$ u = 0.25 cm/s

Modulus of vertical component of velocity (different viscosity)





$$\nu = 0.001\ \mathrm{cm}^2 / s$$

$$\nu = 0.4\ \mathrm{cm}^2 / s$$

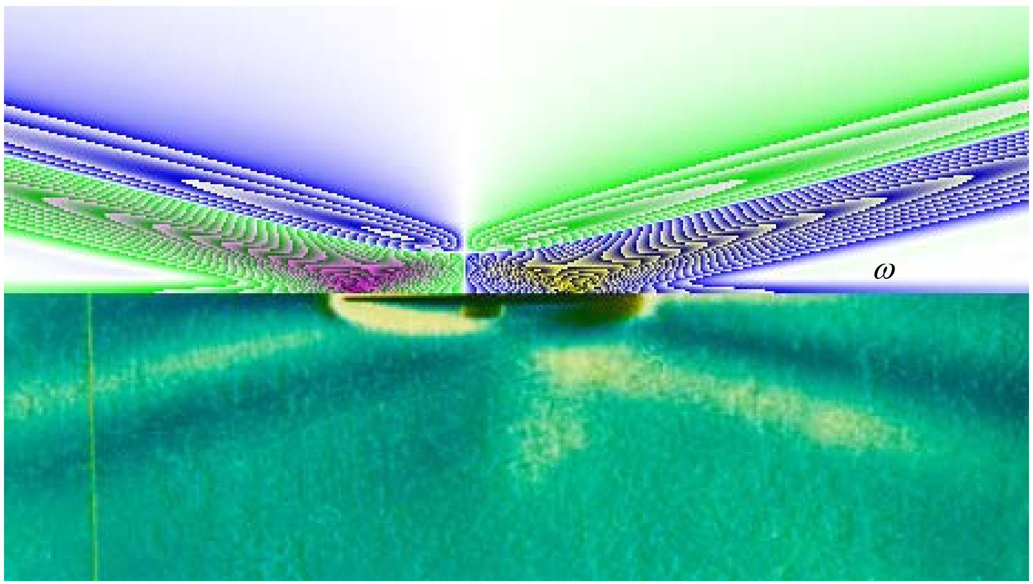
N=1.2 s^{-1}

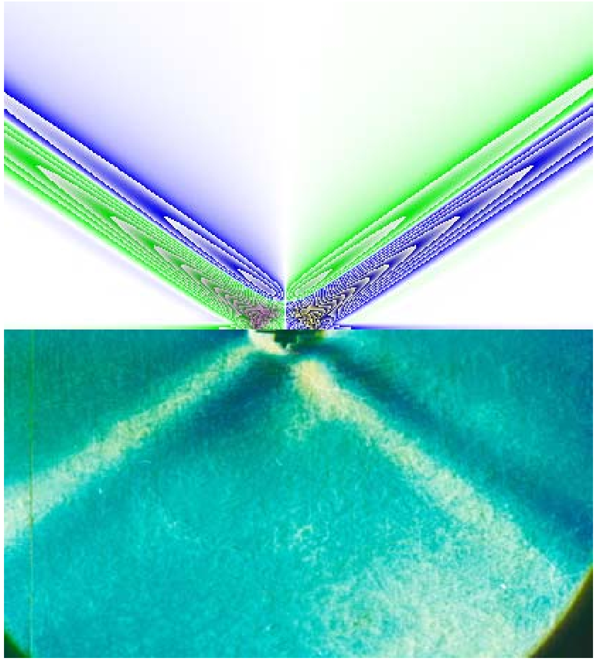
R=4 cm

$\omega = 1.0\ s^{-1}$

u = 0.25 cm/s

Calculated and observed periodic internal waves





$T_b = 6.4\ s,$ $R = 5.0\ \text{cm}$

$\omega = 0.3\ \text{s}^{-1},$ $\omega / N = 0.3,$

$T_b = 6.4\ s,\ R = 2.0\ \text{cm}$

$\omega = 0.6\ \text{s}^{-1},\ \omega / N = 0.6,$

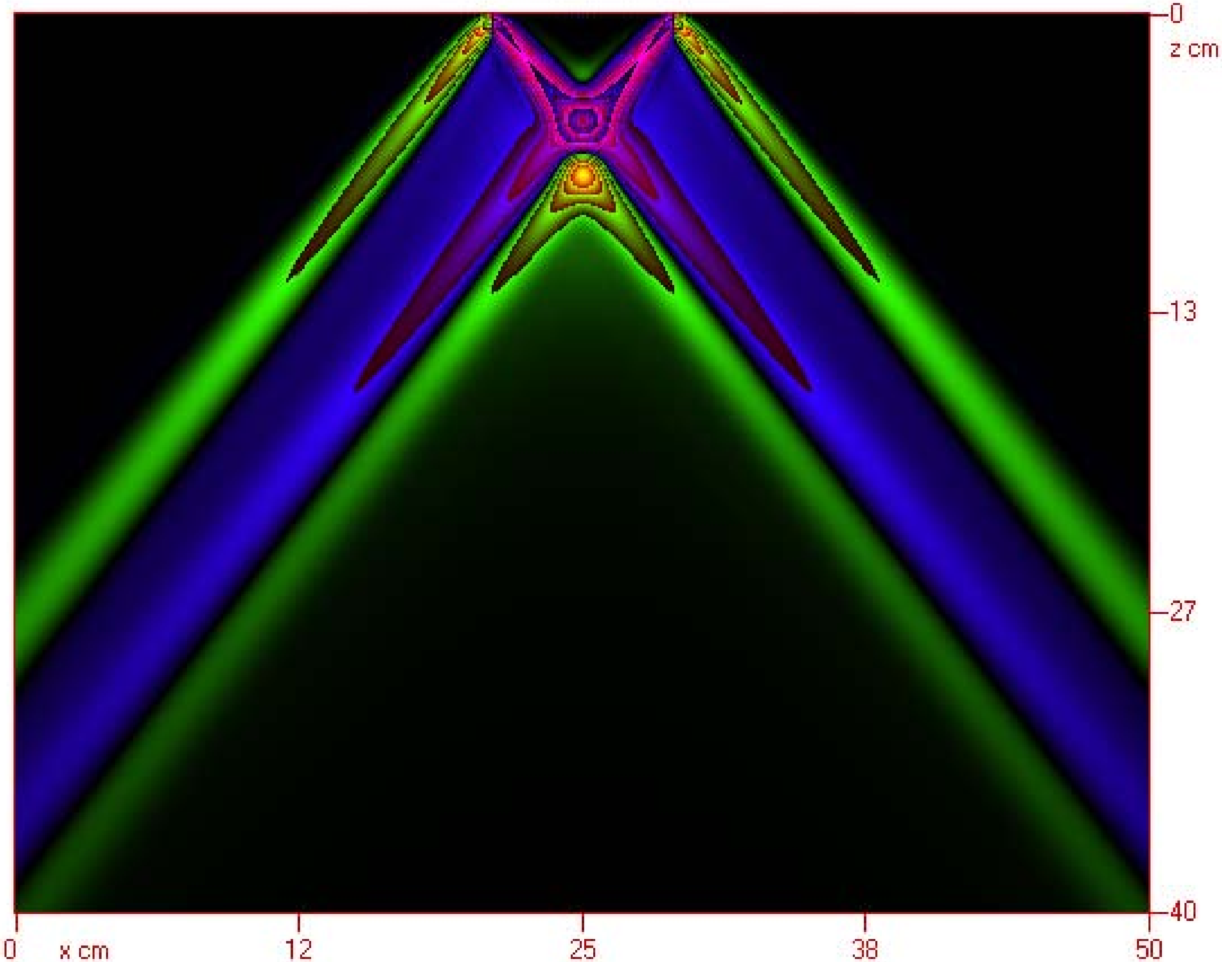
Fine envelope of 3D bi-modal wave beam are twinkled interior singular components

$N = 1.2\ s^{-1}$

$R = 4$ cm

$\omega = 1.0\ s^{-1}$

$u = 0.25$ cm/s



Differential analyzer – pattern of velocity gradients $d\,\mathrm{v}/dr$

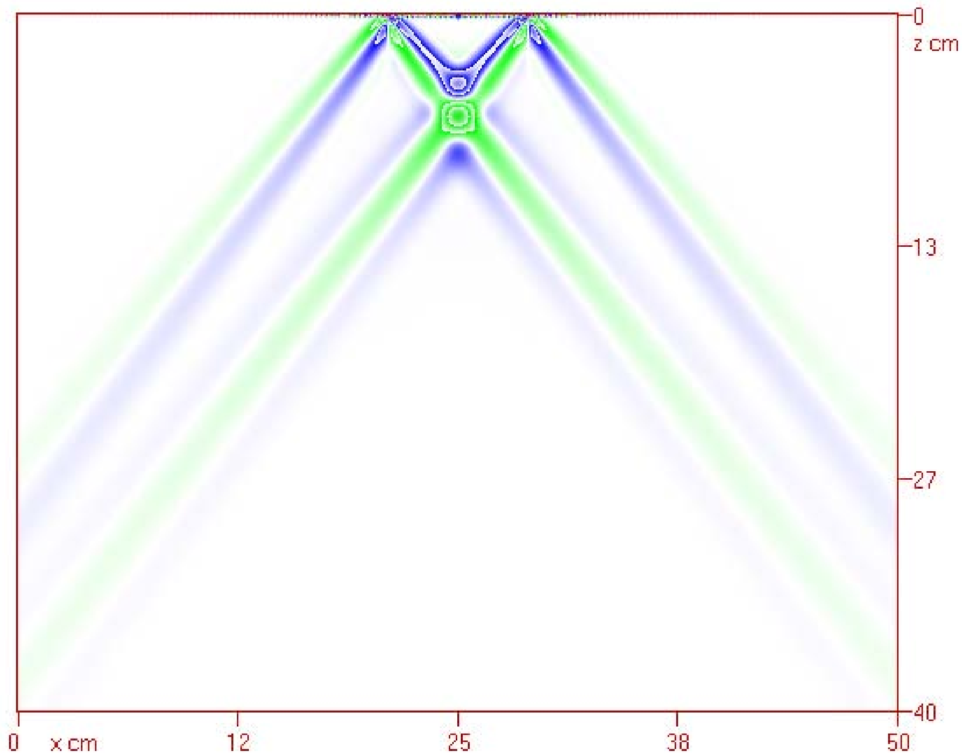
Fine envelope of 3D bi-modal wave beam are twinkled interior singular components

$N=1.2\ s^{-1}$

$R=4$ cm

$\omega = 1.0\ s^{-1}$

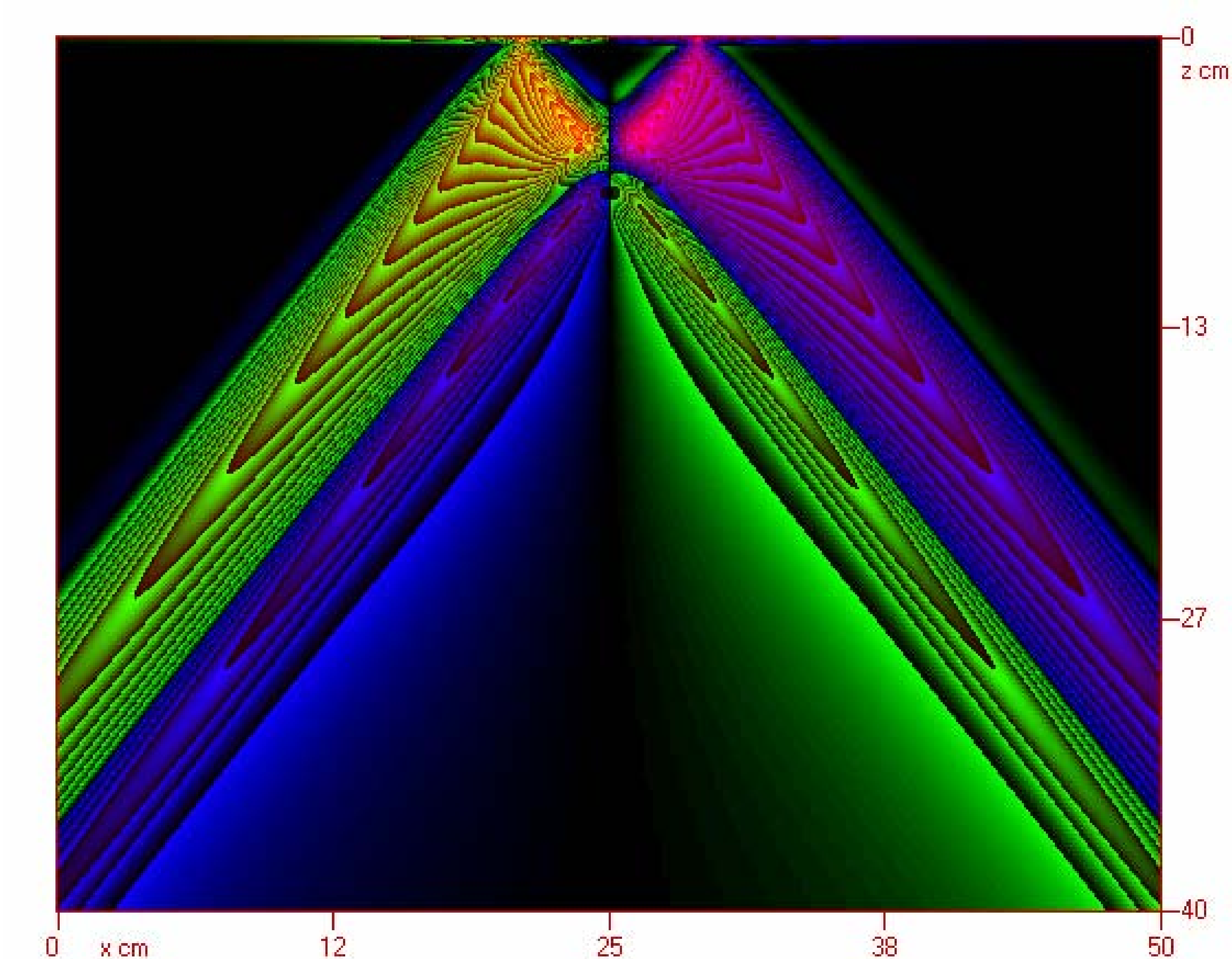
$u=0.25$ cm/s

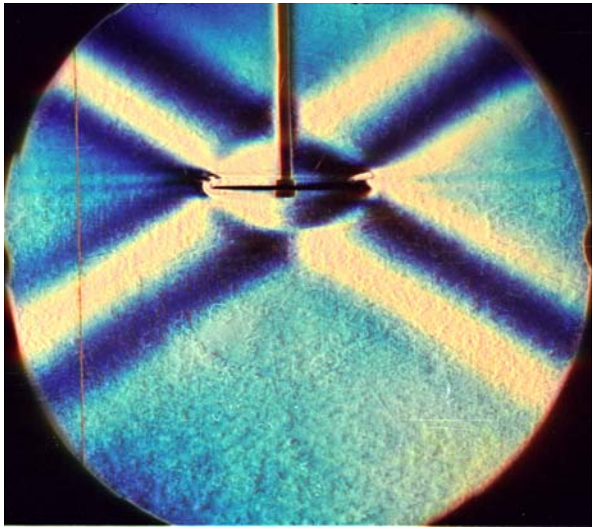


Differential analyzer – pattern of velocity second derivative $d^2 v / dr^2$

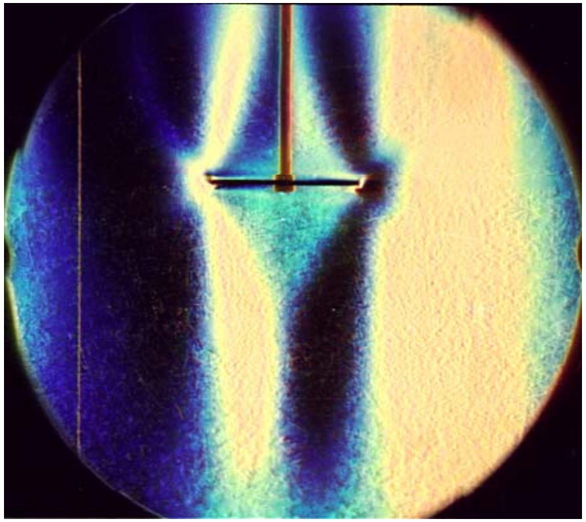
Chashechkin Yu.D. et al. Doklady Earth Sciences. 2004. **397A**.(6). P. 816

Variations of horizontal component of velocity

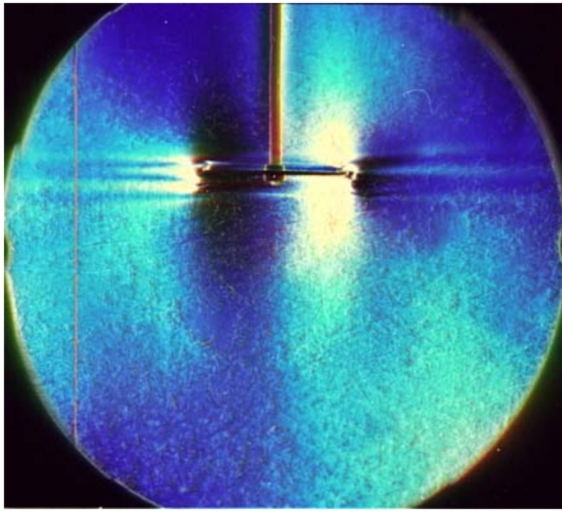




$\omega = 0.48\ \mathrm{s}^{-1}, \quad \omega / N = 0.56$



$\omega = N = 0.86\ \mathrm{s}^{-1}$

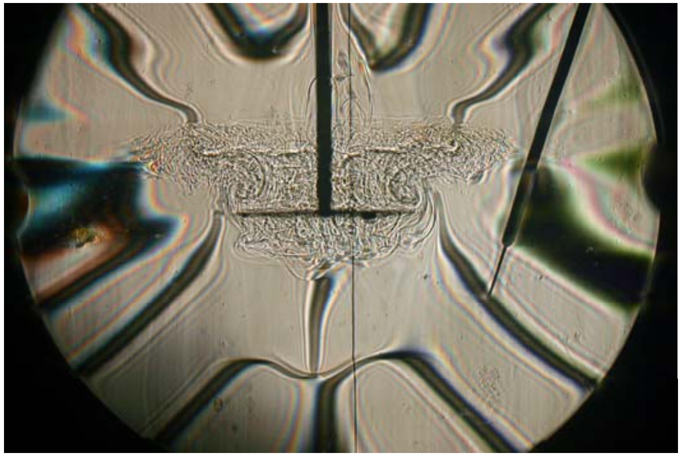


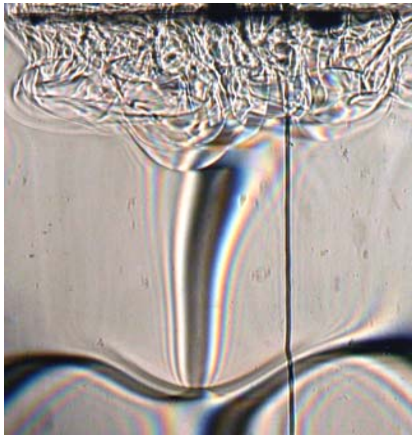
$\omega = 1.2\ \mathrm{s}^{-1}, \quad \omega / N = 1.4$

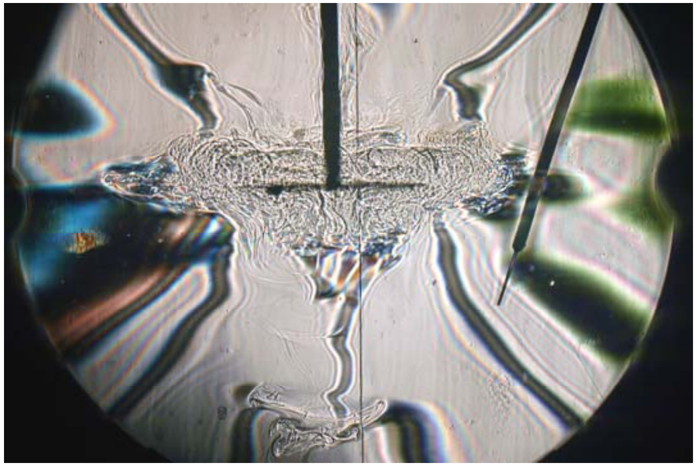
Chashechkin Yu.D. Visualization of singular components of periodic motions in a continuously stratified fluid (Review report) // Journal of Visualization 2007. V. 10. No. 1. P. 17-20.

Yuli D. Chashechkin

Singular envelopes and auto cumulative jets in the periodic internal wave cone

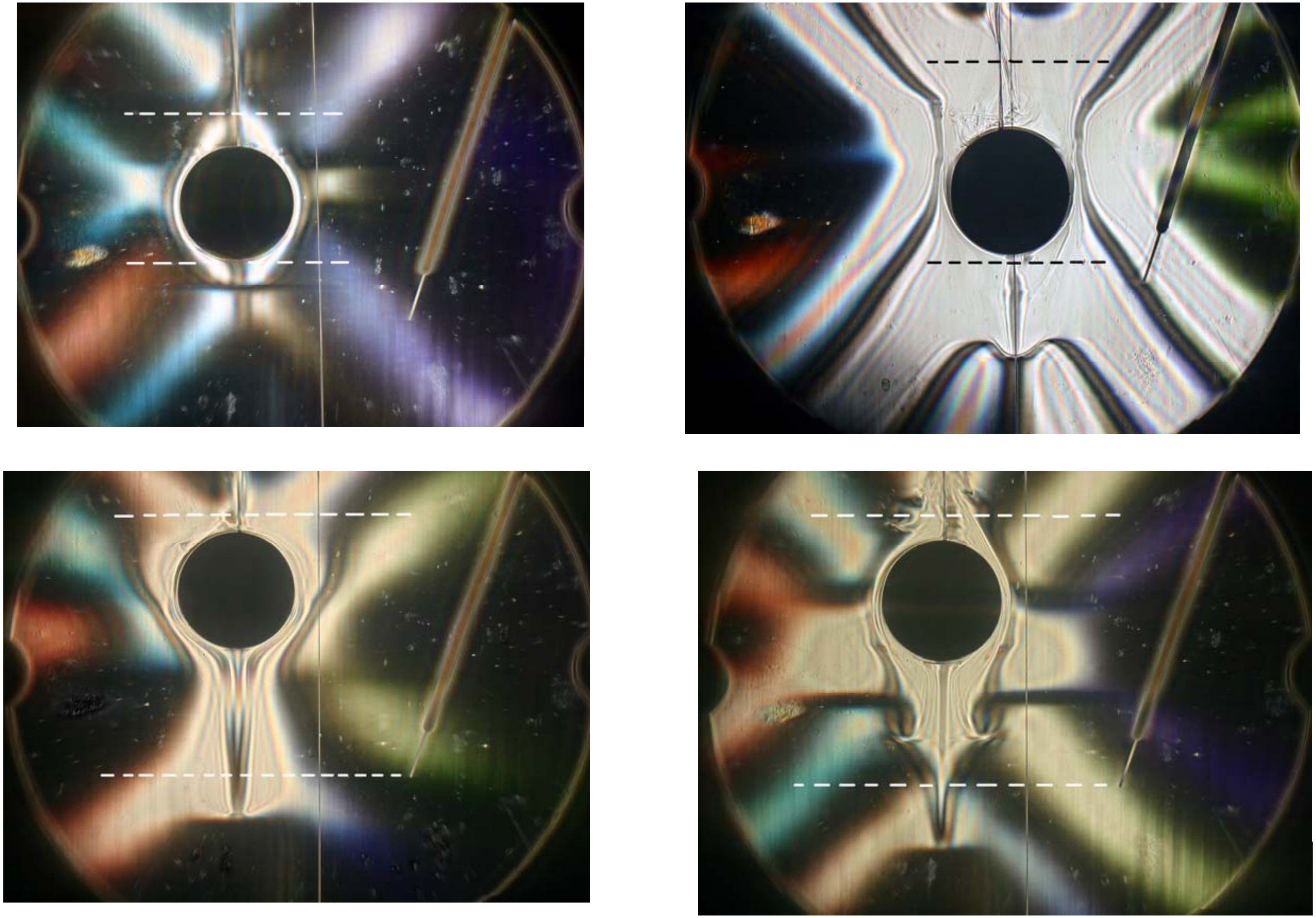






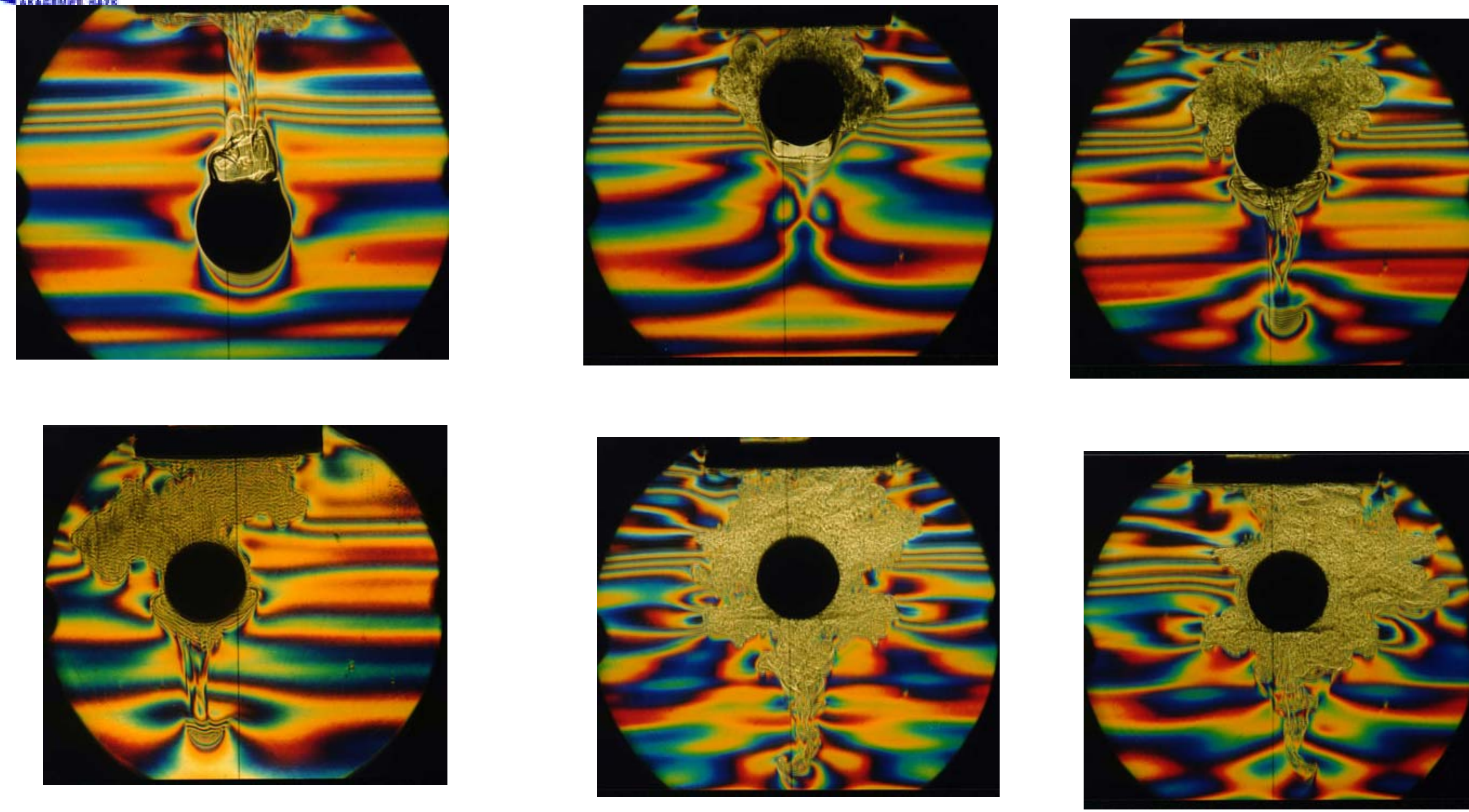
Chashechkin Yu.D. J. of Visualization 2007. V. 10. No. 1. P. 17-20.

Formation of singular singular components inside the periodic wave cone with increasing amplitude of the the source oscillations



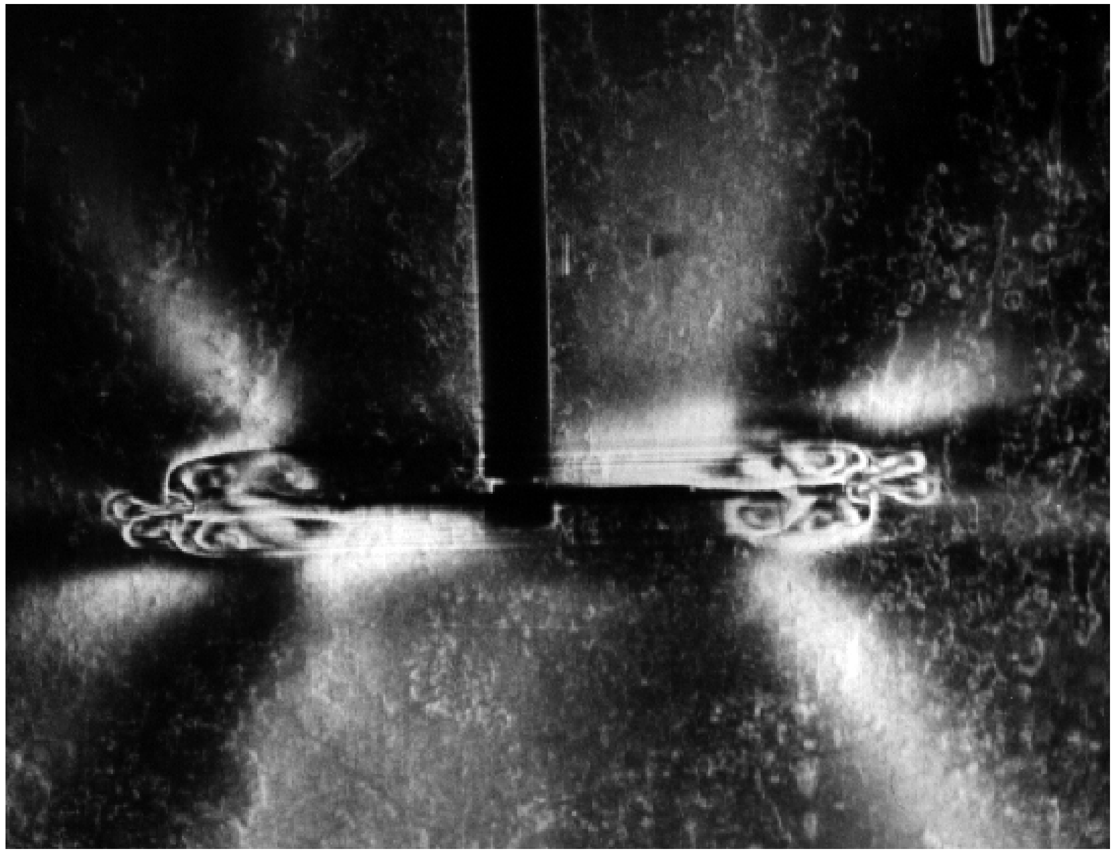
$T_b = 11.2\ s$ $D = 4.5$ cm $A = 2.7$ cm $\omega / N = 0.68$

Singular envelopes and auto cumulative jets in the periodic internal wave cone



Colour images of “natural rainbow” schlieren image of the flow with auto cumulative jets around sphere (D = 4.5 cm, T_b = 7,9 c, H = 12 cm): *a*, *b*) – sinking and first rising of the sphere; *c*, *d*) and *e*, *f*) – forming the first and decaying the second jets

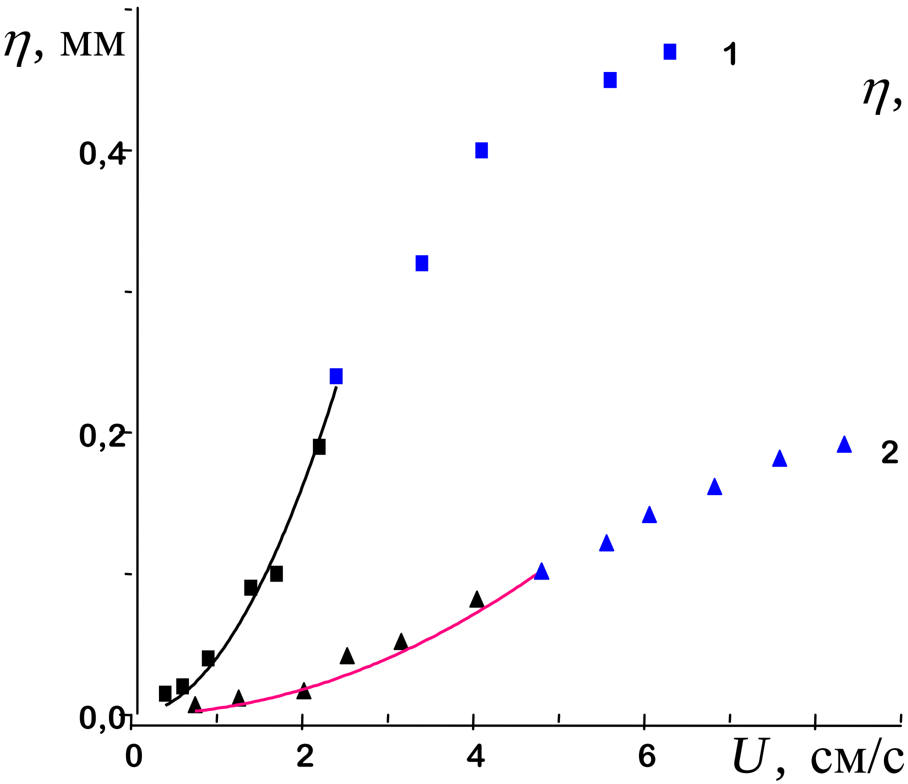
Internal waves produced by torsion oscillations of the disc

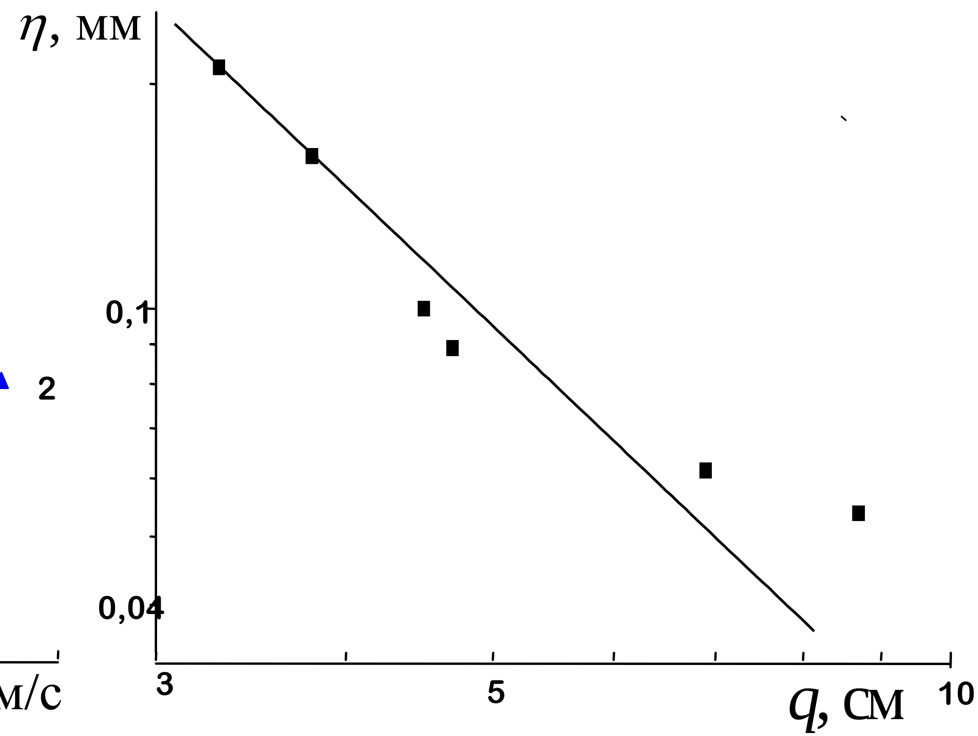


$$\omega_W = 2\Omega_{osc}$$

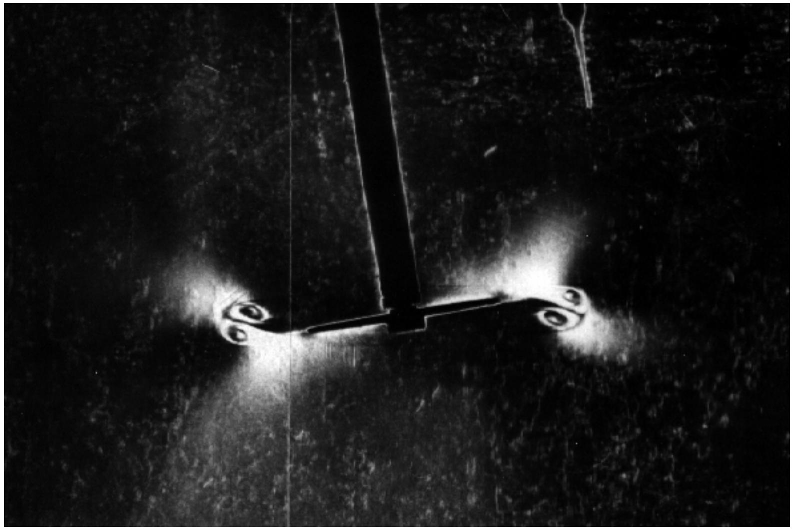
Il'inykh Yu.S., Chashechkin Yu.D. Generation of periodic motions by a disk performing torsional oscillations in a viscous continuously stratified fluid // Fluid Dynamics. 2004. 39(1).

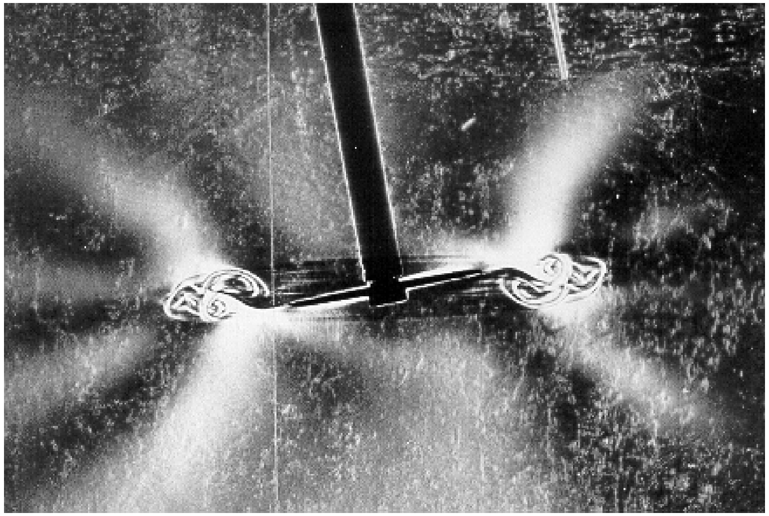
Wave amplitudes in the centre of beam





Il'inykh Yu.S., Chashechkin Yu.D. Fluid Dynamics. 2004. **39**(1). 148.





$$\omega_W = 2\Omega_{osc}$$

Il'inykh Yu.S., Chashechkin Yu.D. Fluid Dynamics. 2004. **39**(1). 148.

Homogeneous fluid:. is underdetermined insoluble system with merged singular components of flow.

Two different 3D boundary layers become identical and are merged into the unique, twice degenerated boundary layer

$$k^2 \left(\omega + i\nu k^2\right)^2 = 0$$

Double boundary layer (transversal divergence free isobaric motion)

$$\left(\omega + i\nu k^2\right)^2 = 0$$

$$k_2 = -k_3 = \sqrt{\frac{i\omega}{\nu} - k_\xi{}^2 - k_\eta{}^2} \qquad \mathbf{k}\cdot\mathbf{u} = 0 \qquad \delta_\nu = \sqrt{2\nu/\omega}$$

$v_x = 0,\ v_y,\ v_z$ -- are independent components of velocity

Large scale periodic longitudinal motion

$$k^2 = 0,\ k_1 = i\sqrt{k_\xi{}^2 + k_\eta{}^2},\ \mathbf{v} = \mathbf{k}\sqrt{\frac{\tilde{P}_k}{\rho}},\ (\mathbf{v} \parallel \mathbf{k})$$

Motion of a strip along a solid horizontal plane

$$\rho_0 = \rho_0(z) \qquad \Lambda = \left| d \ln \rho / dz \right|^{-1} \qquad N^2 = g / \Lambda \qquad T_b = 2\pi\sqrt{\Lambda/g}$$

$$\left[\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) + N^2 \frac{\partial^2}{\partial x^2} - \nu \frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right)^2 \right] \Psi = 0$$

Boundary conditions

$$\left.\frac{\partial \Psi}{\partial z}\right|_{z=0} = U\,\vartheta\left(x + \frac{a}{2} - Ut \right) \vartheta\left(\frac{a}{2} + Ut - x \right), \quad \left.\frac{\partial \Psi}{\partial x}\right|_{z=0} = 0.$$

Dispersionи relation $\quad \omega = \mathbf{ku}$

1. Bardakov R.N., Chashechkin Yu.D. A stratified flow fine structure near a horizontally moving strip // JSME International Joиurnal. 2006. V. 49. No. 3. P. 601-604.

Complete solution

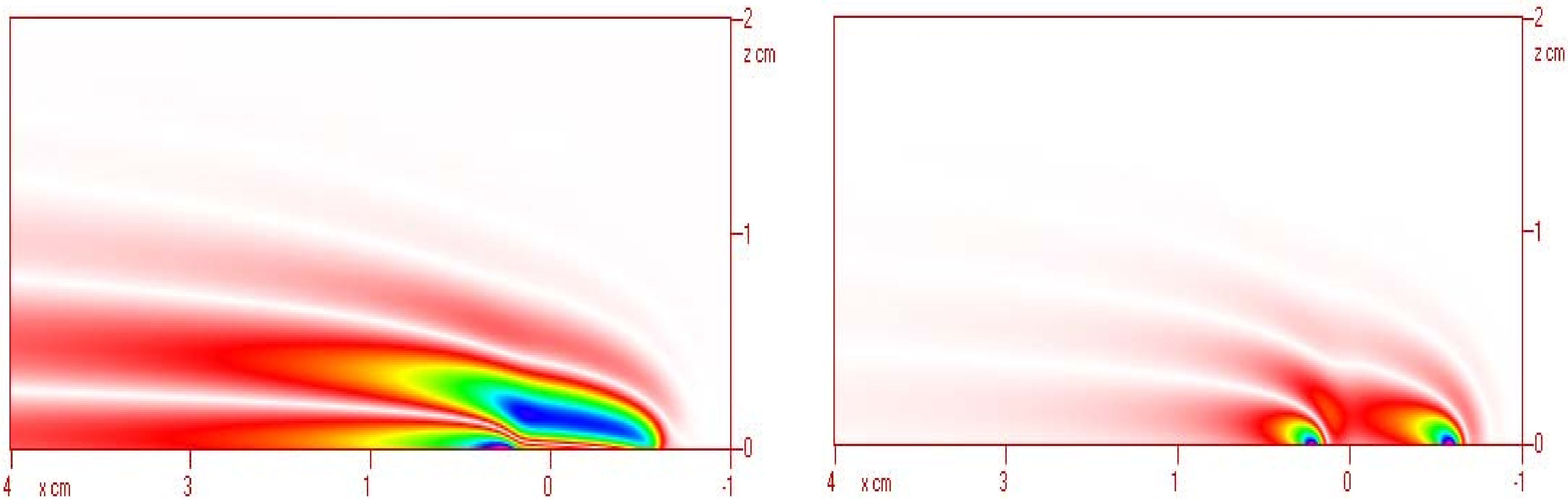
$$\Psi(x,z,t)=\int_{-\infty}^{\infty} e^{-i\omega t}\int_{-\infty}^{\infty}\left[A_w(\omega,k)e^{ik_w(\omega,k)z}+B_i(\omega,k)e^{ik_i(\omega,k)z}\right]e^{ikx}\,dk\,d\omega$$

$$\Psi(x,z,t)=\frac{iU}{\pi}\int_{-\infty}^{\infty}\frac{1}{k}\sin\frac{ka}{2}e^{ik(x-Ut)}\frac{e^{ik_w(kU,k)z}-e^{ik_i(kU,k)z}}{k_w(kU,k)-k_i(kU,k)}dk$$

Solutions of Dispersion equation

$$\omega^2\left(k^2+k_z^2\right)-N^2k^2+i\omega\nu\left(k^2+k_z^2\right)^2=0$$

$$k_w^2(\omega,k)=-k^2+\frac{i\omega}{2\nu}\left[1-\sqrt{1+\frac{4i\nu k^2N^2}{\omega^3}}\right] \qquad k_i^2(\omega,k)=-k^2+\frac{i\omega}{2\nu}\left[1+\sqrt{1+\frac{4i\nu k^2N^2}{\omega^3}}\right]$$



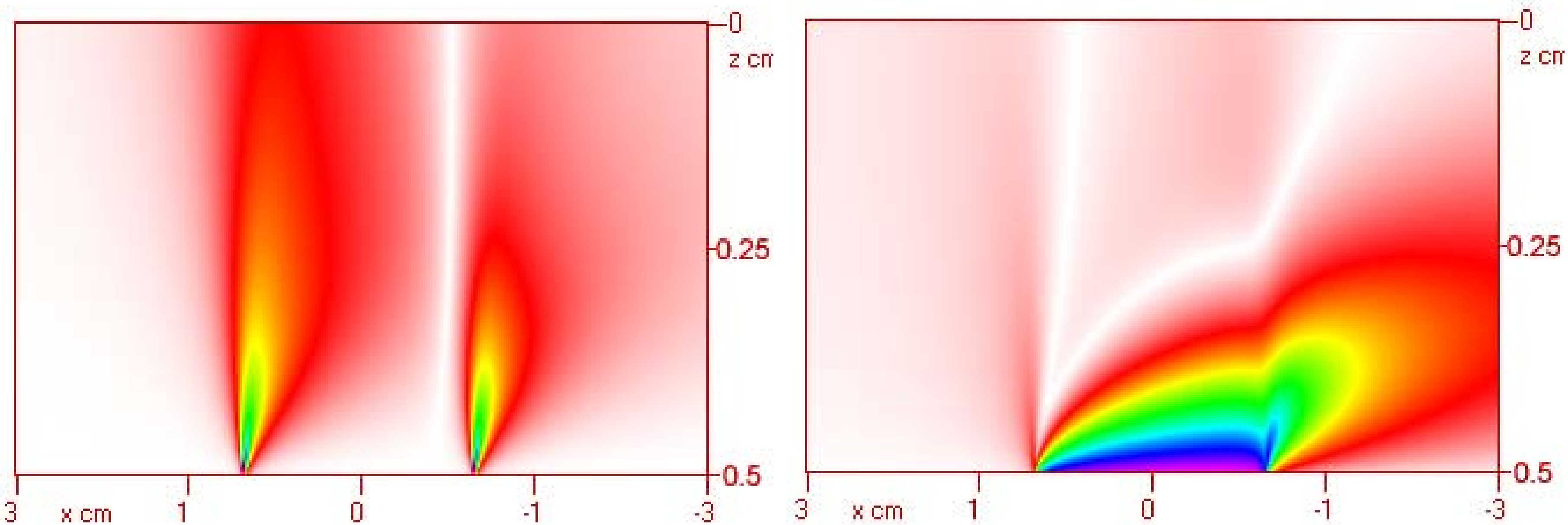
Slowly moving strip *a, b*) horizontal and vertical velocity components

l = 1 cm, *U* = 0.01 cm/s: *T* = 14 s, λ = 0.14 cm,

$\delta_u = \nu / U$ = 1 cm, Fr = 0.02, Re = 1

Bardakov R.N., Chashechkin Yu.D. JSME International Joиurnal. 2006. 49 (3). 601.

Structure of boundary layer on a horizontal strip – singularities of leading and trailing edges

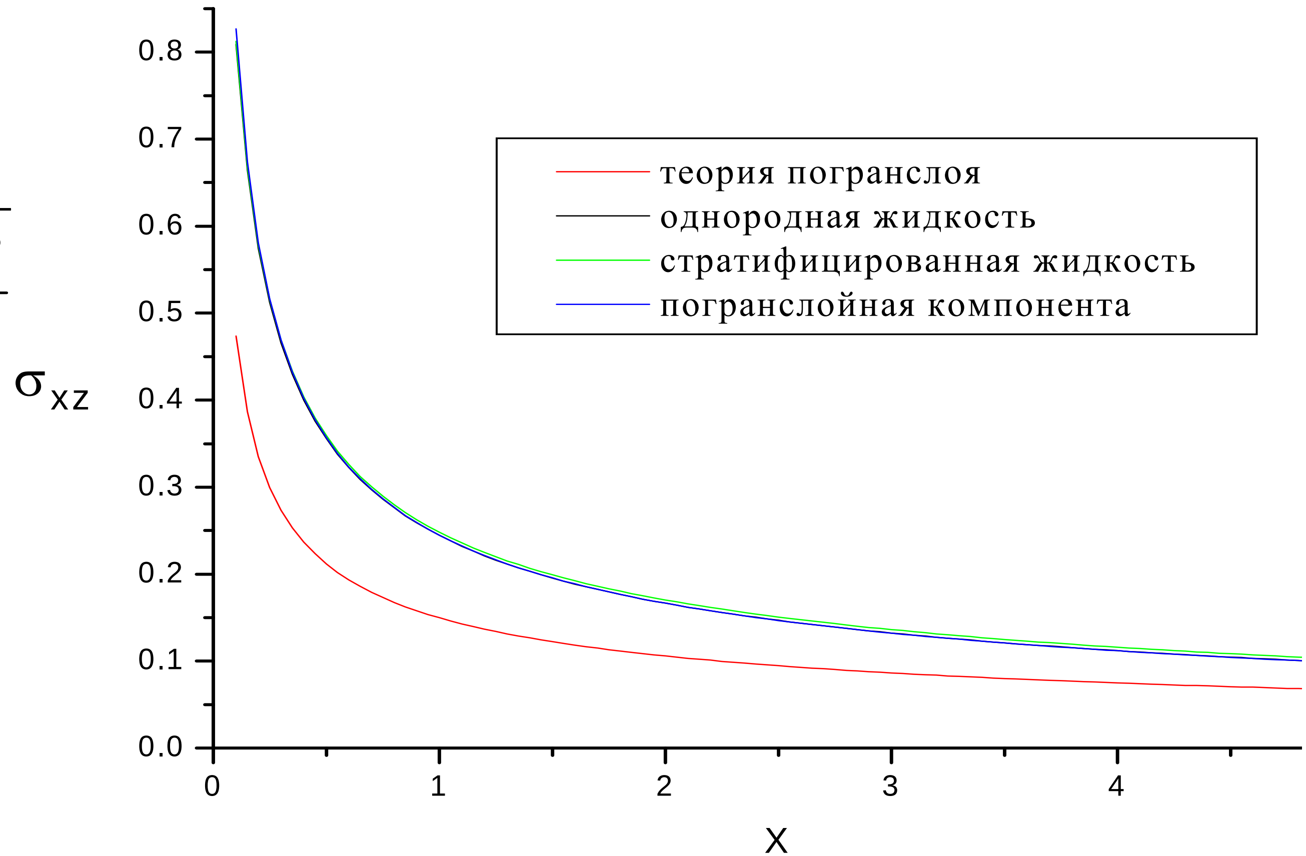


Modulus of vertical and horizontal components of velocity

Bardakov R.N., Chashechkin Yu.D. JSME International Journal. 2006. **49** (3). 601.

Drag acting on unit square

$$\sigma_{xz}^{(b.l.)} = 0.332\sqrt{\frac{\nu\,\rho\,U^3}{x}}$$



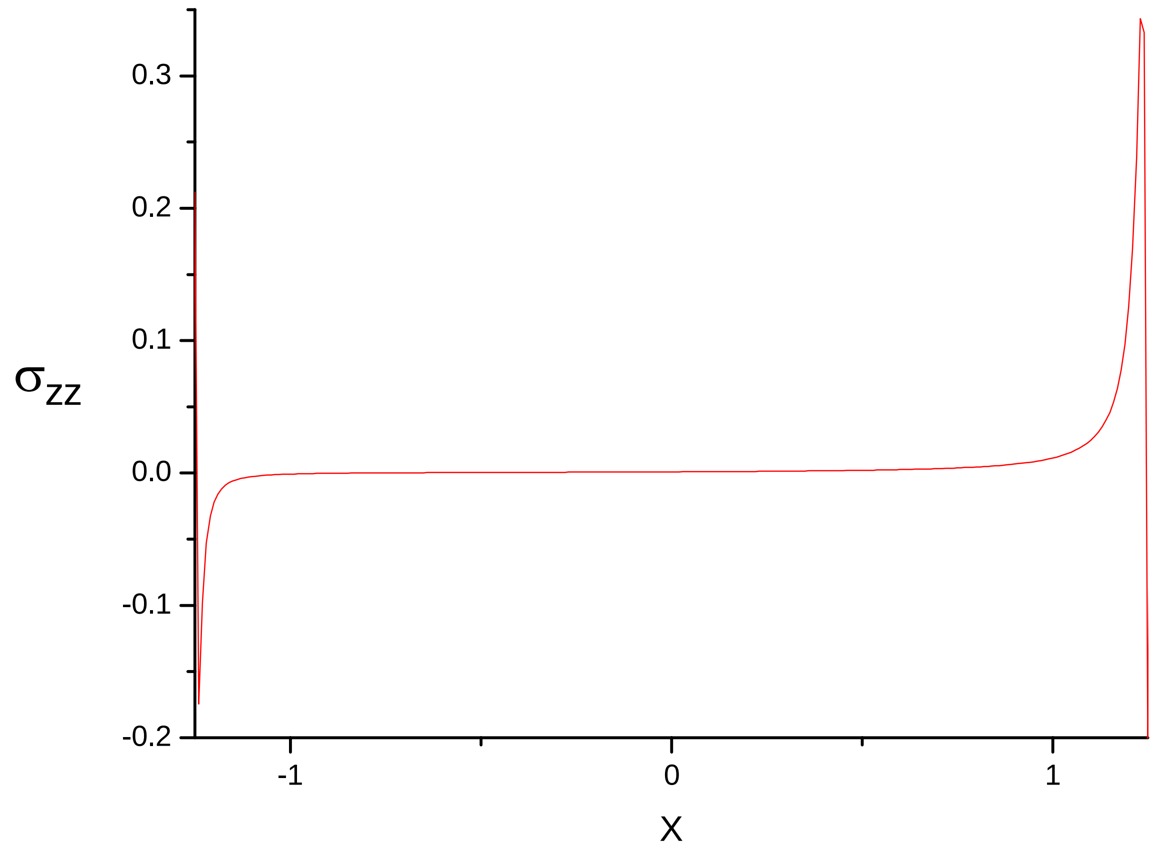
$T_b = 7.55$ c,
L = 400 cm
U = 0.1 7 см/с;

$$\sigma_{xz}(x,z,t) = -\frac{i\nu U}{\pi}\int_{-\infty}^{\infty}\frac{1}{k}\sin\frac{ka}{2}e^{ik(x-Ut)}\frac{k_w^2(kU,k)e^{ik_w(kU,k)z}-k_i^2(kU,k)e^{ik_i(kU,k)z}}{k_w(kU,k)-k_i(kU,k)}dk$$

Bardakov R.N., Chashechkin Yu.D. JSME International Journal. 2006. **49** (3). 601.

Lift acting on unit square

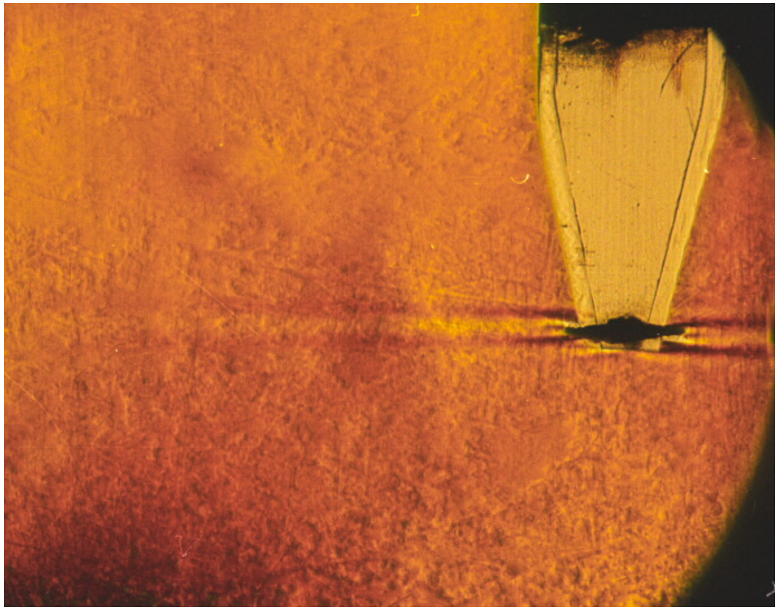
T_b = 7.55 с,
L = 2.5 см
U = 0.1 7 см/с;



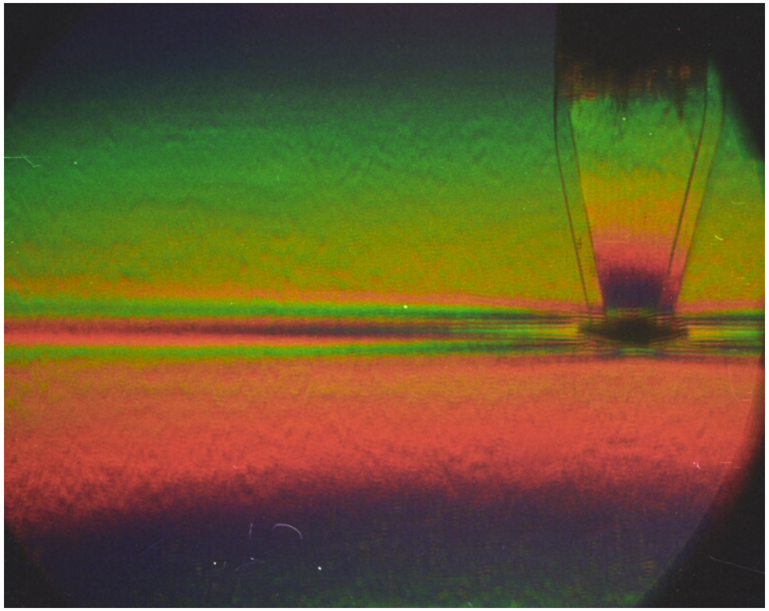
$$\sigma_{zz}(x,z,t)=\frac{i\nu U}{\pi}\int_{-\infty}^{\infty}\sin\frac{ka}{2}e^{ik(x-Ut)}\frac{k_w(kU,k)e^{ik_w(kU,k)z}-k_i(kU,k)e^{ik_i(kU,k)z}}{k_w(kU,k)-k_i(kU,k)}dk$$

Bardakov R.N., Chashechkin Yu.D. JSME International Journal. 2006. **49** (3). 601.

Diffusion induced boundary currents on the motionless obstacle in a stratified fluid at rest



Vertical knife



Colour schlieren method

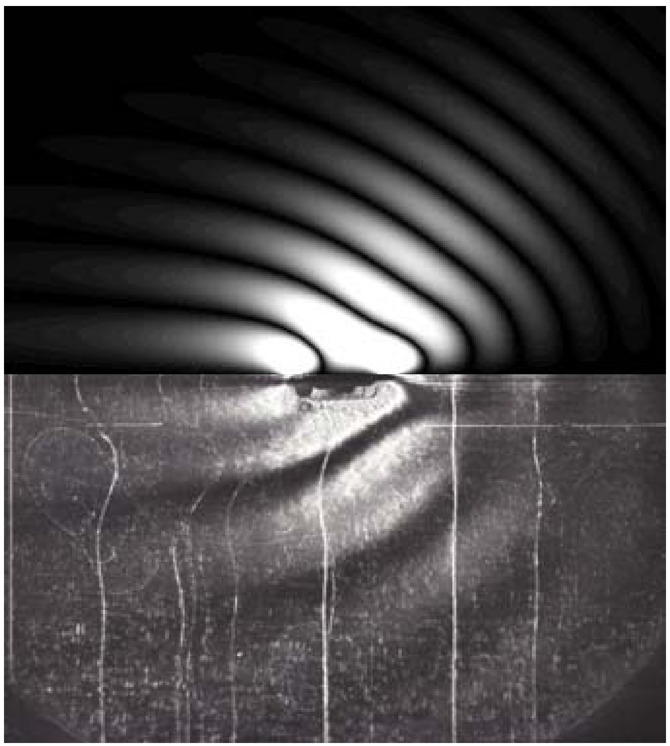
Visualization of the density gradient field

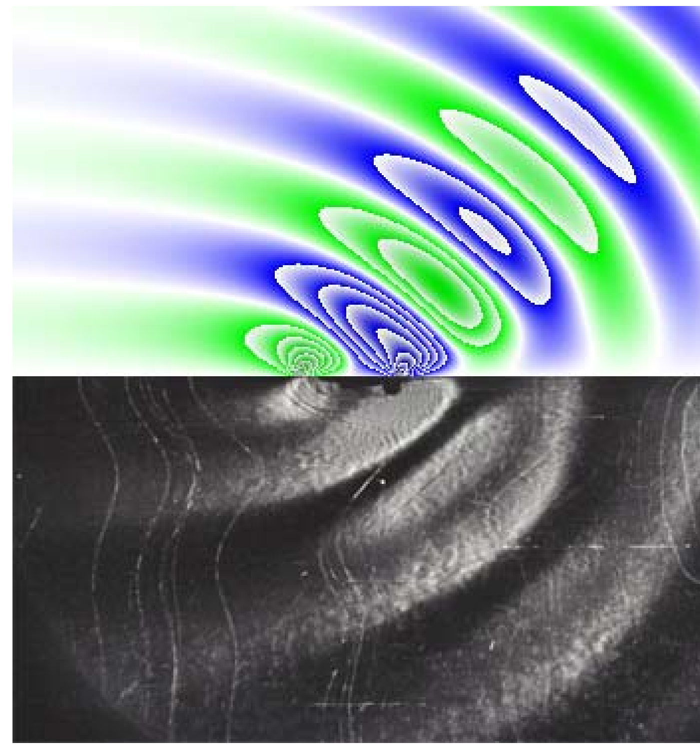
Yuli D. Chashechkin, Roman N Bardakov

L = 2.5 cm, T = 14 s:

a, b) - U = 0,17 cm/s, λ = 2.4 cm, Fr = 0.15, Re = 42;

U = 0,25 cm/s λ = 3.5 cm, Fr = 0.22, Re = 63.





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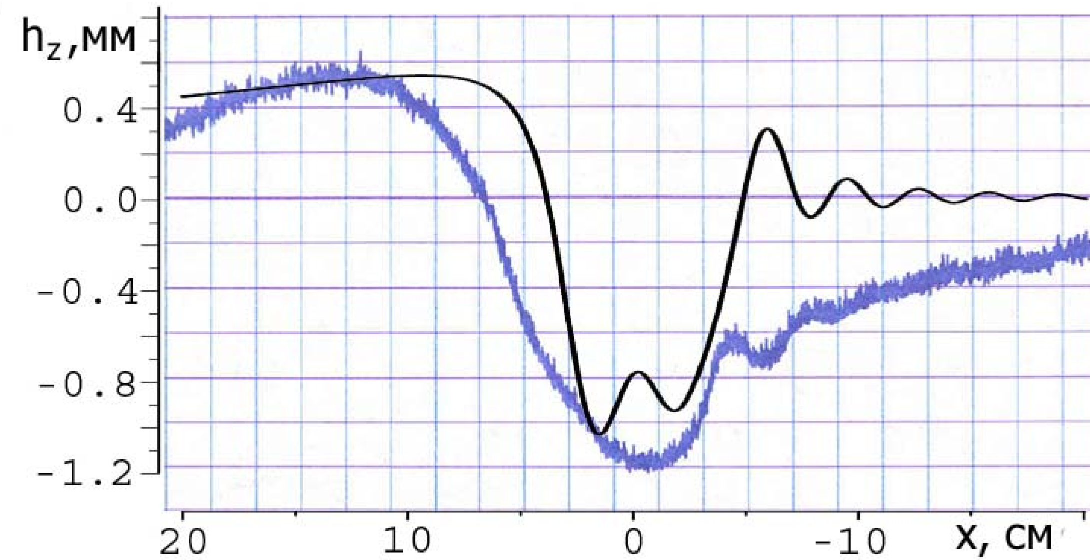
Wave field produced by a horizontally moving strip



Flow pattern and particle displacements past strip

$$T_b = 7.6\ \text{s} \quad L_x = 7.5\ \text{cm}, \quad U = 3.97\ \text{cm/s}$$

$$\text{Re} = 2780 \qquad \text{Fr} = 0.63$$

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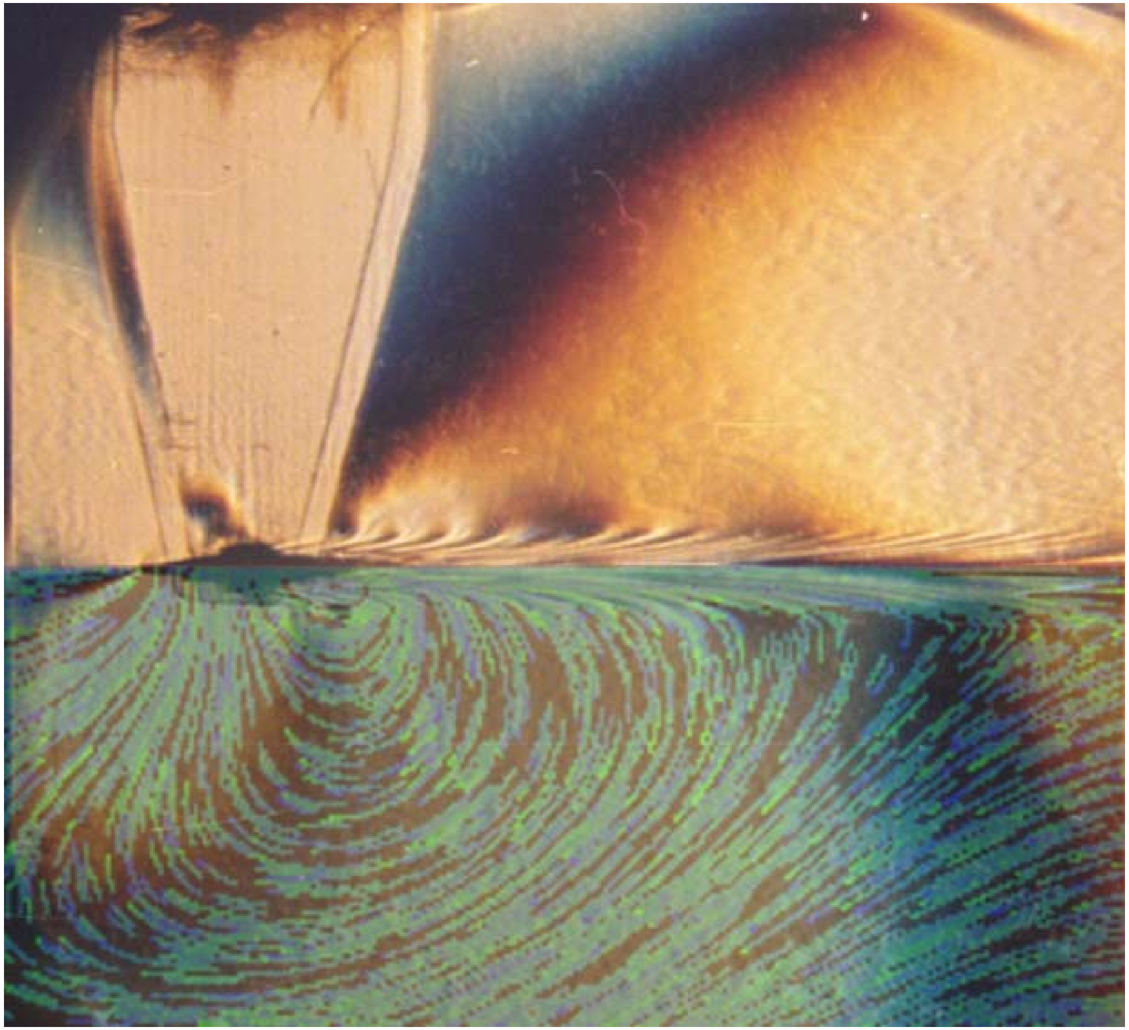
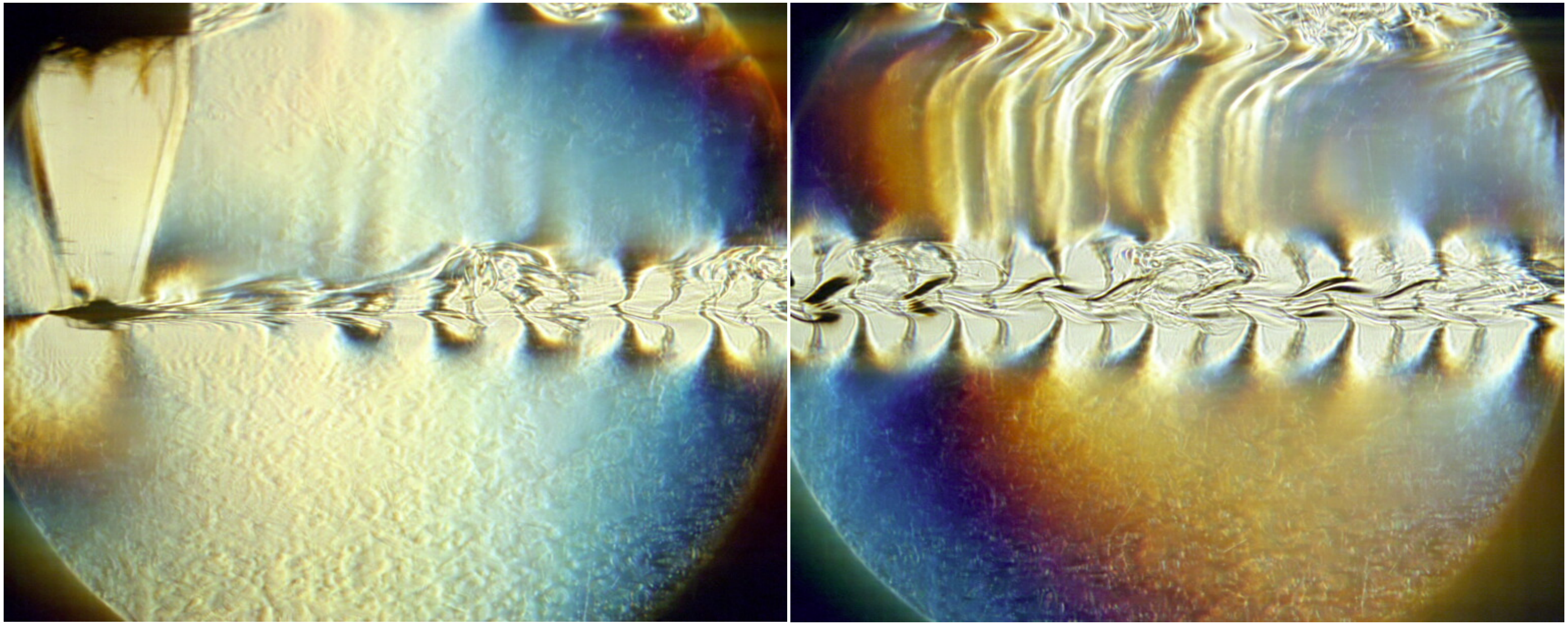
Streaky structures on a horizontally moving strip



T_b = 7.5 s, L_x = 2.5 cm, U = 2.3 cm/s;

Reconnection of streaks into clusters and formation of the vortex street



T_b = 7.5 s, L = 2.5 cm, U = 4.9 cm/s;

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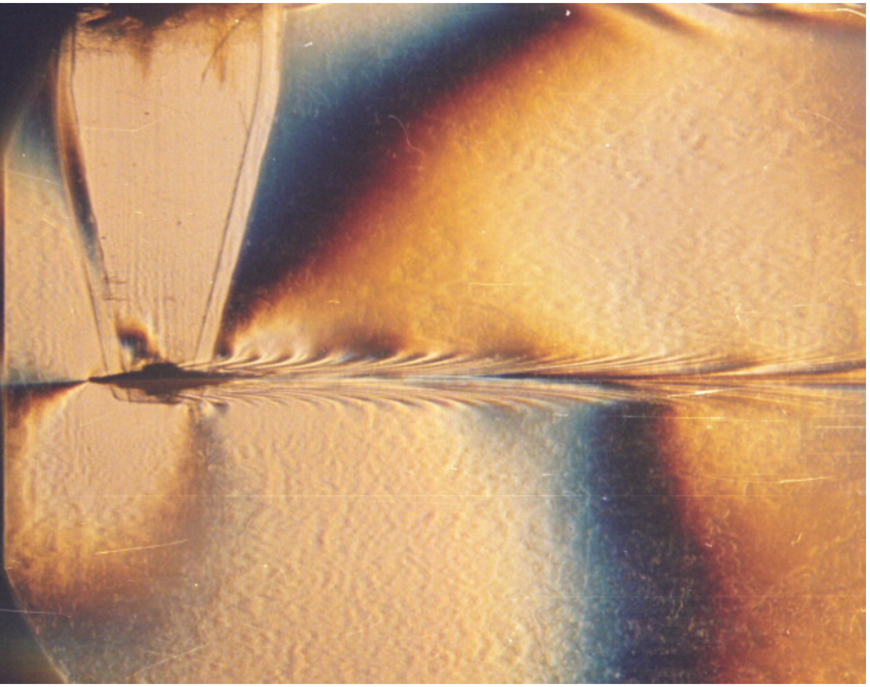
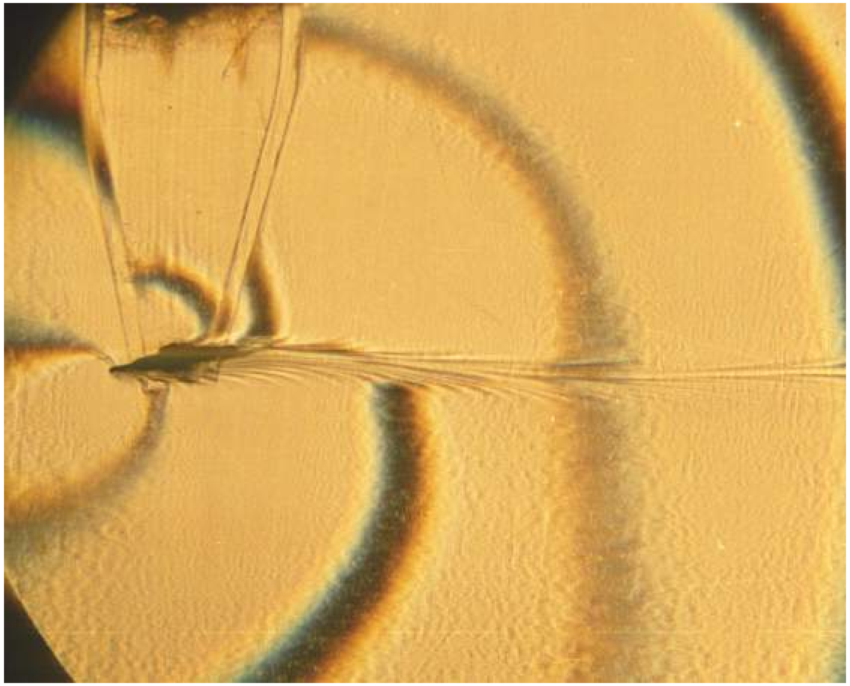
Streaky structures on horizontal and sloping strip



T_b = 7.5 s, L = 2.5 cm, U = 2.3 cm/s;



T_b = 7.5 s, L = 2.5 cm, U = 1.4 cm/s;
Угол атаки α = **12.5°**

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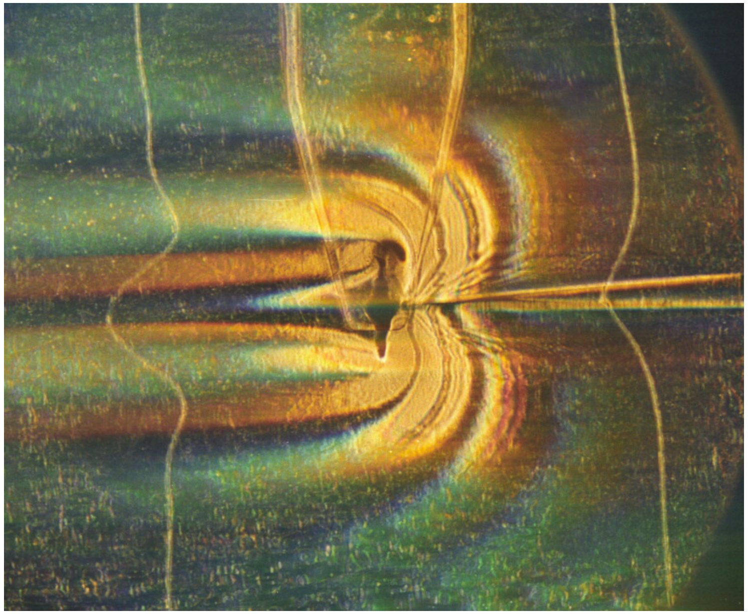
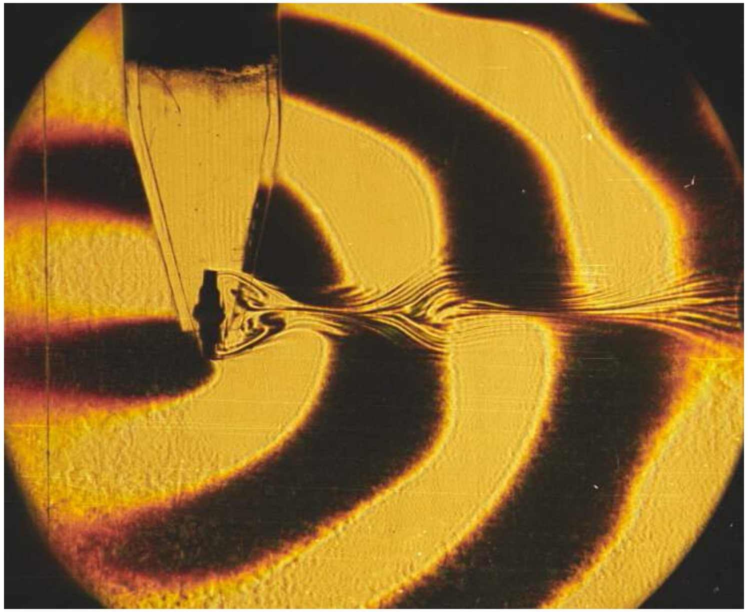
Fine structure of the flow around the vertical strip



$T = 12.5$ s, $U = 0.1$ cm/s



$T = 17.4$ s, $U = 0.3$ cm/s

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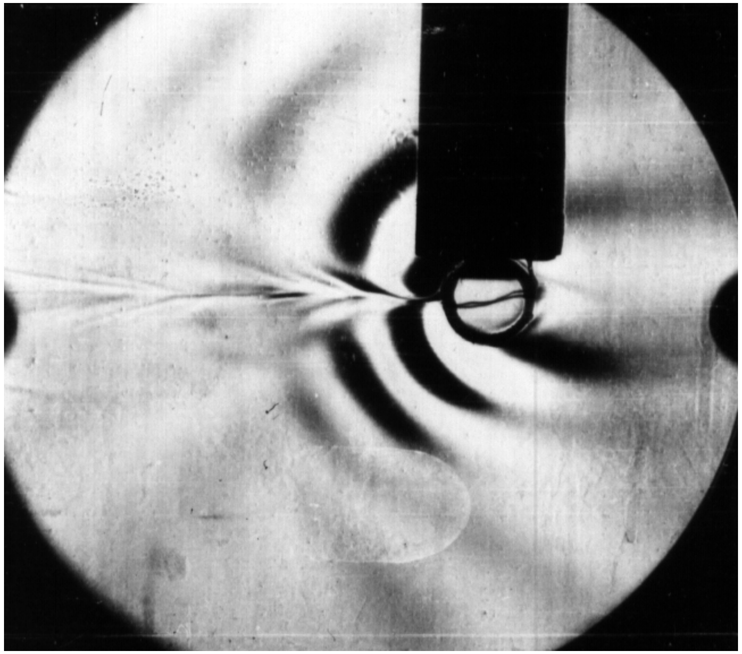
Attached downstream (lee) waves and split density wake past a cylinder



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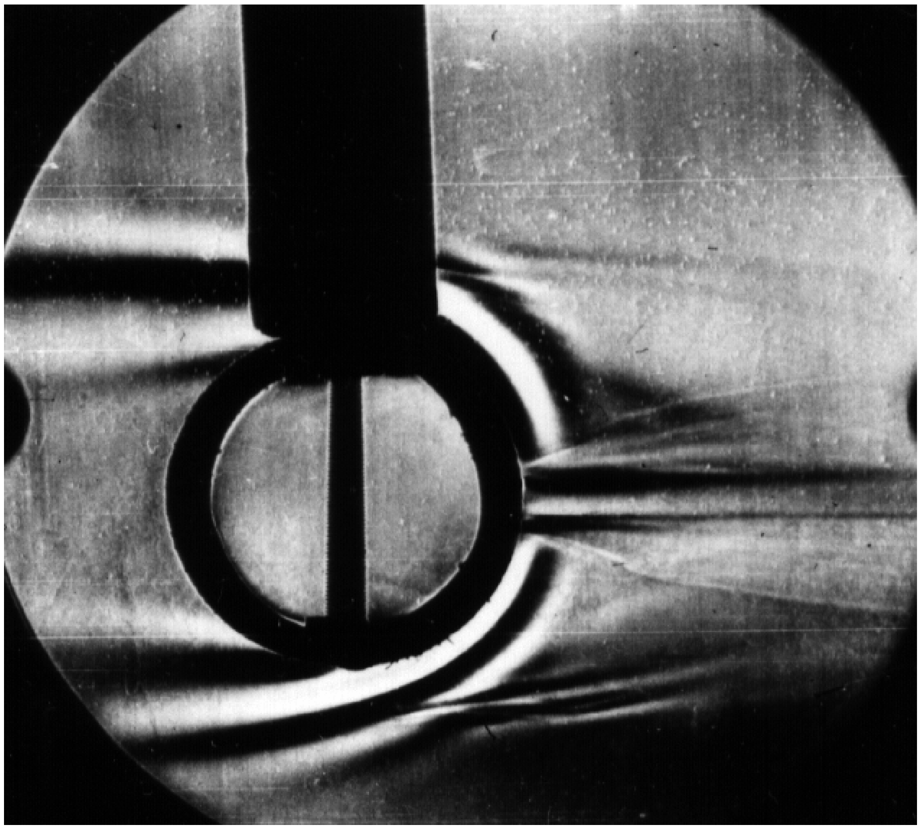
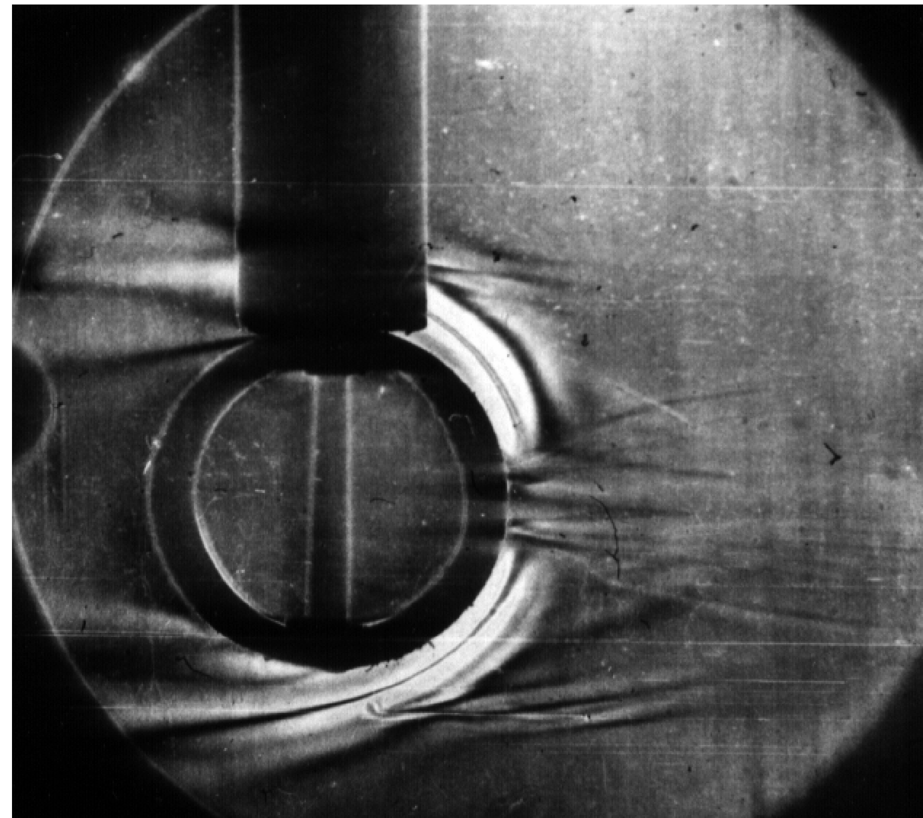
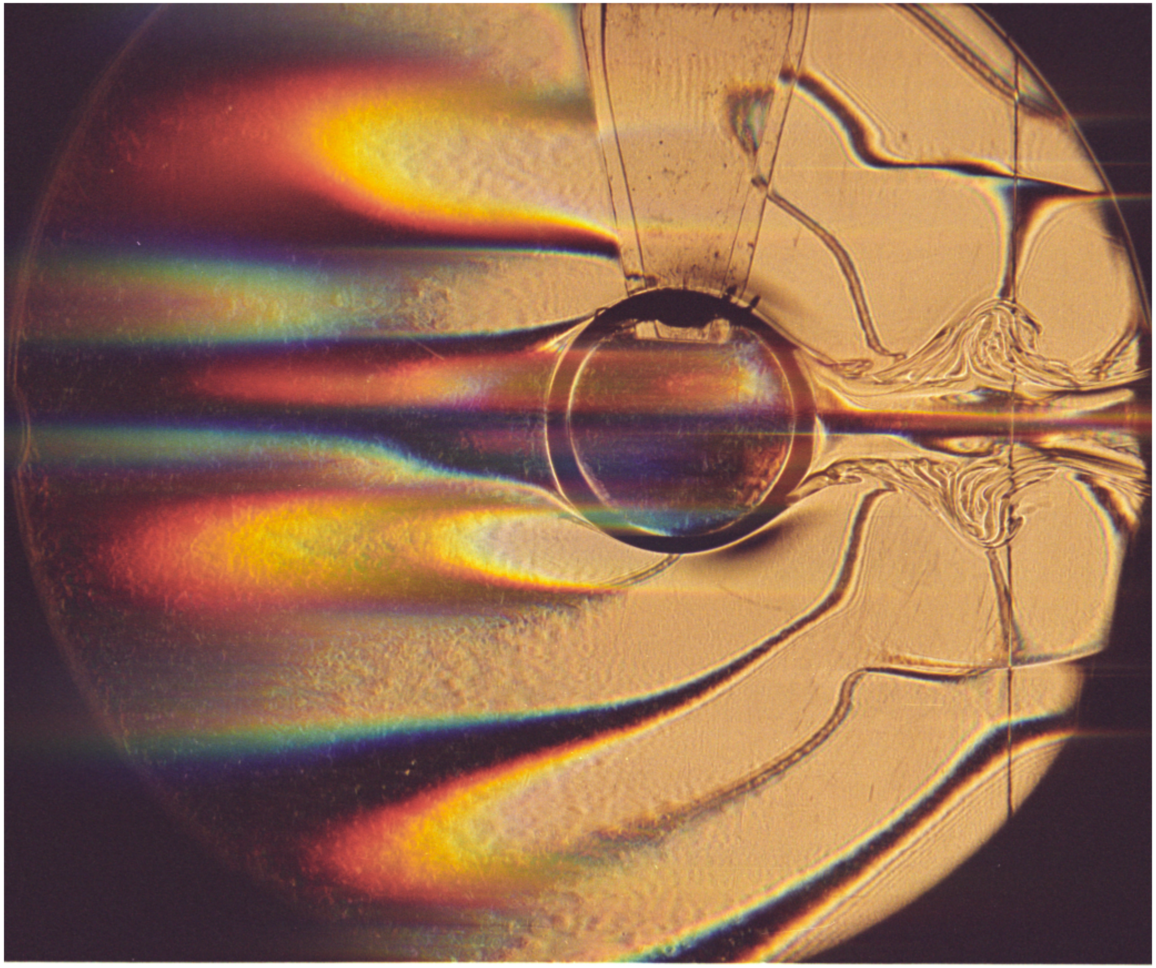
Pattern of flow with a singular interfaces past a towing cylinder

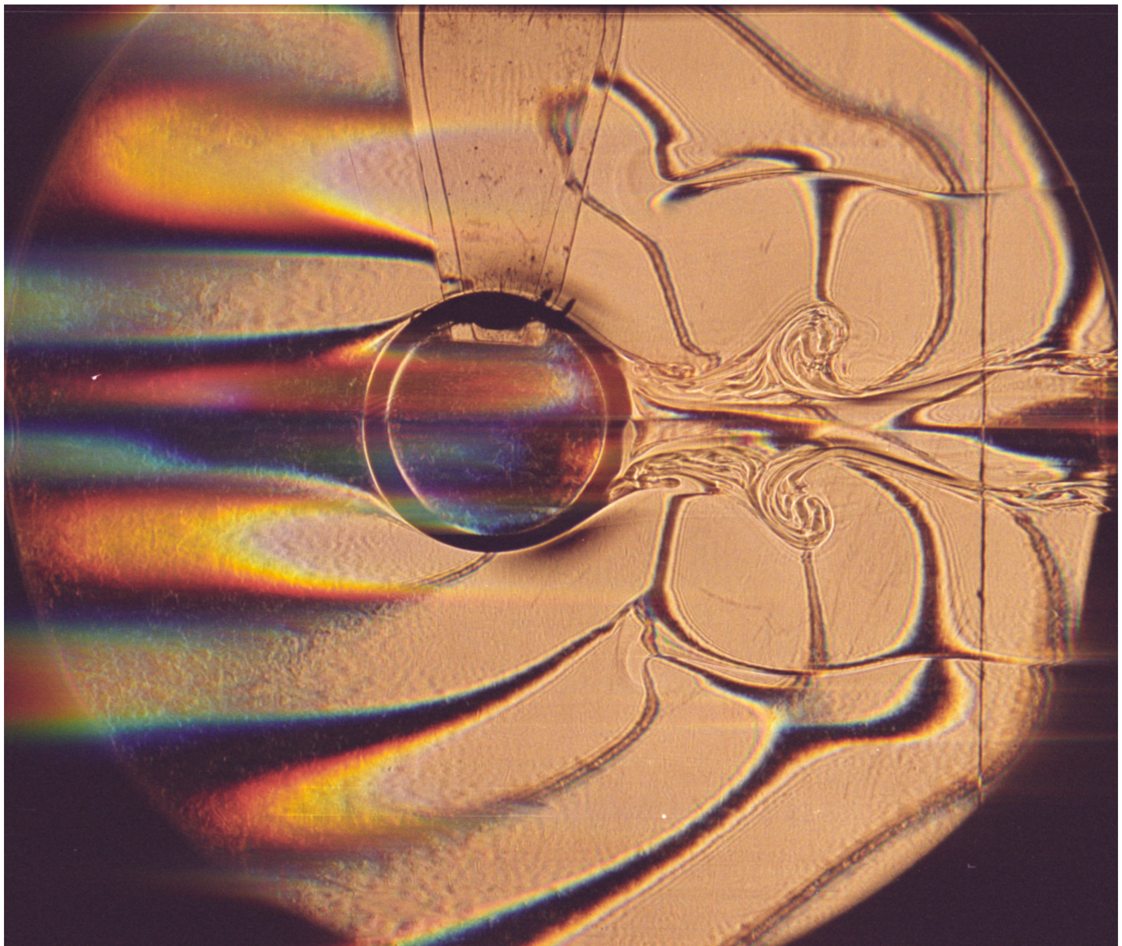
Conventional (left) and Maksoutov (right) schlieren images of the flow past a cylinder; D = 7.6 cm, T_b = 20.5 s, U = 0.04 cm/s, Re = 30, Fr = 0.017, C = 1370.



Formation of interior boundary currents and soaring vortices after the beginning of the motion

$\tau = 2.7$

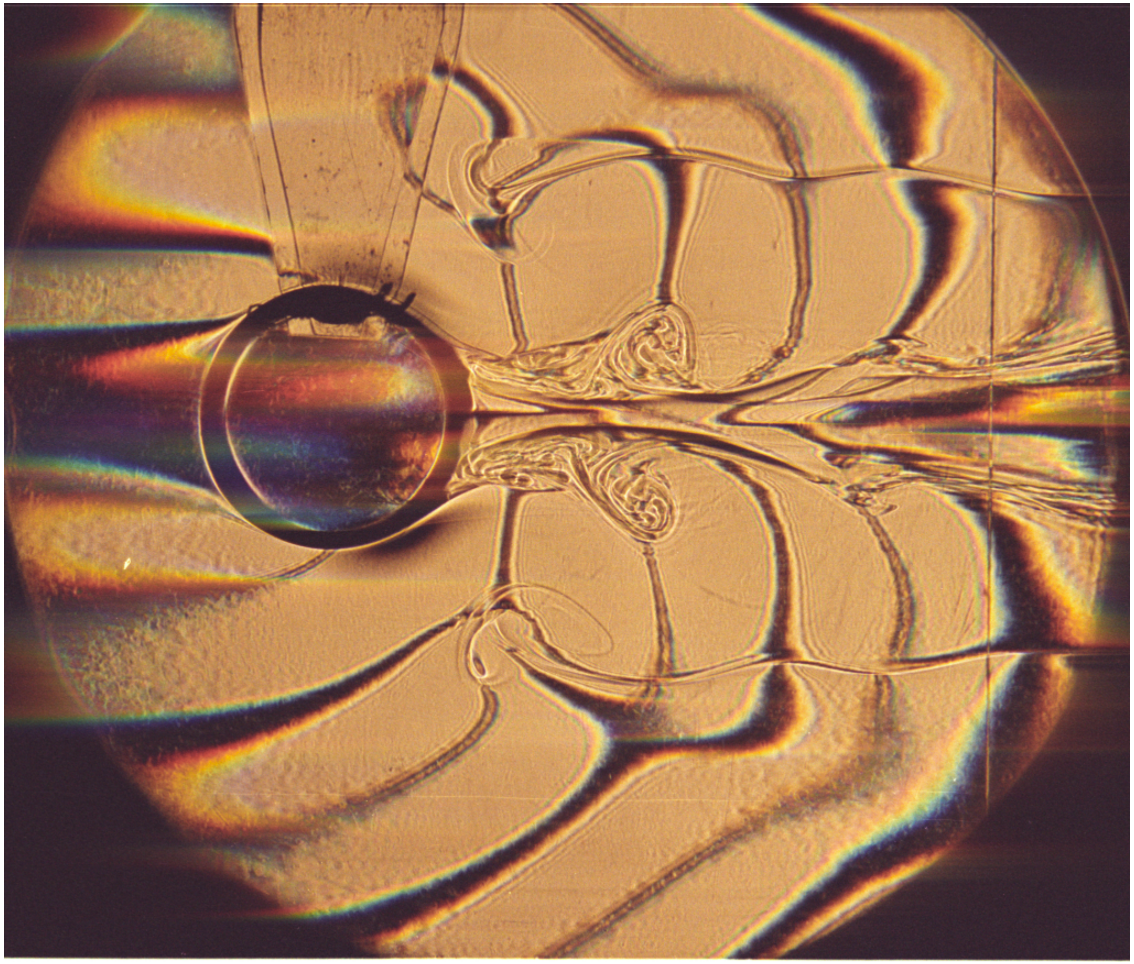
D = 5 см, U = 0,35 cm/c, T = 13 c, Fr = 0.14, Re = 165



Formation of interior boundary currents and soaring vortices after the beginning of the motion

$\tau = 3.9$

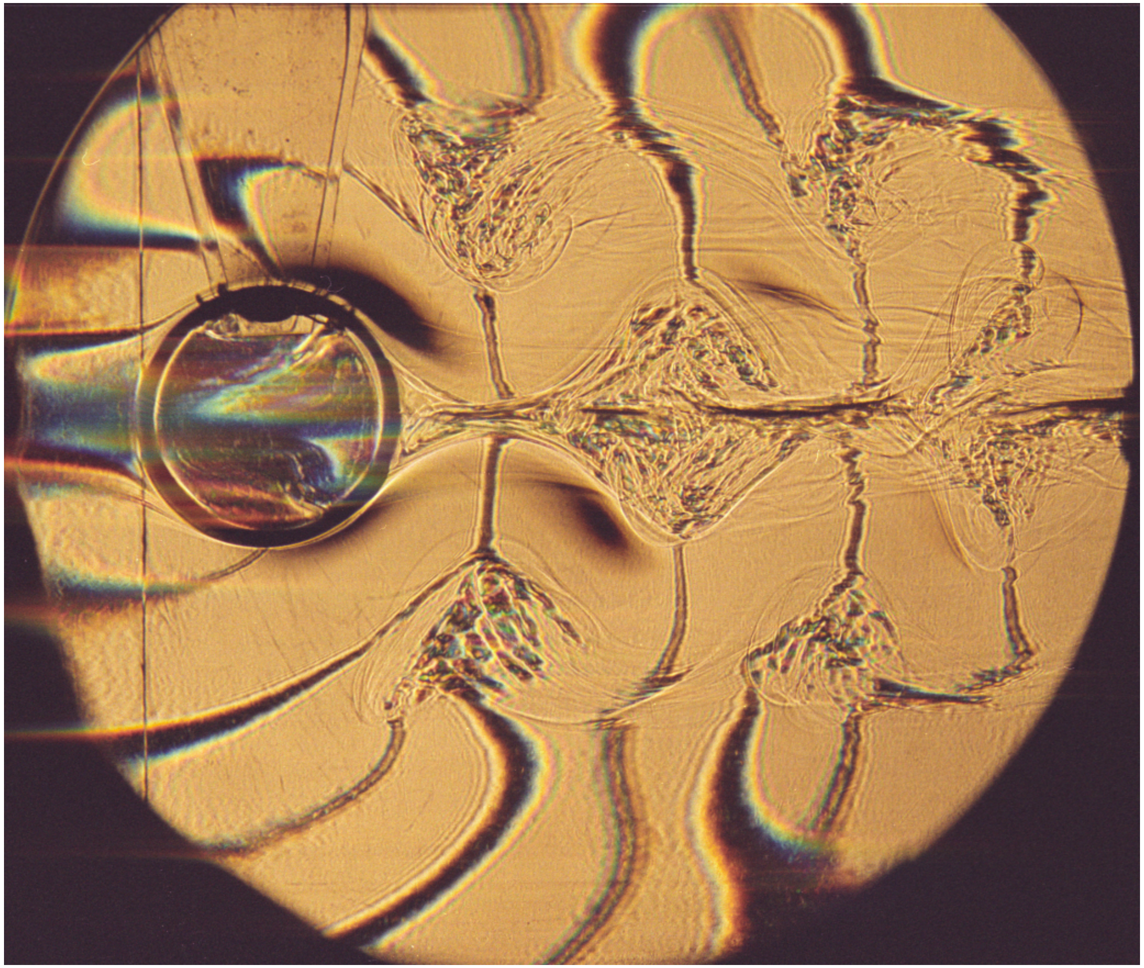
D = 5 см, U = 0,35 cm/c, T = 13 c, Fr = 0.14, Re = 165



Formation of interior boundary currents and soaring vortices after the beginning of the motion

$\tau = 5.8$

D = 5 см, U = 0,35 cm/c, T = 13 c, Fr = 0.14, Re = 165



Formation of interior boundary currents and soaring vortices after the beginning of the motion

$\tau = 65$

$D = 5$ см, $U = 0{,}35$ cm/c, $T = 13$ c, $Fr = 0.14$, $Re = 165$

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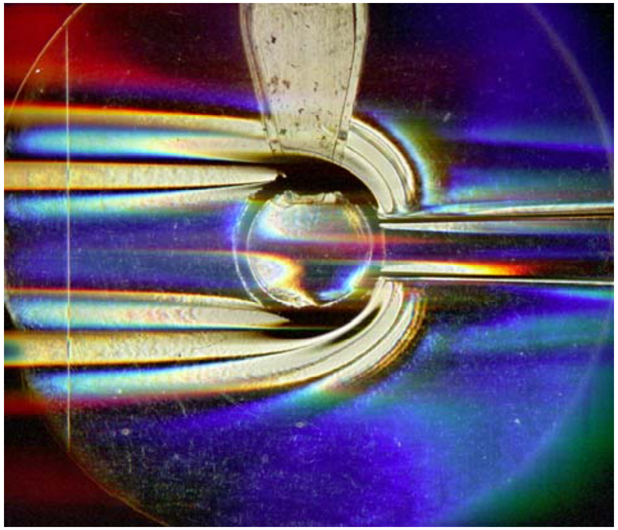
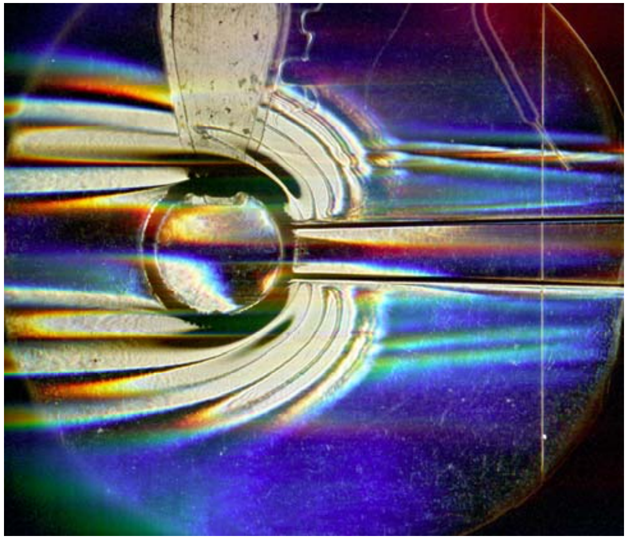
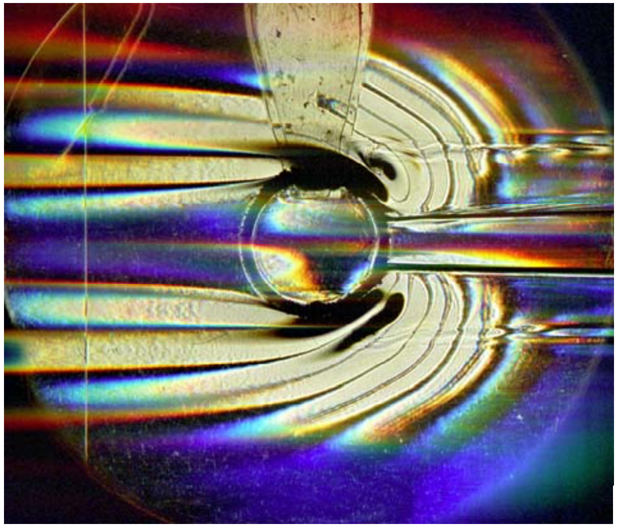
U=0.1cm/s,
Fr=0,024;
Re=51,5

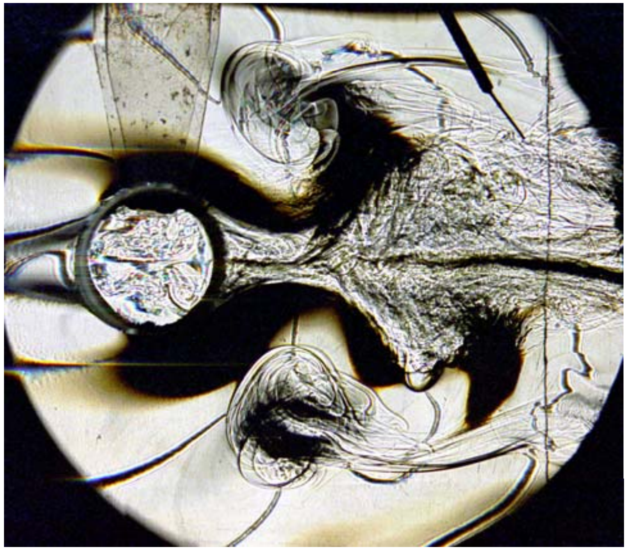




U=0.14cm/s,
Fr = 0,032;
Re =69

U=0.16 cm/s,
Fr = 0,037;
Re = 78

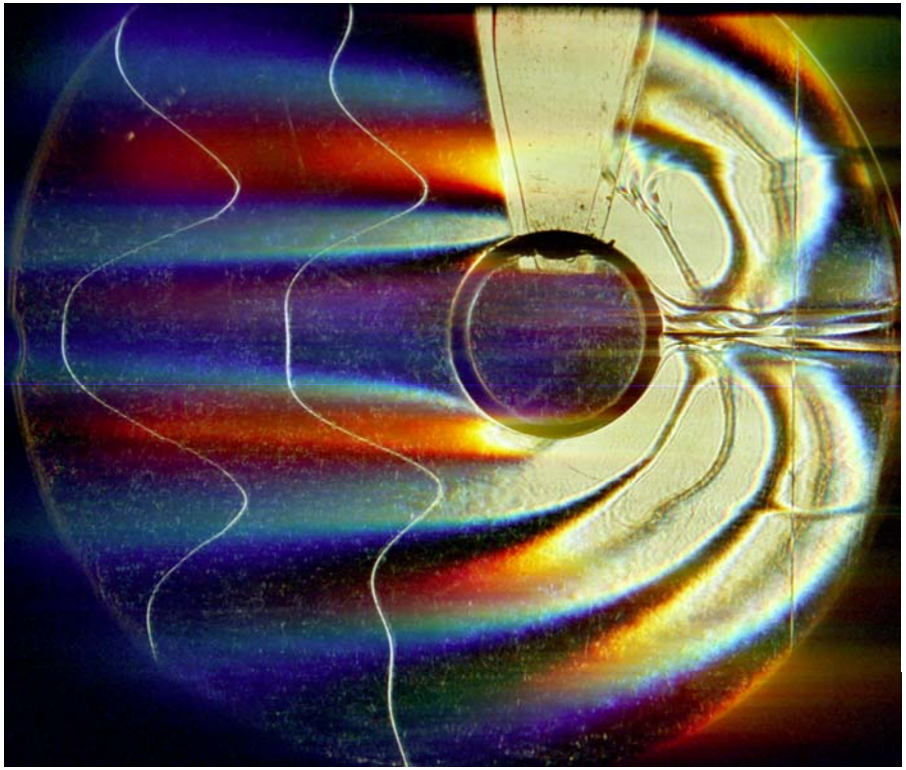


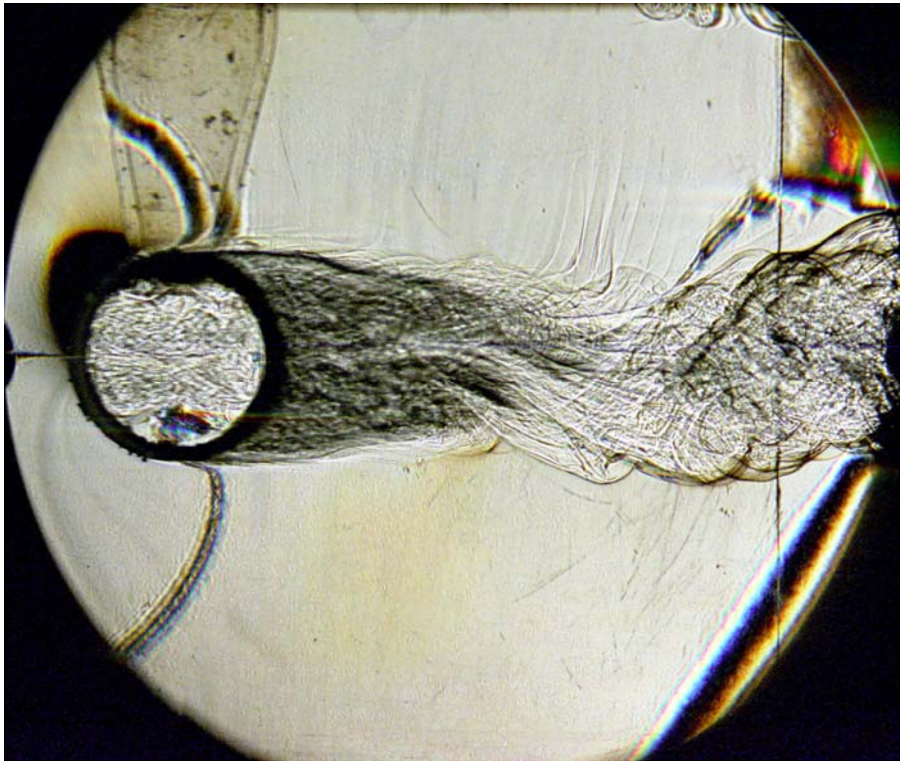


U=1.01 cm/s
Fr = 0,24;
Re = 505

T = 7.4 s

Yuli D. Chashechkin





T = 20.1s,   U=0.1 cm/s,
Fr = 0,072; Re = 54

T = 7.4 s U=3.48 cm/s,
Fr = 0,82; Re = 1470

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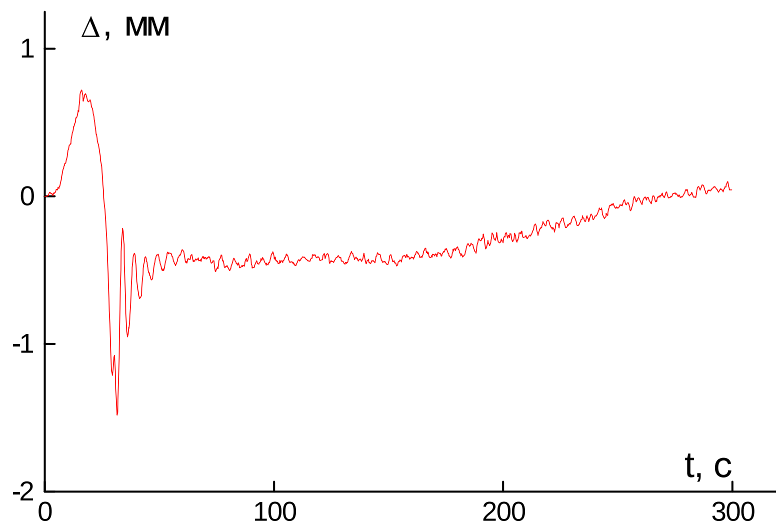
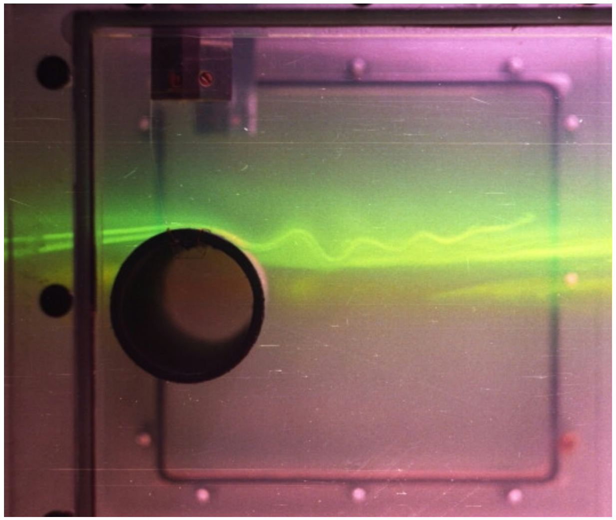
Accumulation of contaminants on interfaces in the wake



а)



б)

D = 1,5 см; *U* = 0,39 см/с;
T_b = 5,2 с;
Fr = 0,22; *Re* = 59

D = 7,6 см; T_b = 7,0 с; *U* = 0,31 см/с;
Fr = 0,045; *Re* = 236

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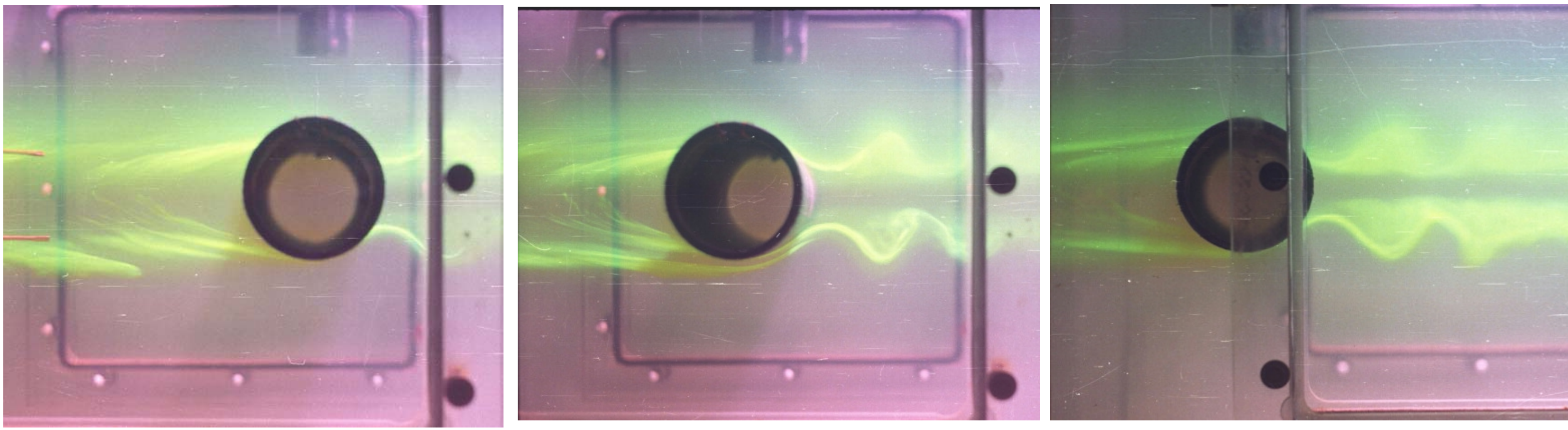
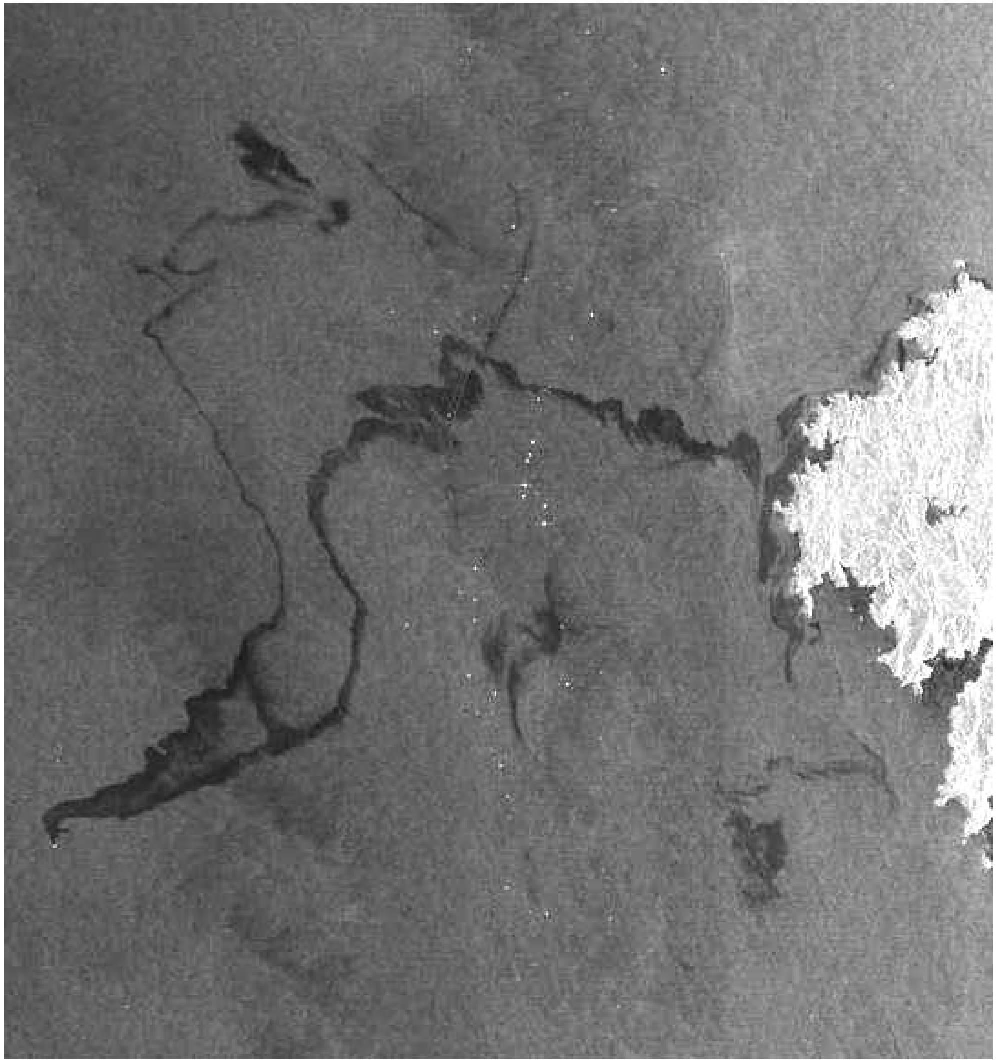
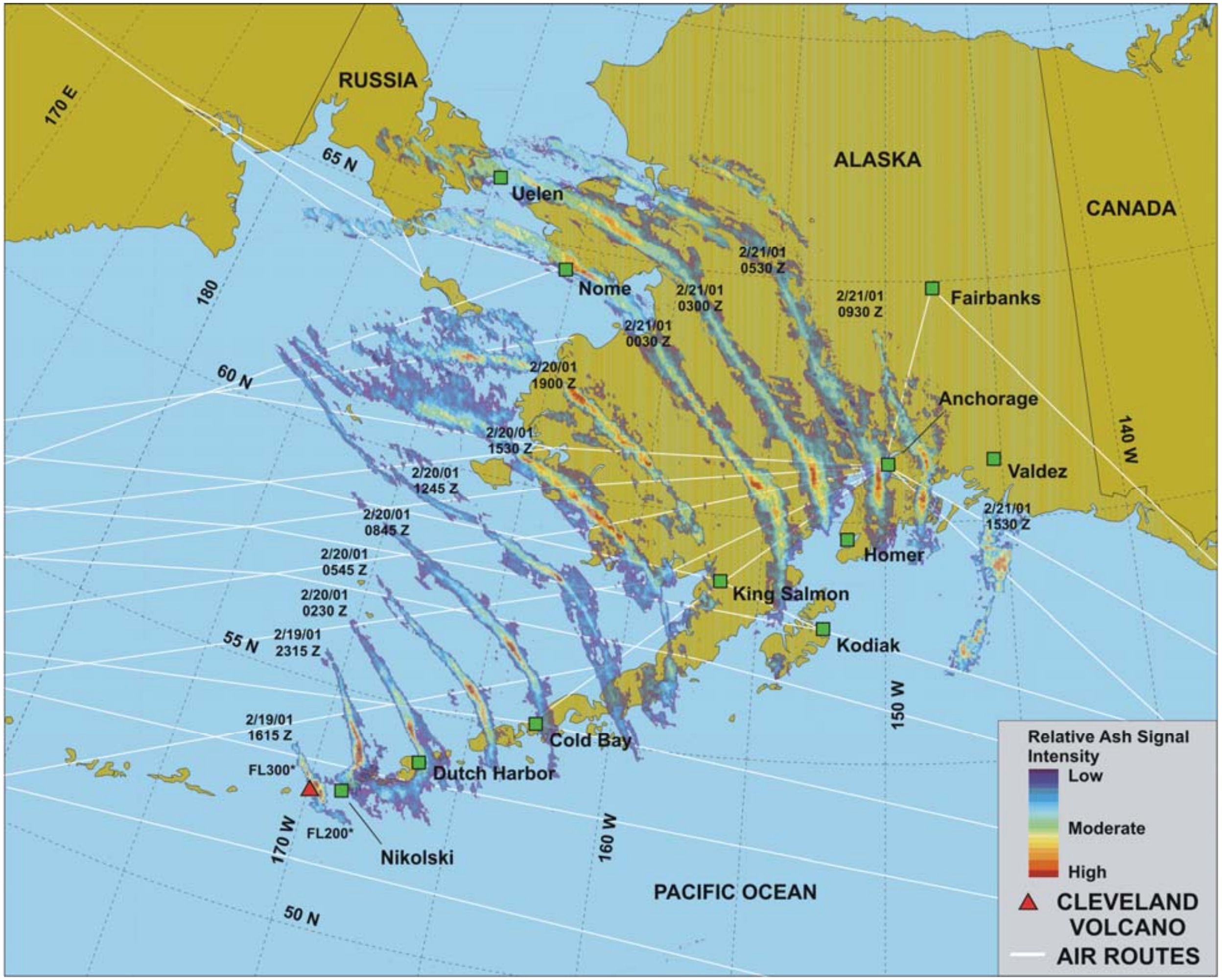


Re-distribution of a released dye ahead and past the cylinder and formation of bright envelopes on a high gradient interfaces inside the density wake (D = 7.6 см; T_b = 7. s; U = 0.69 см/с; Fr = 0.1; Re = 524.

$$\tau = t / T_b = 3{,}6; 5{,}0; 8{,}6$$



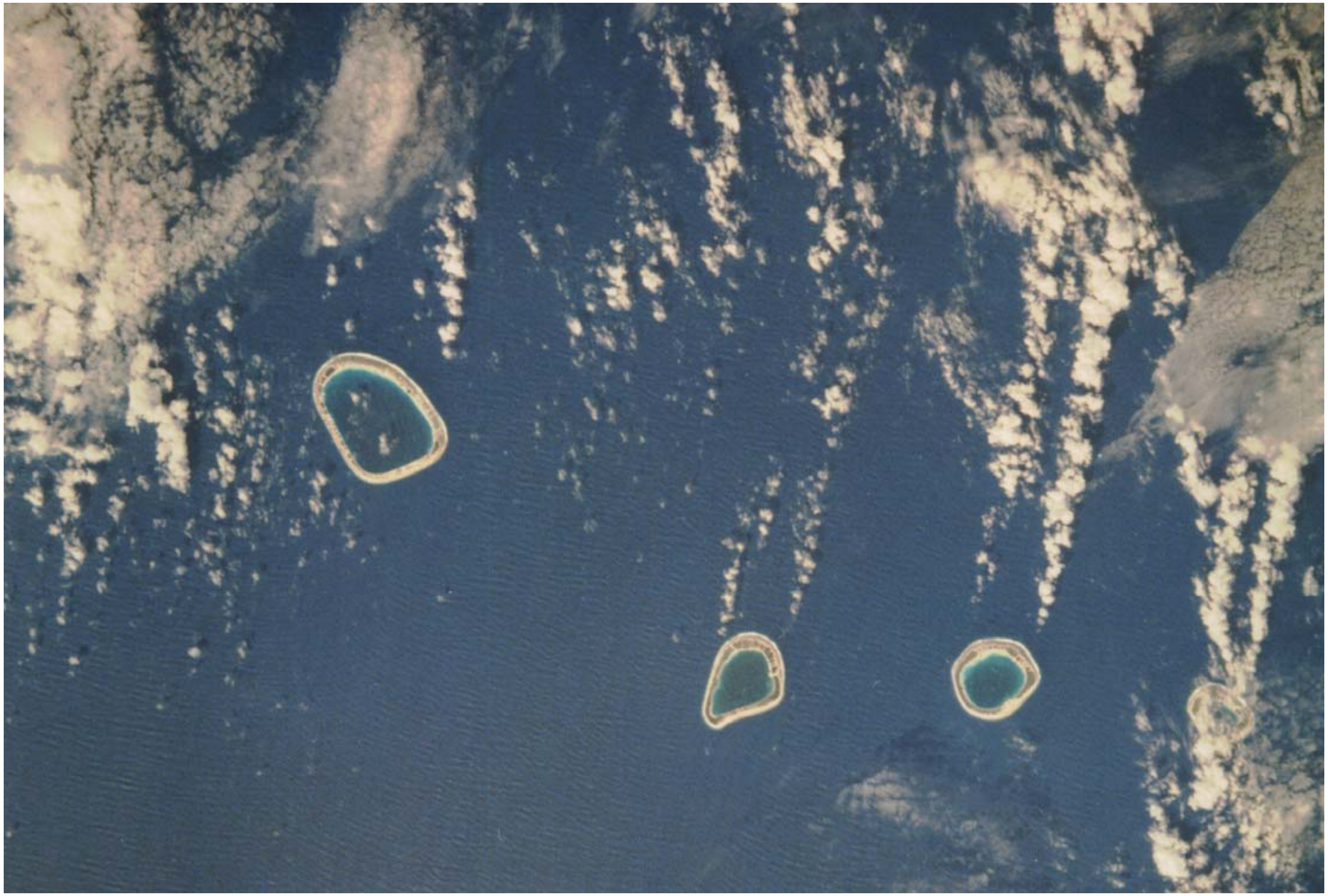
“Prestige” oil spills (SAR image on November 17, 2002)



Ash plume transfer after Cleveland volcano eruption

Conclusion

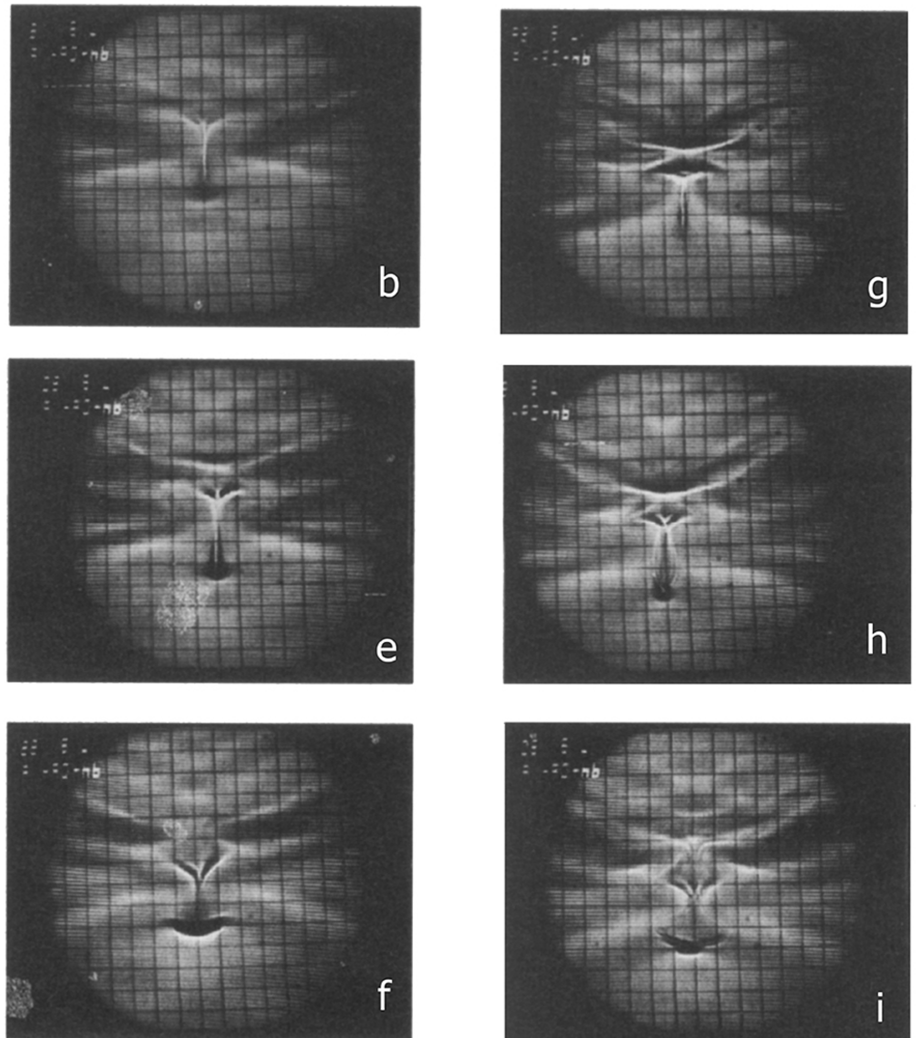
- Complete well-posed and self-consistent governing equations sets form basis for adequate modeling, forecasting and control of flows.
- Regular periodic flows co-exist with a rich set of singular components forming boundary layers on contact surfaces and a high gradient interfaces inside fluids;
- Singular component affect flow energy dissipation, vorticity and mass transport effectively;
- New instruments and protocols for identification of singular components are necessary;
- Classical set of Navier-Stokes and continuity equations for homogeneous fluid is underdetermined, degenerated and insoluble system with merged singular components of flow.



- Thank you very much for your kind attention!

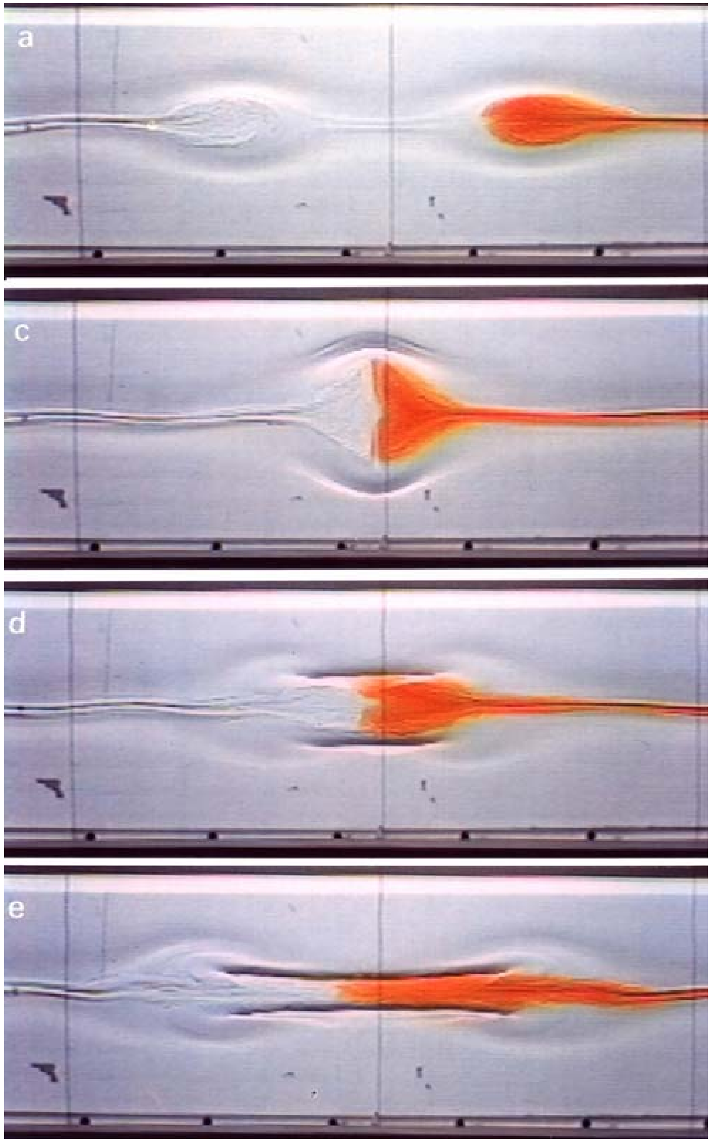
It is time to study practically above mentioned singular components of flows in coral lagoons!

Teoh S.G., Ivey G.N., Imberger J., JFM, 1997



Intersection of two periodic internal wave beams produced by independent sources

Honji H., Matsunaga N., Sugihara Y., Sakai K., FDR, 1995



Collision of solitons on the interface between miscible fluids