5 EDER, R.: Dissertation, T.U. Wien 1987.
6 AMANO, R. S.; GOEL, P.: ASME J. Fluids Eng. **109** (1987), 424–428.
7 ROTTA, J.: Turbulente Strömungen. Teubner-Verlag, Stuttgart 1972.
8 HANJALIĆ, K.; LAUNDER, B. E.: J. Fluid Mech. **52** (1972), 609–638.
9 LAUNDER, B. E.; REECE, G. J.; RODI, W.: J. Fluid Mech. **68** (1975), 537–566.
10 CORMACK, D. E.; LEAL, G. G.; SEINFELD, J. H.: ASME J. Fluids Eng. **100** (1978), 47–54.

Address: Prof. Dr. WILHELM SCHNEIDER, Institut für Strömungslehre und Wärmeübertragung, Technische Universität Wien Wiedner Hauptstr. 7, A-1040 Wien, Österreich

60: MULTIPHASE FLOWS (I) AND CAVITATION

ZAMM · Z. angew. Math. Mech. 69 (1989) 6, T 629 – T 630

HILLER, W. J.; KOWALEWSKI, T. A.

Optical Investigation of Oscillating Liquid Droplets

For a given initial state two quantities describe a particular mode of drop oscillation: oscillation frequency Ω and amplitude decay rate – both depending on physical properties of the fluid i.e. its surface tension, density and viscosity. The purpose of the present work is to apply this phenomenon as a nonintrusive method to measure the surface tension of dispersed droplets in "real time" of the experimental conditions.

An approximate model of small oscillations of a droplet, LAMB [1], gives the following expression for the frequency of the n-th mode of oscillation:

$$\Omega_n^2 = \frac{(n-1)\cdot(n+1)\cdot(n+2)\cdot n\cdot\sigma}{[(n+1)\cdot\varrho_i + n\cdot\varrho_e]\cdot R^3}, \tag{1}$$

where σ is the surface tension, $\varrho_{i,e}$ densities of the internal and external phases and R the radius of the droplet at equilibrium. The dominating mode of oscillation is associated with $n = 2$ and the above formula for a liquid droplet oscillating in vacuum or air will have the following simple form:

$$\Omega_2^2 = \frac{8\cdot\sigma}{\varrho_i\cdot R^3}. \tag{2}$$

This equation is independent of the fluid viscosity and gives the asymptotic value of the oscillation frequency. All aspects of the small-amplitude oscillation of a viscous droplet in another viscous fluid were derived by PROSPERETTI [2] and BROSA [3]. According to their models, the viscous contribution to the oscillation frequency can be proved to be very small in all cases examined in this study, i.e. for water and ethanol droplets larger then 10 μm, oscillating in air. However, due to the viscous damping the validity of the model of periodic oscillation is limited to droplets larger then some critical radius. For a water or ethanol droplet in air this limiting size is about 1 μm and for smaller droplets the aperiodic return to its spherical shape must be assumed.

The experiments were performed with water and alcohol droplets dispersed in air. A laminar jet of a few centimeters of length and 0.1 mm diameter issues from the nozzle and due to capillary instability breaks up into a train of droplets. The experimental set-up, already described in a previous study [4], has been modified by using a nozzle with a piezoelectric modulator. The droplets generated are of 0.2 mm to 0.4 mm diameter and move at velocities of approximately 1 m/s. Their oscillation is observed by a video camera (CCD) mounted on a microscope. Droplets are illuminated by a series of 15–30 light pulses from an LED. The frequency of the strobe illumination is in the range from 10 to 50 kHz. Multiexposed images of the oscillating droplets are then stored by a computer for subsequent analysis. Among the pictures taken of the droplets, only those are selected, where undoubtedly the axis of symmetry of the droplet is in the plane of the observation and the amplitude of their oscillation is relatively small. In such a case one can assume that the droplet has the form of a spheroid, with its principal axes lying in the plane of observation. They coincide with the axes of the ellipse, which is observed on the video frame. The change of length of axis of the ellipse can be described by the following equation:

$$C(t) = R\cdot[1 + A\cdot\sin(\Omega\cdot t + \varphi)], \tag{3}$$

where A is the oscillation amplitude, Ω is the angular oscillation frequency and φ the phase angle.

The length of the axis C was found with the help of a special numerical procedure fitting ellipses to the registered images of a droplet. The time of registration of droplet oscillations is limited to one or at most two periods. Hence, the FFT methods must be excluded from the analysis of the oscillations. Therefore, the oscillation frequency was evaluted by fitting the expression (3) to the measured length C_i of axis of the ellipses, by minimizing the four-par-

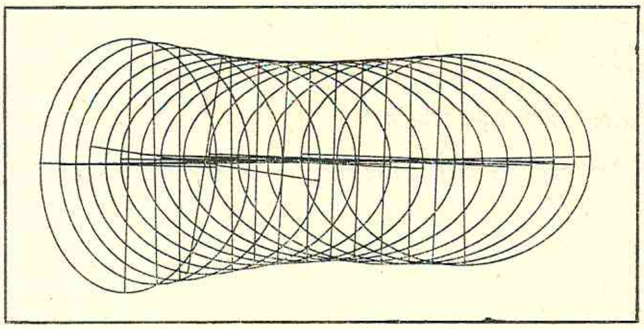

Fig. 1. Example of ellipses fitted to the stored images of an oscillating water droplet (relates to drop nr. 0 in table 1). Strobe frequency 30 kHz

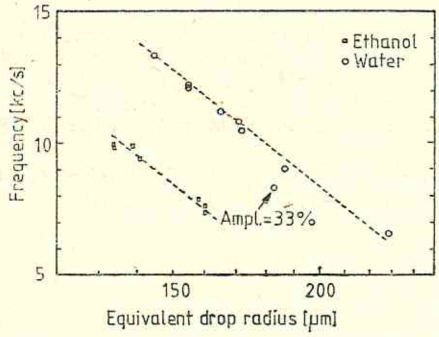

Fig. 2. Measured oscillation frequency as a function of droplets radius

ameter function

$$F(\Omega, \varphi, A, R) = \sum_i [C_i - C(t_i)]^2 , \tag{4}$$

where the lengths C_i of the axes are measured at times $t_i = i/f$, with f being the strobe frequency. The "best fitting" result was then used to calculate the surface tension with the help of formula (2).

In fig. 2 are collected results of measured oscillation frequency as a function of droplet radius. It was noticed that for amplitudes of oscillation below 20–25%, the oscillation frequency can still be well described by the Lamb formula (2). For higher amplitudes the observed frequency of the oscillating droplet has an appreciably lower value. Table 1 collects two sets of measurements for water and ethanol droplets. The calculated surface tension is compared with the value measured by a ring method. It can be noticed however that if the relative error of a single experiment exceeds 10%, the mean value of calculated surface tension agrees very well with the one measured by the ring method. The main source of the error is due to an uncertainty in estimating the droplet radius. We hope that further progress in experimental technique (high resolution CCD camera) will result in an increase of measurement accuracy.

It appears that the method proposed can be developed as a very useful tool for measuring surface tension, especially for all transient processes where the experimental conditions are changing very rapidly.

Table 1. Examples of measured values of surface tension

Nr.	liquid	Ampl. A %	Osc. freq. Ω kHz	Droplet radius R μm	Calc. σ g/s²	Measured by ring method σ g/s²
1	ethanol	11.1	9.84	134	22.76	
2		11.7	9.77	134	22.45	
3		11.6	9.82	139	25.35	
4		14.0	9.33	141	24.20	
5		14.6	7.80	158	23.55	
6		21.2	7.55	160	23.10	
7		20.2	7.29	160	21.35	
Mean value					23.25	23.3
1	water	2.3	13.34	145	68.53	
2		13.0	12.10	155	68.65	
3		7.5	12.19	155	69.82	
4		15.9	11.24	165	71.07	
5		17.5	10.80	171	72.90	
6		15.3	10.45	172	70.19	
7		15.8	10.57	172	70.95	
8		11.8	8.99	187	65.80	
9		7.2	6.60	229	65.07	
0		31.7	8.25	183	52.24*)	
Mean value					69.19	69.3

*) This value, obtained for a large amplitude of oscillations, was not taken into account when calculating mean value

References

1 LAMB, H.: Hydrodynamics. Sixth ed. Cambridge University Press, Cambridge 1932, pp. 473–475.
2 PROSPERETTI, A.: Free oscillations of drops and bubbles: the initial-value problem. J. Fluid Mech. **100** (1980), 333–347.
3 BROSA, U.: Strongly dissipative modes, unpublished (1988).
4 HILLER, W.; KOWALEWSKI, T. A.: Eine einfache Hochgeschwindigkeitskamera mit CCD-Sensor. Bericht Nr. 8/1987, Max-Planck-Institut für Strömungsforschung, Göttingen 1987.

Address: Dipl.-Phys. W. J. HILLER, Dr. T. A. KOWALEWSKI, Max-Planck-Institut für Strömungsforschung Bunsenstraße 10, D-3400 Göttingen, BRD