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UNSTEADY DROPLET EVAPORATION

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SUMMARY

Dynamic and history of vaporization of a single isolated droplet suddenly exposed to an unsaturated gaseous environment has been studied numerically and experimentally. The proposed model of single droplet vaporization includes the effect of the transient cooling in the liquid phase, as well as the effect of variable thermodynamic and transport properties. The results are compared with preliminary observations of 0.2mm ethanol droplets generated by the controlled breakup of a laminar jet discharging into normal atmosphere.

INTRODUCTION

Besides the direct application in combustion droplet evaporation is of importance also in a number of technical processes involving combined momentum, heat and mass transfer to droplets, such as drying and cooling, spray cooling or direct contact condensers, etc. Although intensive research in the area has been undertaken for decades, even "simple" single droplet evaporation problem accounting for the variable transport properties and describing transient effects present at initial stages of droplet life is not yet satisfactorily solved.

In the moment when the liquid of a given temperature is injected into a low pressure environment, the sensible heat contained in the liquid causes its momentary boiling, i.e. flashing evaporation. As a result the liquid temperature (at the surface of the droplet) falls down to the saturation temperature corresponding to the lower environment pressure. Hence the flashing evaporation of the liquid is supposed to terminate within a very short time. Further evaporation of the droplet occurs until its bulk temperature reaches the saturation temperature. During this later phase evaporation is essentially controlled by a heat transfer in the liquid side and as it can be shown the effect of vapour side resistance to the heat transfer (and evaporation) is comparatively small and in most cases negligible.

In the high pressure environment with an neutral gas a similar condition for the evaporation is achieved due to the low partial pressure of the vapor. A high resistance of the neutral gas against created at the droplet surface vapour causes a relatively slow, controlled by diffusion evaporation. Within a short time limited heat transfer to the droplet surface begins also to play deciding role, invoking further decrease of the evaporation rate.

The purpose of this study is to describe, both numerically and experimentally the early phase of droplet evaporation. At the present stage we will analyze evaporation of a spherical liquid droplet in an infinite or constant volume environment of the neutral gas. In the following a simple numerical model allowing to predict an initial history of evaporating droplets is compared with preliminary experiments for slowly evaporating ethanol droplets.

NUMERICAL ANALYSIS

The famous D^2 - law model, with a linear relationship between the droplet surface area and time, assumes a constant temperature at the drop surface. This is only true at the end of the evaporation process, when the so called "wet bulb" temperature of the surface is established. In many processes, especially for relatively large droplets ($R_0 > 50\mu\text{m}$), or high external temperatures, the droplet surface reaches its equilibrium temperature after several hundreds of milliseconds or not at all (it will completely evaporate before). Also when the environmental conditions are changing quickly, for example by spray injection

or shock wave passage, the transient heat conduction in a droplet becomes important.

Therefore to describe the full history of an evaporating droplet a time-dependent droplet temperature must be taken into account. In the present study we solve iteratively the one-dimensional, spherically symmetric model of evaporation of a single liquid sphere in the environment of the neutral gas. The droplet is assumed spherical through its lifetime. The initially uniform temperature distribution in the droplet is changing during the evaporation due to the cooling (or heating) of its surface. The temperature and pressure of the external gas can be modified during evaporation history (finite external gas volume) or assumed to be constant (infinite environment). The properties of the external gas, assumed to be spatially constant, can be quite satisfactory modeled using their values calculated at the reference temperature $T_{1/3} = T_s + (T_{inf} - T_s)/3$ (so called 1/3 rule proven by Hubbard et al. 1975). T_s is temperature of the surface, T_{inf} - gas temperature in the infinity. The history of the evaporating droplet is calculated by solving the total heat equation at the droplets surface, whereas radiative terms in the heat transfer are neglected.

The sum of heat rate transferred from the gas dQ_e/dt through the droplet surface, the rate of change of droplet internal energy dQ_i/dt and the rate of energy used for evaporation of an incremental volume of the liquid phase dQ_l/dt have to be equal zero, i.e.:

$$dQ_e/dt + dQ_i/dt + dQ_l/dt = dQ_{sum}(T_s)/dt = 0 \quad (1)$$

Beginning at time $t_1=0$ with T_s equal to the initial droplet temperature, equation (1) is solved step-by-step. At each time step a new temperature distribution in the droplet

$$f(r)_i = T(r, t_{i-1}). \quad (2)$$

and equation of mass transfer

$$dm_i/dt = \text{Evaporation_model}(T_s, P_s, P_{inf}, \dots) \quad (3)$$

are solved.

The latent heat rate is given by evaporation rate dm/dt :

$$dQ_l/dt = \text{Latent_heat}(T_s, P_s, \dots) \cdot dm_i/dt \quad (4)$$

The functions $\text{Latent_heat}()$ and $\text{Evaporation_model}()$ depend on the surface temperature (T_s), vapour pressure at the surface (P_s), temperature and pressure in the infinity (T_{inf} , P_{inf}), and also on variable properties of fluid and gas, what is indicated in symbolic form by three dots.

Solution of equation (1) results in the temperature T_s , evaporation mass rate dm/dt and pressure P_s . After evaporating a part of the droplet its new mass

$$m_i = m_{i-1} - (t_i - t_{i-1}) \cdot dm_i/dt$$

and new droplet radius R_i are used in the next time step.

Parallel to these equations also relative droplet velocity V_i (momentum equation), thermodynamic properties of liquid and gas are calculated from algebraic equations.

Modelling of the heat and mass transfer. The external heat transfer is given by the following formula:

$$Q_e = 2\pi R_i \lambda (k_T T_{inf} - T_s) (1 + Kn Nu_H / B_H) Nu_H \quad (5)$$

where λ is heat conduction coefficient, Kn - Knudsen number, k_T and B_H are functions only important at large Mach and Knudsen numbers; Nu_H is the Nusselt number modified by Ranz and Marshall (1952) for a droplet moving relatively to the gas:

$$Nu_H = 2(1 + 0.3 Re^{1/2} Pr^{1/3}) \quad (\text{for } Re Pr^{2/3} < 200) \quad (6)$$

Pr and Re are the Prandtl and Reynolds numbers for the droplet.

Heat transfer in the droplet is calculated using solutions found by Carslaw & Jaeger (1959) for the flow of heat in a solid sphere. The temperature distribution in the droplet at time t is given as:

$$T(r, t) = T_s \left[1 + \frac{2R_i}{\pi r} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} e^{-\alpha n^2 \pi^2 t / R_i^2} \sin \frac{n\pi r}{R_i} \right] + \frac{2}{R_i r} \sum_{n=1}^{\infty} e^{-\alpha n^2 \pi^2 t / R_i^2} \sin \frac{n\pi r}{R_i} \int_0^{R_i} r' f(r') \sin \frac{n\pi r'}{R_i} dr' \quad (7)$$

where α is thermal diffusivity of the liquid.

The function $f(r)$ - describes the initial temperature distribution in the droplet (for previous time step). At $t=0$ the function $f(r)$ is equal initial temperature of the droplet.

As the surface temperature and the droplet radius are functions of time, $T(r, t_i)$ must be recalculated for each time step, using previous distribution $T(r, t_{i-1})$ as the initial temperature function $f(r)$. This is the most time consuming step of the presented scheme.

The heat transfer to the droplet interior is found from:

$$dQ_i/dt = 4\pi R_i^2 \lambda dT/dr|_{r=R_i} \quad (8)$$

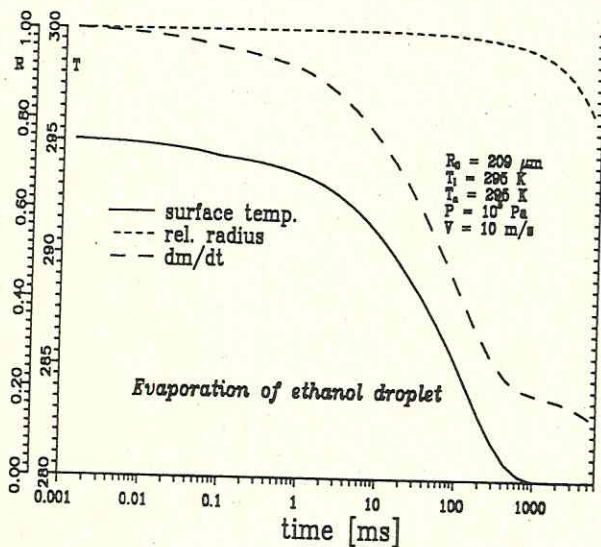


Figure 1 Calculated time history of droplet evaporating in the atmosphere at s.t.p. Evaporation mass rate dm/dt normalized with its maximum value of $1.58 \cdot 10^{-8}$ kg/s.

The calculation of the evaporation rate is done with help of the multirange (wide range of Knudsen number) formula given by Gyarmathy (1982):

$$dm/dt = \frac{2\pi R_i D \rho_{\infty} (\bar{P}_{vinf} - \bar{P}_s) Nu_M}{1 - \bar{P}_{vinf} + Nu_M Kn / B_M} \quad (9)$$

Dashed symbols indicate pressures normalized with the ambient pressure P_{inf} , D is diffusion constant (vapour in gas), P_{vinf} - vapor pressure in the infinity and ρ_{∞} - gas density in the infinity (far from the droplet).

Function B_M is defined as:

$$B_M = \sqrt{\frac{2}{\pi}} \sqrt{\frac{Rv}{Rm}} Sc \left[k_M + 1 - \sqrt{\frac{T_{inf}}{T_s}} \frac{\bar{P}_s}{\bar{P}_{vinf} - \bar{P}_s} \right] \quad (10)$$

Rv and Rm - are vapour and mean (evaluated for external gas-vapour mixture) gas constants, k_M is a function of Mach and Knudsen numbers and can be assumed equal 1 for small Mach numbers.

The modified Nusselt number for the mass transfer (called also Sherwood number) depends on the Schmidt number Sc , Reynolds number and normalized gas pressure \bar{P}_{ginf} :

$$Nu_M = 2(1 + 0.3 Re^{1/2} Sc^{1/3} \bar{P}_{ginf}^{1/3}) \quad (11)$$

For a pure gas environment vapour pressure in the infinity $P_{vinf}=0$ and (9) reduces to Maxwell-Stephan model for diffusive evaporation.

Internal mixing in a droplet. The heat transfer inside the droplet may be additionally modified by internal circulation of liquid (Hill's vortex). The intensity of this movement depends on the relative viscosity of the surrounding atmosphere and liquid and on the relative velocity of the droplet V . It can be shown that the Hill's vortex solution gives the maximal velocity in the droplet:

$$U_{max} = V / (2k+1), \text{ where } k - \text{ is liquid - gas viscosity ratio.}$$

For typical liquids the viscosity ratio k is about 500. Hence, the velocity of internal circulation is below 10^{-3} of the droplet relative velocity V and becomes important for internal mixing after the droplet has passed a distance of more than 1000 diameters. In our experiments, where the typical droplet diameter is $400 \mu m$, first effects of internal mixing could appear at distances larger than 40cm from the nozzle. In the performed experiments droplets are observed typically along distances of a few centimeters. Therefore, the influence of the Hill-Vortex on the internal heat transfer will be neglected in the present study.

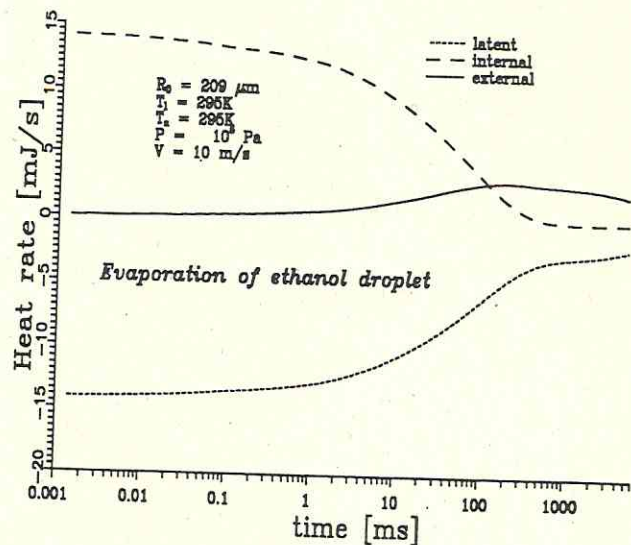


Figure 2. Time dependence of the internal, external and latent heat fluxes for the evaporating droplet from figure 1.

In figure 1 an example of calculated history of ethanol droplet evaporating under normal atmospheric pressure is shown. The initial droplet radius is $209 \mu\text{m}$ and initial temperatures of the droplet and the surrounding neutral gas at infinity are equal and amount to 295 K . The droplet initial velocity is 10 m/s . The variation of the droplet diameter with time is shown together with its surface temperature and evaporation mass rate. It can be seen that temperature of the droplet surface decreases almost linearly with time and approaches its equilibrium value after the relatively long relaxation time of about 0.5s . During this period evaporation is mainly controlled by inner heat transport. Time variations of all three heat fluxes concerned in the present analysis can be seen in figure 2.

It can be seen that at the initial time the evaporation heat rate is almost completely determined by internal heat transfer. At the final stage, (time longer than 0.5s) cooling of the droplet interior diminishes and the external heat flux becomes to control the evaporation rate. The temperature distribution in the droplet, which is shown in figure 3, becomes after this relaxation time practically uniform and the steady evaporation process, described by the D^2 law begins. After about 6 seconds the droplet is evaporated completely. The initial stage of evaporation which is controlled by the droplet thermal relaxation amounts to only about 20% of its total life-time. However, in most cases this transient time is of prime interest (e.g. evaporation of fuel in automotive engines, where the whole evaporation time is determined by the engine cycle).

In the following preliminary experiment this initial behavior of evaporating ethanol droplet will be analyzed during the first 15ms and compared with the proposed numerical modeling.

determination of small changes of the droplet radius (relative error below 0.2%). A detailed description of the apparatus and registration technique has been given elsewhere (Hiller & Kowalewski 1989, Becker et al. 1991).

The images of the drop are recorded periodically by an image processor and stored on a hard disk of the computer. A series of about 300 images allow observation of droplet evaporation and simultaneous recording of several periods of drop oscillations (during 5 - 30ms). The measured time variation of the droplet diameter gives directly the droplet evaporation rate. An additional analysis of the droplet oscillation frequency is used to measure time dependent variation of the surface tension of the droplet. The variation of the surface tension with temperature for the liquid used, shown in figure 4, offers an unique possibility of measuring the surface temperature of an evaporating droplet. The precise measurement of this temperature, one of the most important parameters of all evaporation models, is very difficult. One of our further tasks is to use observed oscillations of the droplet to calculate their surface temperature.

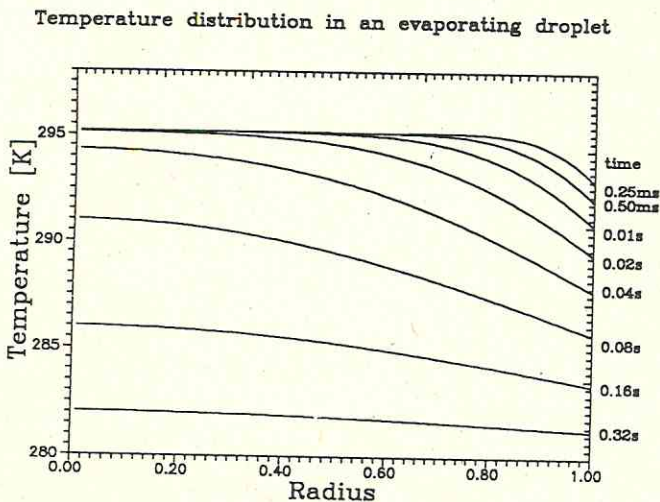


Figure 3. Development of the temperature distribution in the droplet from figure 1.

EXPERIMENTAL

The drops are generated by the breakup of a small-diameter laminar jet discharging into an infinite gaseous environment from a convergent nozzle. The diameter of the nozzle can be chosen in the range from 100 to $300 \mu\text{m}$ so that droplets with diameters in the range from 200 to $500 \mu\text{m}$ can be generated. The pressure inside the plenum chamber of the nozzle is modulated by a piezoceramic device. By a proper choice of the modulation frequency one achieves nearly monodispersed droplets which are oscillating in axially symmetrical modes. The droplets are observed through a microscope in bright field illumination. Their images are registered by a CCD camera, digitized and stored in a computer for further processing. A specially developed image processing software allows very accurate

Surface tension measured by the ring method

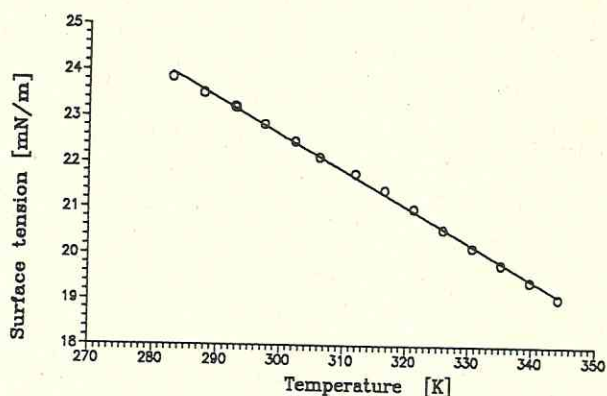


Figure 4. Variation of the surface tension with temperature measured by the ring method for 95% ethanol.

Preliminary experimental results. The experiments have been performed with ethanol (95%) (denatured with methyl-ethyl-ketone) as droplet medium. Droplets have room temperature 295K and are dispersed in air of normal atmospheric pressure. The relative droplet velocity is 10 m/s .

Figures 5 and 6 show variation of the droplet radius and its oscillation amplitude measured from 270 droplet images. The observed decrease of the droplet radius is very small (weak evaporation). However, it can be seen that the observed initial evaporation rate compared with predicted values is almost twice as high. After about 10ms the measured contraction rate of the droplet radius decreases to the calculated values, which are of order 10^{-4} m/s . The observed initial intensification of the evaporation rate is perhaps caused by large oscillation amplitudes of the droplets (see figure 6). The relative motion of the droplet surface induces internal and external circulation of the fluid. It modifies the heat transfer in the droplet and also the heat and mass transfer between droplet and gas. It would be valuable to find out in further studies, how strong droplet oscillations influence evaporation process.

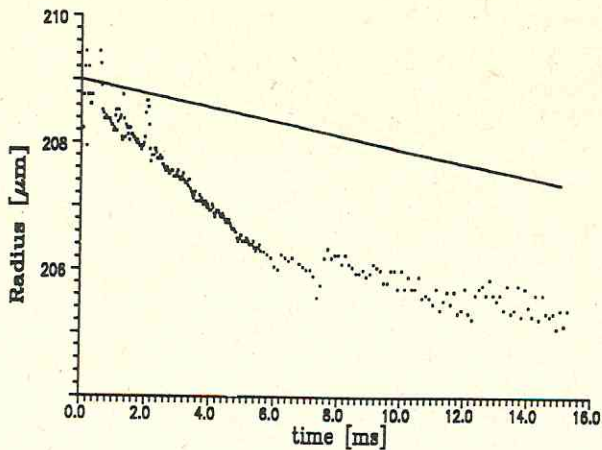


Figure 5. Measured radius variation for the oscillating ethanol droplet, evaporating in normal atmosphere. Initial parameters (droplet radius, temperatures and pressure) are the same as in the calculated example (figure 1). Solid line - calculated variation of the droplet radius (enlarged segment of figure 1).

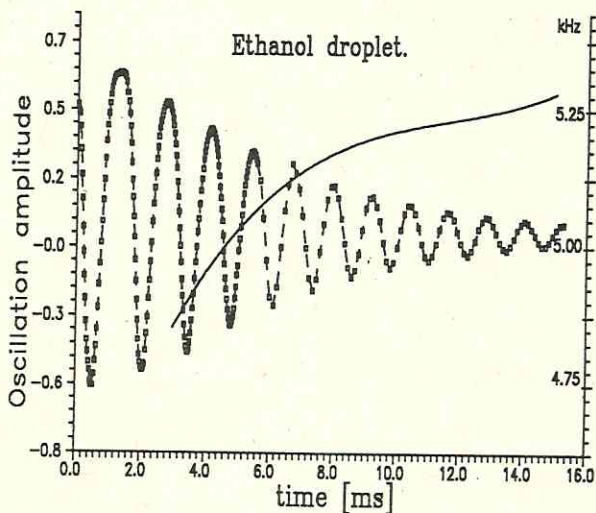


Figure 6. Measured amplitude of the fundamental oscillation mode for the evaporating ethanol droplet. Solid line - approximate variation with time of the oscillation frequency.

It can be noticed that the measured frequency of the droplet oscillations (figure 6) increases during analyzed time. It suggests an increment of the surface tension, meaning cooling of the droplet surface. However a detailed analysis of the droplet oscillation needs to take into account also nonlinear effects, present at large amplitudes. Therefore only part of the measured oscillation frequency shift is due to the variable surface temperature (i.e surface tension) of ethanol. The already developed nonlinear theory of droplet oscillations (Becker 1991) gives us the hope to separate these effects in the near future.

FINAL REMARKS

The described numerical model of an evaporating droplet allows the prediction of its initial history (decrease of droplet radius and surface temperature). Further experiments and the application of a newly developed nonlinear theory of droplet oscillation will hopefully also permit verification of the calculated surface temperature of evaporating droplets.

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