METRO
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Statistics in foundry process control

An overview of basic methods and the SixSigma™ approach

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Education and Culture

Statistical Process Control
Definition and basic elements

Statistical Process Control (SPC) is a complex of activities within an optimisation philosophy concerned with continuous process improvements, using a collection of statistical tools.

Ultimately, SPC seeks to maximise profit by:

- improving product quality
- improving productivity
- streamlining process
- reducing wastage
- reducing emissions
- improving customer service

Statistical Process Control
Definition and basic elements

Basic steps within SPC:

- Make a detailed flow chart of the production process
- Sample of process or product at regular time intervals and make appropriate measurements of the controlled quantities.
- Detect and analyse of process faults and irregularities.
- Use results of the analysis to determine the causes of the process faults and irregularities.
- Eliminate these causes.

Tools for Statistical Process Control

The most important tools used in SPC:

- Flow charts
- Run charts
- Pareto charts and analysis
- Cause and effect diagrams
- Frequency histograms
- Control charts
- Process capability indices

SPC tools
Flow charts

Flow charts have no statistical basis, but are excellent visualisation tools.

They can show:

- progress of work
- flow of material
- flow of information

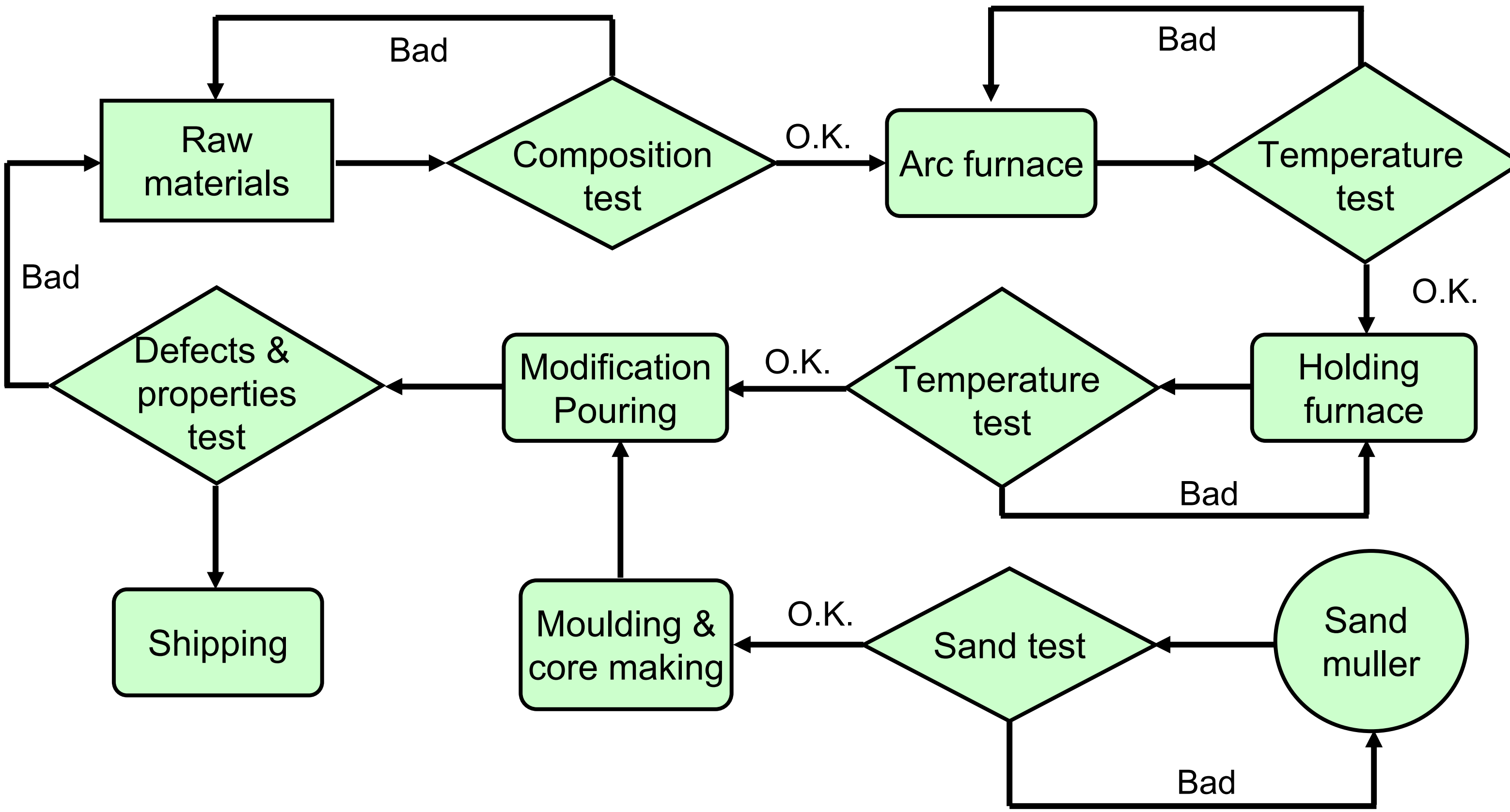
Flow charts can be constructed for:

- designing of product
- designing of manufacturing process
- manufacturing process itself
- control and inspection procedures

SPC tools
Flow charts

- Flow charts are used in an initial process analysis.
- Flow charts should be complemented by process flow sheets or process flow diagrams (more detailed) if available.
- Everyone involved in the project should draw a flow chart of the process part of his/her interest.

SPC tools
Example of a flow chart



Grey cast iron casting production in Crane Valves foundry (USA)

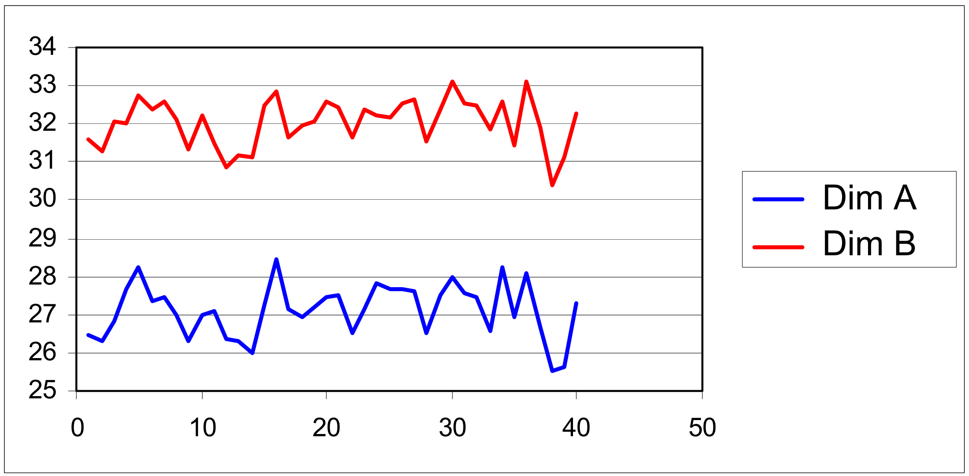
SPC tools
Run charts

Run charts are simply plots of process characteristics against time or in chronological sequence. They do not have statistical basis, but are useful in revealing:

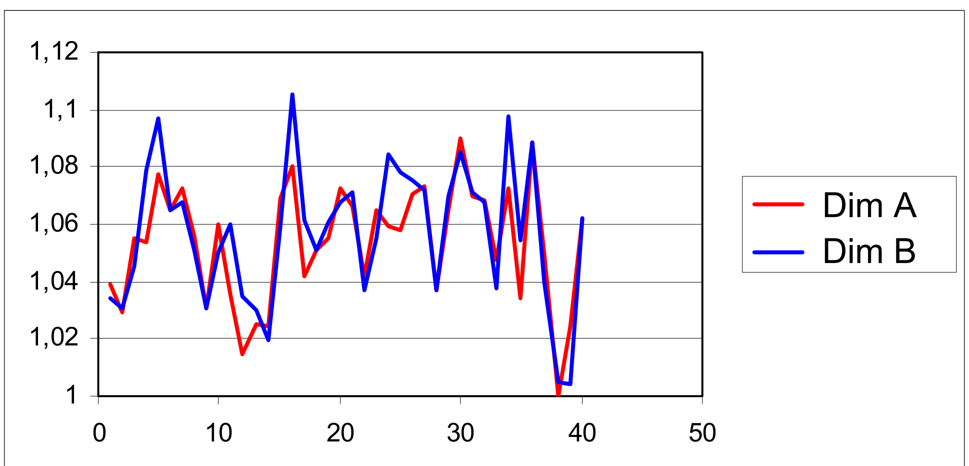
- trends of process or product parameters
- relationships between variables

When used for analysis of a relationship between variables their values are plotted on the same chart. It is useful to convert their actual values to standardised ones.

SPC tools
Run charts



Example of a run chart for two dimensions of a casting (A = 32 mm and B = 27 mm)



Example of a run chart for two dimensions of a casting
in a standardised version
(obtained by dividing their actual values by the maximum ones)

SPC tools
Pareto charts and analysis

The Pareto Principle states that „not all of the causes of a particular phenomenon occur with the same frequency or with the same impact”.

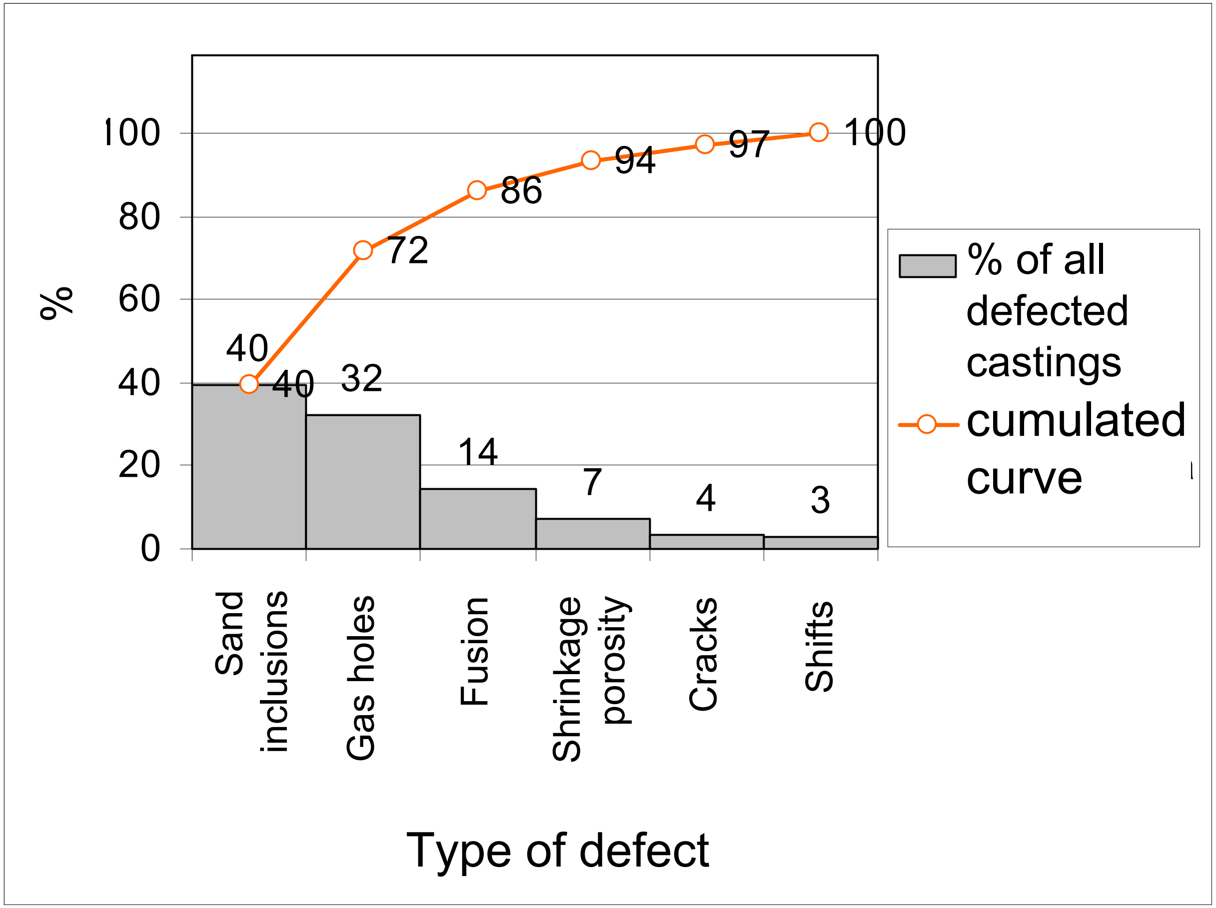
Such characteristics can be presented using Pareto charts, which:

- show the most frequently occurring factors
- help to make best use of limited resources by pointing at the most important problems to tackle

Example: a product may suffer from different defects, but:

- the defects occur at different frequency
- only a few account for most of the defects
  present different defects incur different costs

SPC tools
Example of a Pareto chart



From the chart it can be concluded that the foundry staff should concentrate on reducing 2 defects: ‘sand inclusions ’ and ‘gas holes ’, which make up 72% of all defects

Cause-and-effect (Ishikawa) diagrams

Cause-and-effect diagram is a kind of putting together of factors affecting a process.

Because of its shape sometimes it is also called *fishbone diagram*, or *Ishikawa diagram* (due to the name of its author, professor Kaoru Ishikawa from Tokyo University).

These type of diagrams do not have a statistical basis, but are excellent aids for problem solving and trouble-shooting.

SPC tools
Cause-and-effect (Ishikawa) diagrams

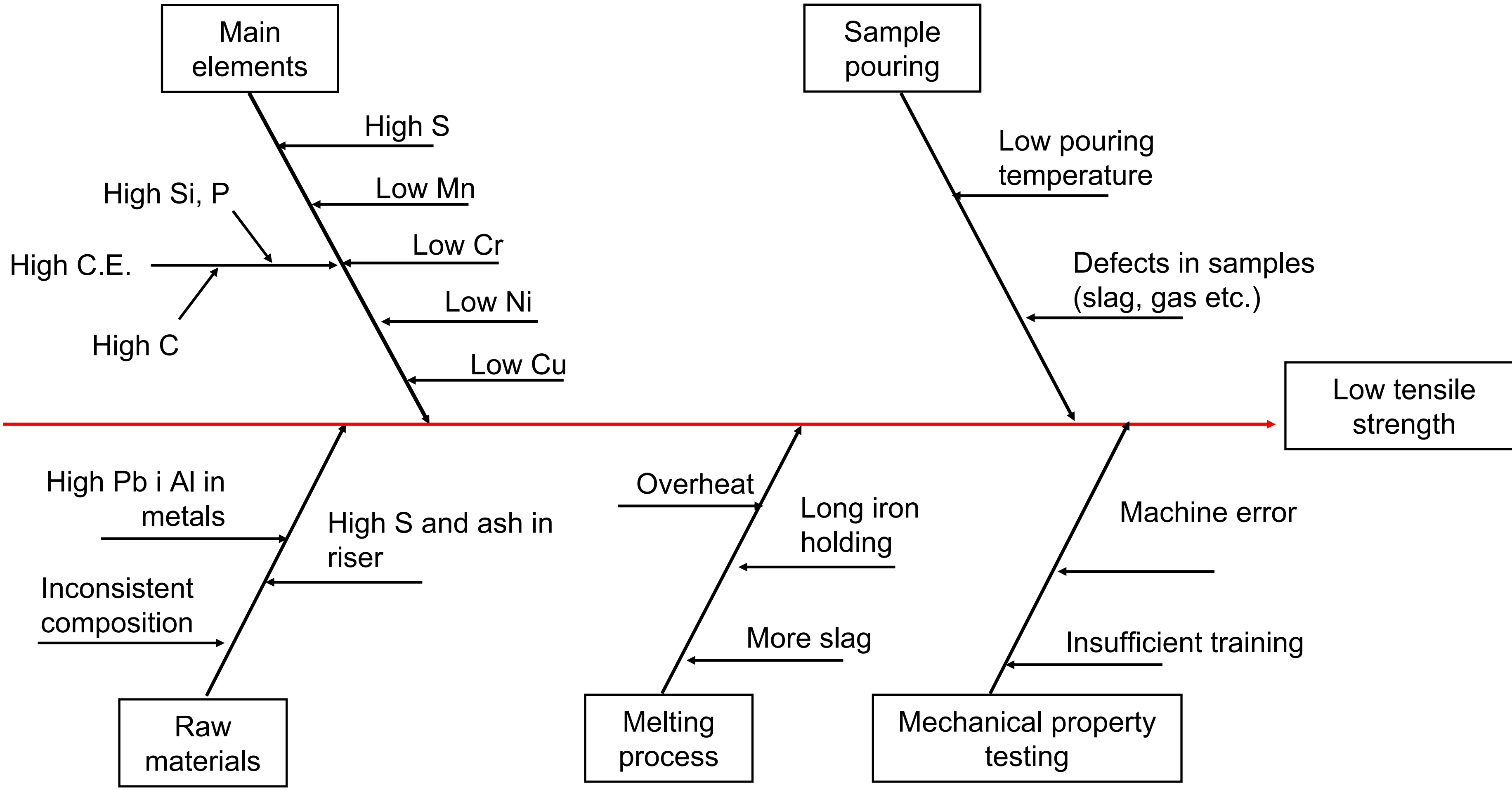
Cause-and-effect (Ishikawa) diagrams can:

- reveal important relationships among various process variables and possible causes of faults
- provide additional insight into process behaviour

Construction of the diagram requires the following consecutively taken actions:

- Make up a flow chart of the process.
- Define the problem to be solved.
- Find all possible causes of the problem (brain storm technique can be used).
- Group these causes into categories.
- Build the diagram which illustrates the relationships between the causes.

SPC tools Example of Ishikawa diagram used for determination of causes of lowered grey cast iron strength (Crane Valves foundry, USA)



SPC tools
Frequency histograms

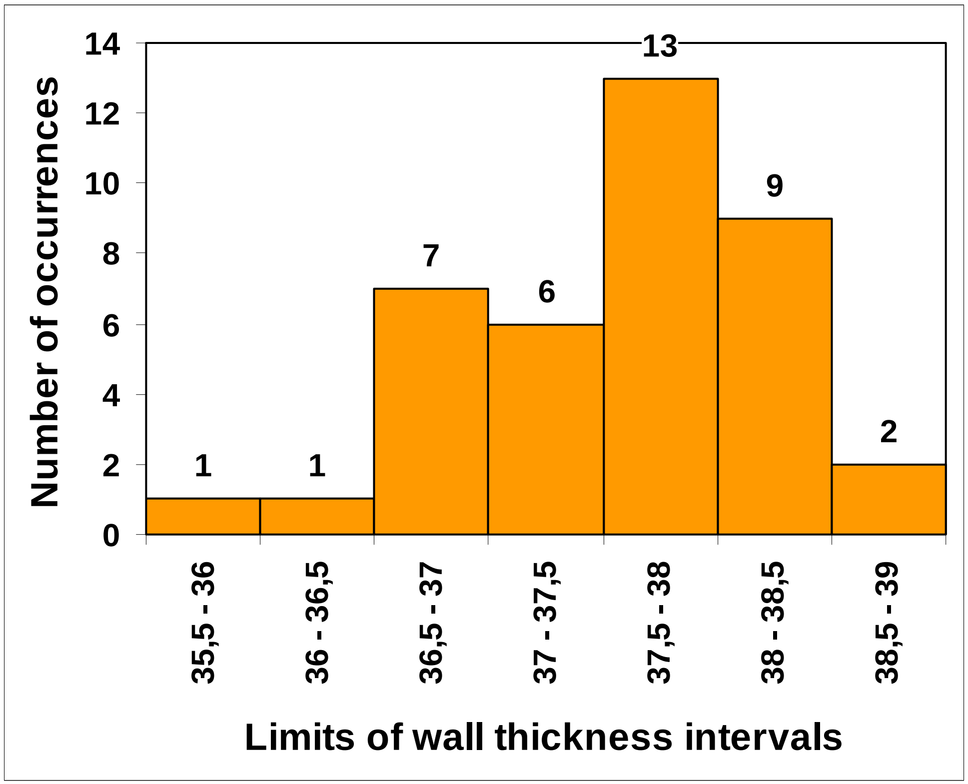
Frequency histogram is a very effective graphical and easily interpreted method for summarising data.

Is a fundamental statistical tool of SPC.

Is a graphical presentation of the selected variable distribution over predefined intervals (classes).

Number of occurrences in a class is illustrated by the height of a bar representing that class.

SPC tools
Example of a frequency histogram



Frequency histogram for 40 measurements of casting wall thickness of its nominal value equal to 38 mm

SPC tools
Frequency histograms

Frequency histograms provide information about:

- the average (mean) of the data
- the variation present in the data
- the pattern of variation
- whether the process is within specifications.

Principles of constructing histograms:

- Intervals should be equally spaced
- Number of intervals is usually between 6 to 20, and
  - small amounts of data require fewer intervals
  - 10 intervals is appropriate for 50 to 200 readings

SPC tools: control charts
Stability and variability of processes

Processes that are not statistically stable can exhibit:

- excessive variations
- variations that change with time

Control charts are used to detect whether a process is statistically stable.

Two types of variations can be distinguished:

- that are normally expected of the process due chance or common causes
- that change over time due to assignable or special causes.

SPC tools: control charts
Stability and variability of processes

Variations due to common causes:

- have small effect on the process
- are inherent to the process because of:
  - the nature of the manufacturing system
  - the way the system is managed
- can only be removed by:
  - making modifications to the process
  - changing the process to a different one
- are the responsibility of higher management

SPC tools: control charts
Stability and variability of processes

Variations due to special causes are:

- localised in nature
- exceptions to the system
- considered abnormalities
- often specific to a:
  - certain operator or team
  - certain machine or device
  - certain batch of material, etc.

Investigation and removal of variations due to special causes are key to process improvement, aided by control charts.

SPC tools: control charts
Definition and construction basics

The principles behind the construction and application of control charts are simple and are based on the combined use of run charts and statistical analysis, in particular hypothesis testing.

A control chart is constructed according to the following procedure:

- sample the process at regular intervals
- plot the chosen parameter (statistic or some measure of performance) for each sample, in a function of its number, e.g.:
  - mean
  - range
  - standard deviation
  - number of defects, etc.
- check (graphically) if the process is under statistical control.

SPC tools
Main types of control charts

Control charts can be used for two types of data: numerical continuous and discrete (attributes and countable).

- Numerical data apply to broadly understood measurements (e.g. dimensions, hardness, chemical composition).
  They are usually of continuous character.
- Discrete data usually come from an acceptance control based on alternative valuation, i.e. where products are classified as ‘good’ or ‘bad’.

SPC tools
Types of control charts for continuous data

Most often used control charts for continuous data are:

- Shewhart sample mean ($\bar{X}$ - chart)
- Shewhart sample range (R - chart)
- Shewhart sample standard deviation (alternative for R - chart)
- Cumulative sum (CUSUM)
- Shewhart cumulative sum (CUSUM)
- Exponentially Weighted Moving Average (EWMA) chart

Basic types of charts are ($\bar{X}$ - chart) and (R - chart)

CUSUM and EWMA charts, using so called sequential tests, facilitate detection of a shift of mean or variance.

SPC tools
Types of control charts for discrete data

Most often used control charts for discrete (attributes and countable) data are:

- sample proportion defective (p-chart)
- sample number of defectives (np-chart)
- sample number of defects (c-chart)

Discrete (attributes and countable) data is simple to collect but they provide less information about process than continuous data.

For discrete data the control charts based on sequential tests are also applicable.

SPC tools
Assumptions for control charts

Constructing of control charts requires two basic assumptions to be made, concerning the plotted statistic (e.g. sample mean or sample range):

- independence assumption (of the current value on previous values and about absence of its influence on the next values)
- assumption about normal distribution of the statistic

Normal distribution is characterised by the following probabilities *P*, that a point will fall outside of an interval determined by the following limits (expressed as a multiple of the standard deviation of the distribution σ):

Interval limits	*P*	Remarks
$\pm\sigma$	31,7 %	
$\pm 2\sigma$	4,55 %	$\bar{x}$ - chart warning limits position
$\pm 3\sigma$	0,27 %	$\bar{x}$ - chart control limits position
$\pm 6\sigma$	0,0000002 %	Used in SixSigma method

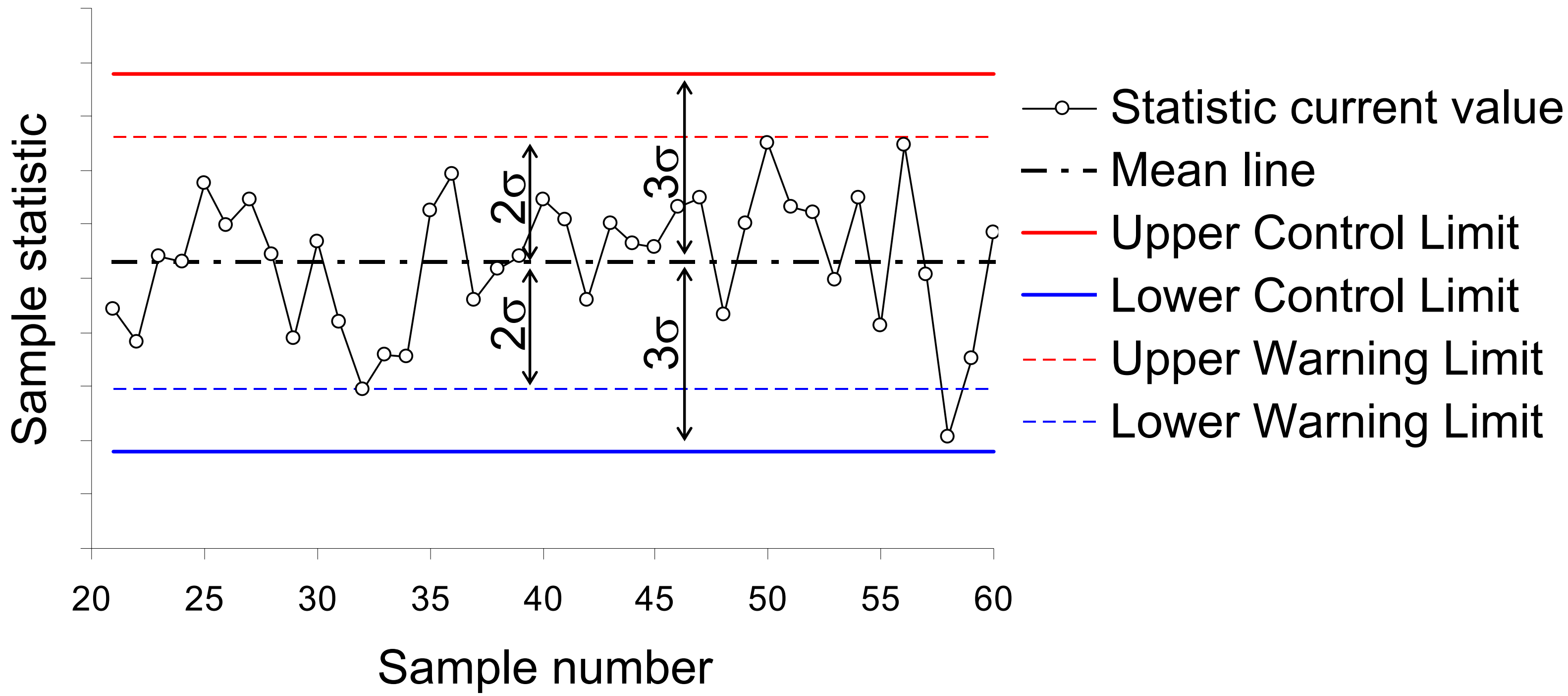
SPC tools: control charts
Principles of data collection

The sample size and frequency of sampling should be determined taking into account that both of them increase:

- probability of detection of process instability
- cost of measurements

Typical sample size is between 3 and 10. Usually all samples have equal sizes, however different sizes are also possible.

SPC tools: control charts
Main elements



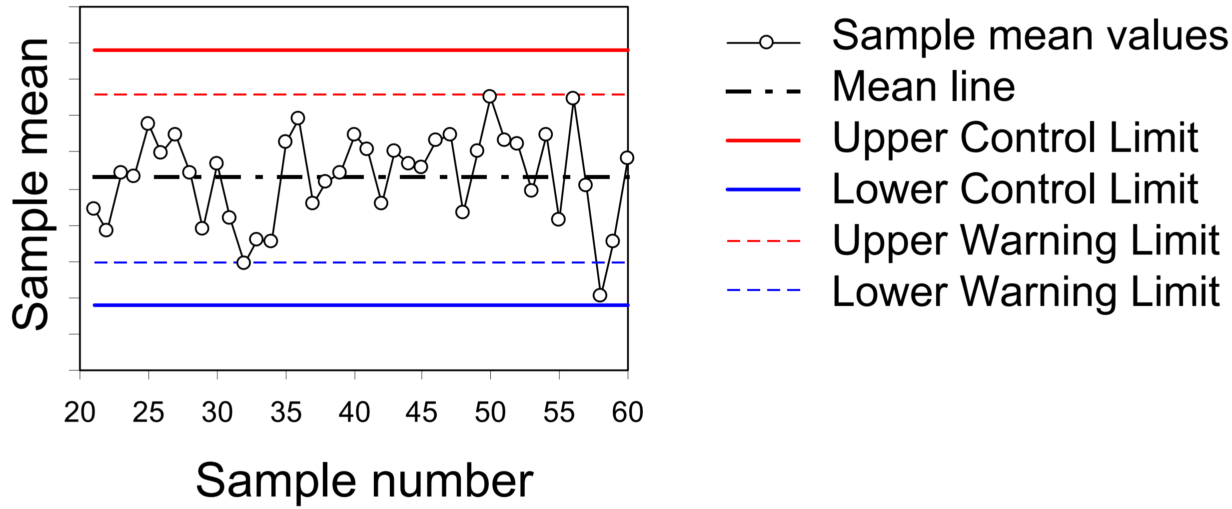
NOTICE:

Position of the mean line and the standard deviation value are calculated for all statistic values on the chart. For example, if the statistic is sample mean, then the mean line position is determined by the mean of the sample means.

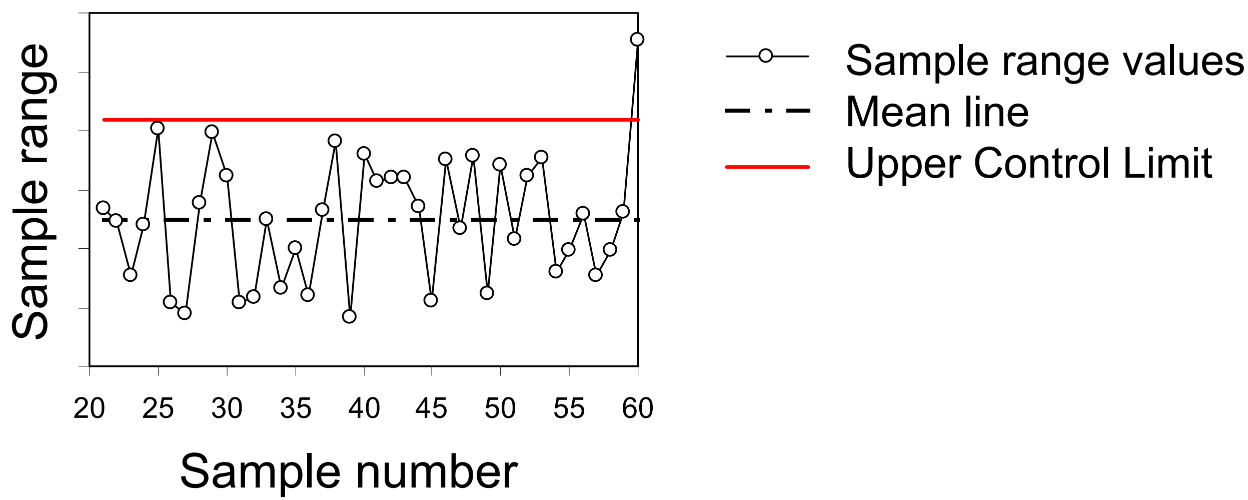
SPC tools: control charts

Examples of basic charts

Example of
$\overline{x}$ - chart



Example of
R - chart



SPC tools: control charts
Run rules

Run rules are rules that are used to indicate *out-of-statistical control situations*. Typical run rules for Shewhart $\overline{X}$- charts with control and warning limits are:

- a point is lying beyond the control limits
- 2 consecutive points lying beyond the warning limits
- 7 or more consecutive points lying on one side of the mean (indicates a shift in the mean of the process)
- 5 or 6 consecutive points going in the same direction (indicates a trend in the process)
- 14 consecutive points going commutatively up and down (two inverse factors are influencing the process, e.g. two material suppliers).

SPC tools
Control charts based on sequential tests

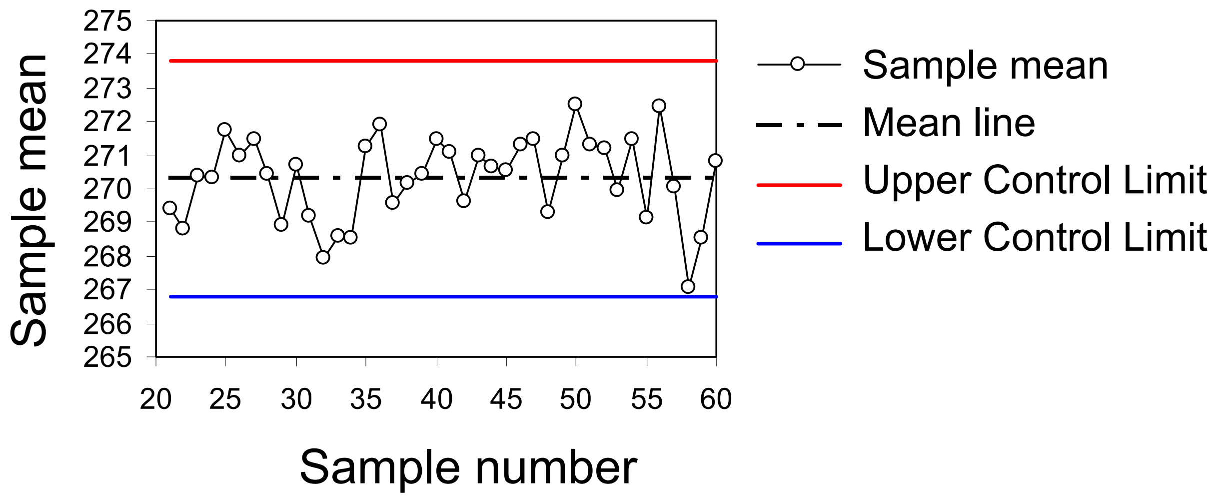
Control charts of CUSUM and EWMA types, using sequential tests, facilitate detection of a shift of mean or variance.

The especially defined statistics are plotted on vertical axis.

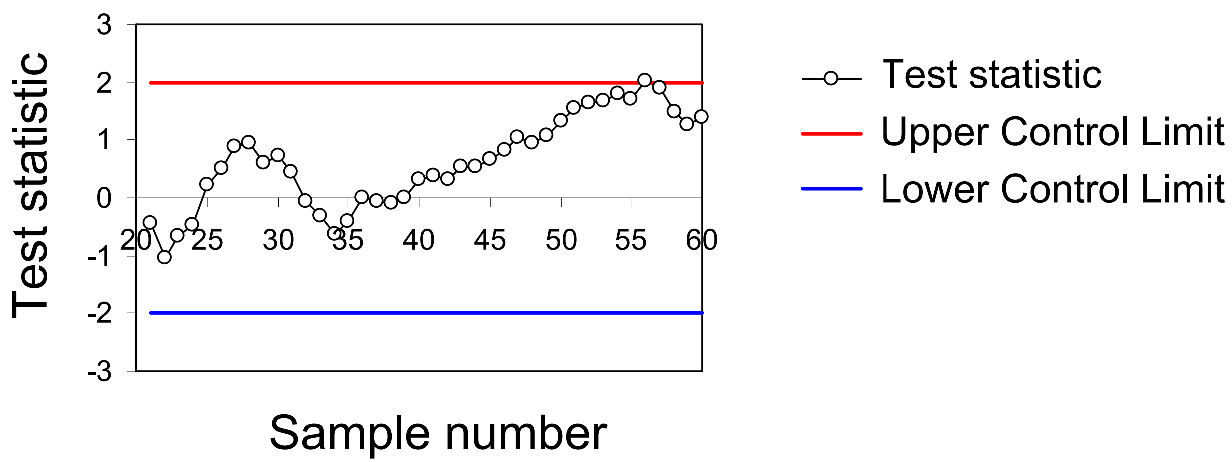
For each point the statistic is calculated on the basis of the previous point or a number of previous points.

SPC tools: control charts
Comparison of capabilities of $\bar{x}$ – chart and Shewhart CUSUM chart

$\bar{x}$ - chart for exemplary data. Detection of the mean shift is practically impossible.



Shewhart CUSUM chart for the same data.
The curve clearly indicates periods of the mean shifts.



SPC tools: control charts
Comparison of capabilities of various types of control charts

Relative merits of different chart types when applied to detect the different changes in a process

Type of change	Chart type		
	Mean	Range	CUSUM
Gross error	+ + +	+ +	+
Shift in mean	+ +		+ + +
Shift in variability		+ + +	
Slow fluctuation	+ +		+ + +
Rapid fluctuation		+ + +	

SPC tools
Process capability analysis

Process capability analysis is aimed at examination whether the process is capable of producing products which conforms to specifications (e.g. for dimension, strength) as well as the variability in process characteristics.

There are essential differences between conformance to control limits (plotted on mean control charts) and conformance to *specification limits* (also called *tolerance limits*):

- Specification limits are usually dictated by a customer, while control limits result from the actual variability (standard deviation) of the process.
- Staying within control limits does not necessarily mean that specification limits are satisfied.
- Staying within specification limits does not necessarily mean that the process is statistically stable.

SPC tools
Process capability index

The *capability index* is defined as a ratio of the tolerance interval to the dispersion of measurements. The latter is assumed to be 6σ ($\pm 3\sigma$), i.e. the distance between control limits.

$$C_p = \frac{T_u - T_l}{6\sigma}$$

where: T_u – upper tolerance limit
T_l – lower tolerance limit
σ – standard deviation

SPC tools
Process capability index

The higher the value of the capability index C_p, the more capable is the process. Characteristic values are:

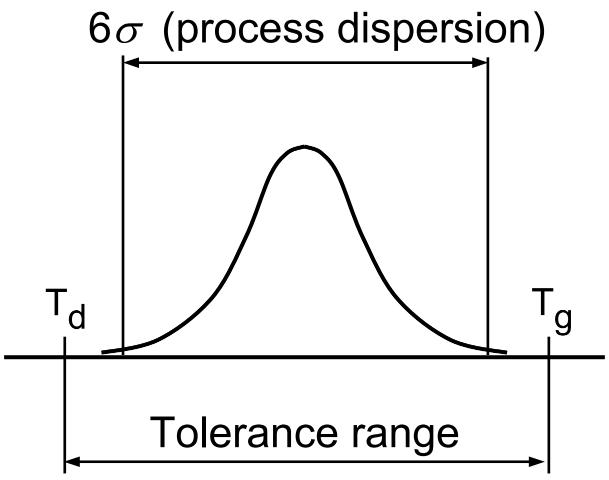
- $C_p < 1$ – process is unsatisfactory
- $1 < C_p < 1.6$ – process is of medium relative capability
- $C_p > 1.6$ – process shows high relative capability.

The value $C_p = 1$ means 0.27 % fraction defective (equal to the % of results beyond control lines). A world standard is $C_p = 1.33$, which corresponds to 0.0063% fraction defective.

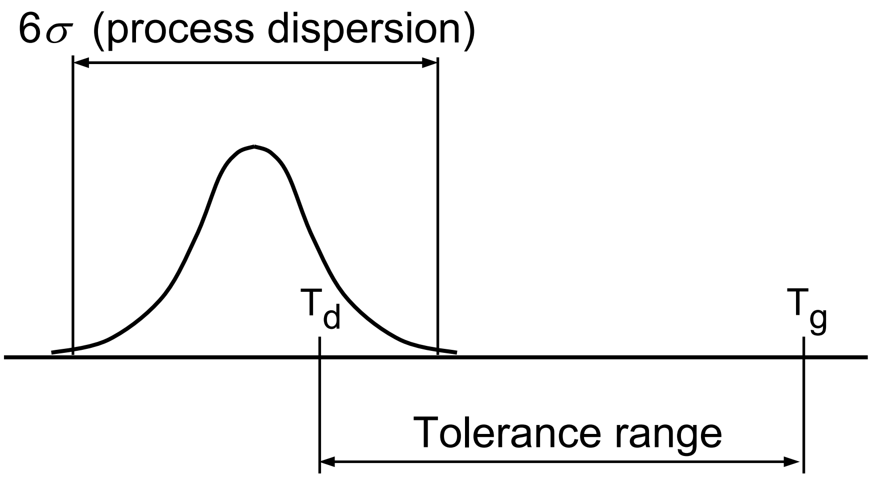
SPC tools
Process capability index

The formula for C_p is simple and intuitive, but it is not satisfactory for evaluation of process performance.

For example, the process having C_p = 1.33, can produce 80% of defective products, if its mean is shifted far beyond the tolerance range:



Centred process C_p = 1.33
0,0063% defective



Shifted process C_p = 1.33
80% defective

SPC tools
Process performance index

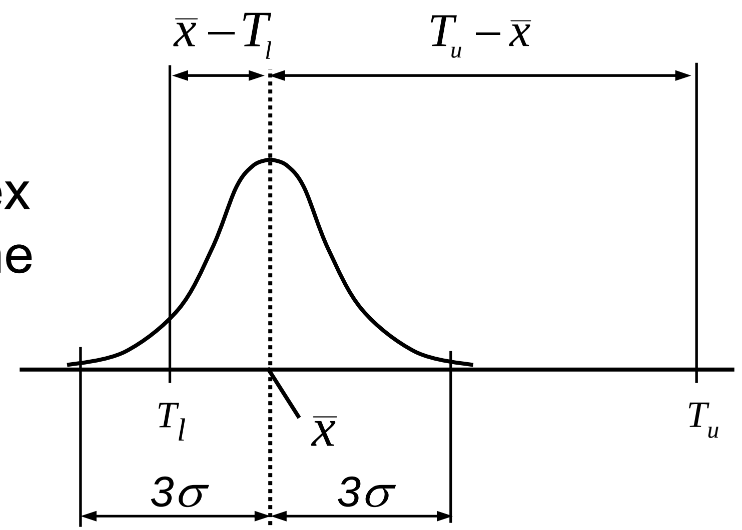
To control the process shift, another index is used, which takes into account the actual process mean: *process performance index* C_{pk}. It is determined according the following procedure:

Two auxiliary indices are calculated, separately for lower and upper tolerance limits:

$$C_{pkl} = \frac{\bar{x} - T_l}{3\sigma} \qquad C_{pku} = \frac{T_u - \bar{x}}{3\sigma}$$

The value of process performance index C_{pk} is defined as the *smaller* (worse) one of those two (C_{pkd} and C_{pkg}).

Indices C_p and C_{pk} are equal to each other, when the process mean is at the center of the tolerance range.



Statistical Process Control
Definition of SixSigma^{TM}

SixSigma^{TM} is a quality oriented system first introduced and formally registered as a trade mark by Motorola in USA.

The general target of the system is to increase profit by substantial *reduction of variablility*, eliminating defects and emissions in manufacturing and service-related processes.

This is closely related to improvement of customers confidence and morale of employees.

Statistical Process Control
Definition of SixSigma^{TM} (continued)

SixSigma^{TM} is a rigorous and disciplined methodology that uses data and statistical analysis.

The reduction of the variability is considered in two aspects:

- Defining measures of the variability and defectiveness as well as methods of their calculation
- Methodology of achievement of those goals, including:
  - organisational issues
  - statistical tools

SixSigma™ methodology
Definition of process variability

Fraction of defective products to be achieved (called SixSigma level) is 0,00034%, i.e. 3.4 defects per million opportunities. A defective product is the one which has any parameter beyond limits specified by a customer.

That target is achieved when the current (i.e. short term) value of standard deviation of the parameter satisfies the following relationship:

$$2 \cdot 6\sigma = T_g - T_d$$

i.e. doubled six sigma is equal to the tolerance interval.

SixSigma^{TM} methodology
Definition of process variability

From the properties of normal distribution it results, that beyond the ± 6σ limits there is only 0,0000002% of population, i.e. 2 products per milliard!

However, it has been found that a typical process is likely to deviate from its natural centring condition by approximately ± 1,5σ, on either side. Such a shift is not included in calculations of the current σ, which means that the process average can approach the tolerance limit by:

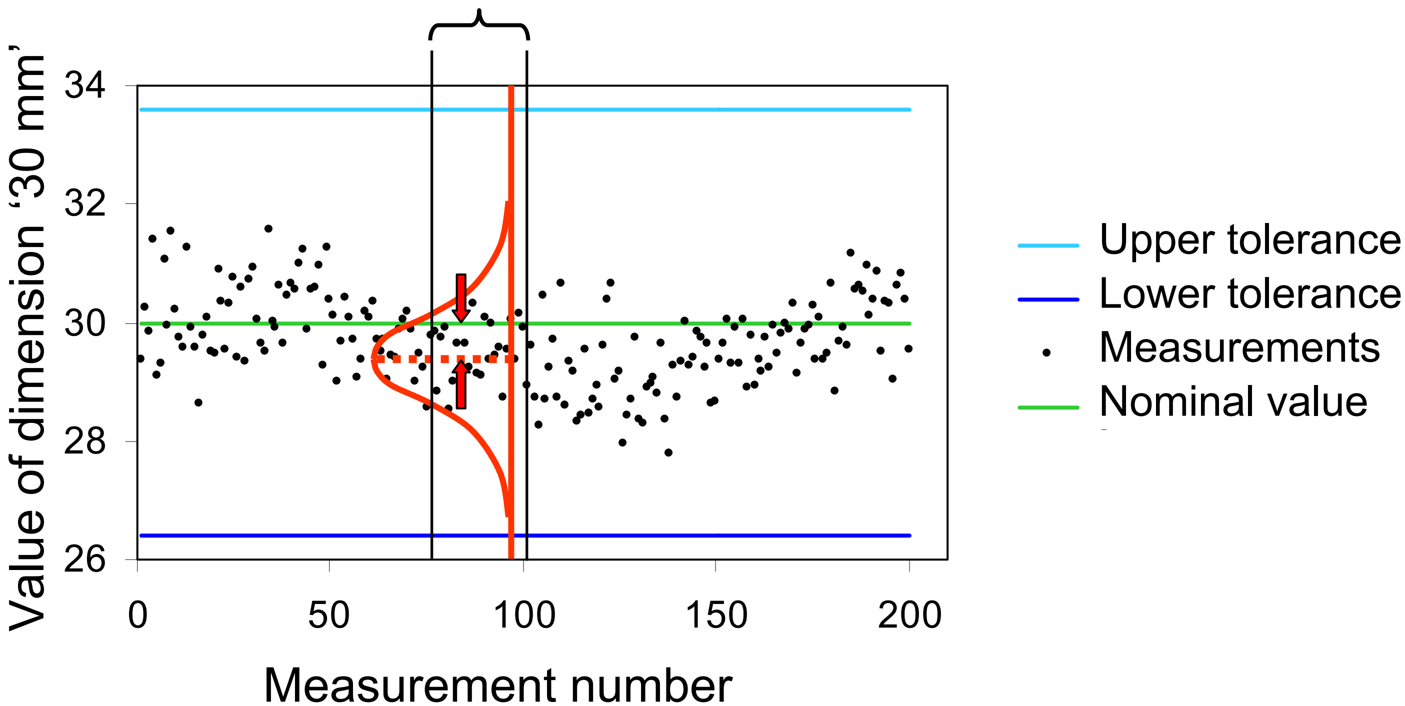
$$6\sigma - 1,5\,\sigma = 4,5\sigma$$

For normal distribution it means that beyond that limit will fall 0,00034% of population, assumed in the Six Sigma methodology.

SixSigma™ methodology
Process control illustration

A series of measurements for calculation of current σ and mean

Exemplary chart presenting the principle of process control in SixSigma™

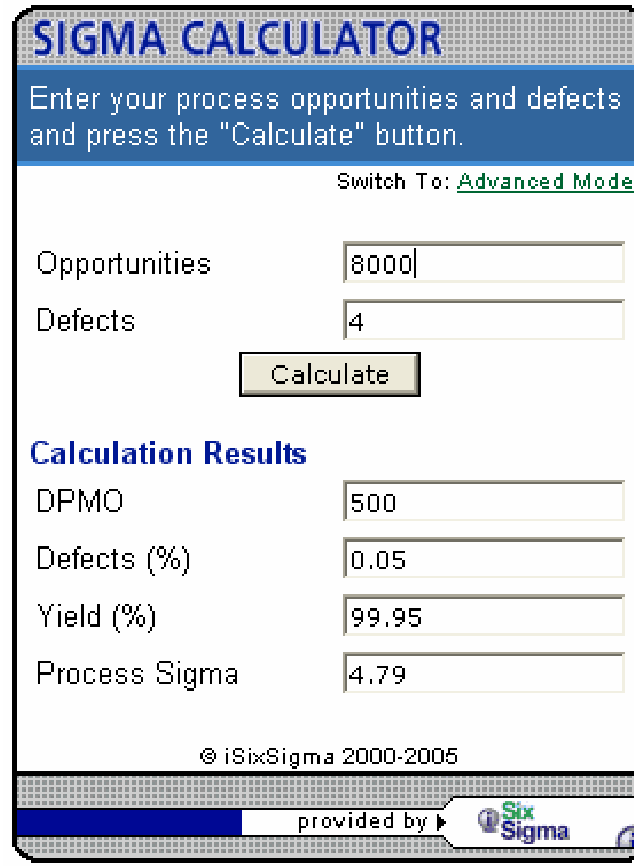


Shift of the process mean in direction of the tolerance limit (max by 1,5σ)

SixSigma^{TM} methodology
Six Sigma calculator

To facilitate calculations of the *achieved nσ* from the fraction of defective products and vice versa, a simple tool called a Six Sigma calculator can be used.

It utilises of the normal distribution function in such a way that the value of nσ reduces to (n–1,5)σ.



SixSigma^{TM} methodology
Implementing the system

SixSigma^{TM} is implemented through the application of an improvement project. This is accomplished through the use of two sub-methodologies:

- DMAIC (Define, Measure, Analyse, Improve, Control)
- DMADV (Define, Measure, Analyse, Design, Verify)

SixSigma™ methodology
Implementing the system

The DMAIC methodology is used for the products or processes already existing in the company, when they fall below specification and need incremental improvement. DMAIC includes the following steps:

- *Define* the project goals and customer (internal and external) deliverables
- *Measure* the process to determine current performance; quantify the problem
- *Analyse* and determine the root cause(s) of the defects
- *Improve* the process by eliminating defects
- *Control* future process performance

SixSigma^{TM} methodology
Implementing the system

The DMADV methodology, instead of the DMAIC methodology, should be used when a product or process is not in existence at your company and one needs to be developed or when an optimisation of existing process (e.g. using DMAIC) was not successful. DMADV includes the following steps:

- *Define* the project goals and customer (internal and external) deliverables
- *Measure* and determine customer needs and specifications
- *Analyse* the process options to meet the customer needs
- *Design* (detailed) the process to meet the customer needs
- *Verify* the design performance and ability to meet customer needs

SixSigma^{TM} methodology
Implementing the system

The SixSigma^{TM} methodology is realised with a use of the statistical tools like those presented earlier in this lecture.

SixSigma^{TM} is implemented in the company by employees having different tasks and qualifications levels, called:

- Green Belts – the lowest level
- Black Belts
- Master Black Belts - the highest level

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Statistics in foundry process control

End of lecture

Education and Culture