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MATHEMATICAL MODELLING OF HEAT EXCHANGE IN TURBULENT SLURRY FLOW WITH ENHANCED DAMPING OF TURBULENCE

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Abstract

The paper deals with modelling of non-isothermal fully developed turbulent pipe flow of Bingham slurry. The slurry contains fine-particles and its concentration varies from 0% to 45% by volume. Mathematical model constitutes time averaged momentum and energy differential equations. The problem of closure was solved by two equation turbulence model in which a new turbulence damping function was used. The new turbulence damping function was previously examined for isothermal slurry flow with enhanced turbulence damping. As such slurry flow exhibits yield stress the Bingham model was chosen in order to calculate apparent viscosity. Finally, for non-isothermal slurry flow, the mathematical model comprises four partial differential equations, which were solved using fine-difference scheme. The mathematical model is able to predict velocity distribution, frictional head loss, and temperature distribution of fine-dispersive slurry with a yield stress in horizontal pipeline. The paper shows that numerical predictions of fine dispersive slurry flow exhibit substantial influence of solids concentration on temperature profile. Results of numerical prediction demonstrate the importance of turbulence damping near a pipe wall and are presented as figures and conclusions. Possible cause of damping of turbulence in the near-wall region is presented.

Key words: Modelling of slurry flow, Heat exchange in slurry flow, Damping of turbulence

NOMENCLATURE

C_i – constant in Launder and Sharma turbulence model, i=1, 2
C_V – solids concentration (volume fraction of solids averaged in cross section), %
c_P – specific heat at constant pressure, J/(kg K)
D – inner pipe diameter, m
f_μ – turbulence damping function at the pipe wall
k – kinetic energy of turbulence, m^2/s^2
Pr – Prandtl number
p – static pressure, Pa
q – input power of heat per unit pipe length, W/m
r – distance from symmetry axis, m
R – inner pipe radius, m
Re_{ap} – Reynolds number for apparent viscosity
Re_t – turbulent Reynolds number
T – temperature, K

U – velocity component in ox direction, m/s
V – velocity component in or direction, m/s
u’, v’ – fluctuating components of velocity U and V, m/s
x – coordinate for ox direction, m
y – coordinate for oy direction or distance from the pipe wall, m
‾ – time averaged

GREEK SYMBOLS

α – heat transfer coefficient, W/(m^2 K)
λ – thermal conductivity, W/(m K)
ε – rate of dissipation of kinetic energy of turbulence, m^2/s^3
μ_τ – turbulent viscosity, Pa·s
μ_{ap} – apparent viscosity, Pa·s
μ_{PL} – plastic viscosity in Bingham rheological model, Pa·s
ν – kinematic viscosity coefficient, m^2/s
ρ – density, kg/m^3
σ_i – diffusion coefficients in k–ε turbulence model, i = k, ε
τ – shear stress, Pa
τ_o – yield shear stress, Pa
τ_w – wall shear stress, Pa

INDEXES

ap – apparent
b – bulk (cross-section averaged value)
i – index, i = 1, 2
m – slurry (solid-liquid mixture)
t – turbulent
w – solid wall

1. INTRODUCTION

Solid-liquid flow, named as slurry flow, appears frequently in chemical engineering, power plants, food and mining industries and is often strongly influenced by heat exchange between the transported materials and the surrounding, (Rozenblit et al., 2000). Solid-liquid flow is classified as stationary bed, moving bed, heterogeneous, and pseudo–homogeneous, or as settling or non-settling types, (Doron et al., 1987). Settling slurries are formed mainly by coarse particles. However, they can also exist with medium and fine solid particles for sufficiently low bulk velocities. When predicting the frictional head loss of slurry flow with coarse or medium particles, it is reasonable to assume the Newtonian model, as now one can measure rheology in such slurries, (Shook and Roco, 1991).

Non–settling slurries contain fine particles and can form stable homogeneous mixture exhibiting increased apparent viscosity. Such slurries usually exhibit yield stress and require an adequate rheological model. Additionally, they demonstrate thicker viscous sublayer, resulting in increased damping of turbulence, which appears in the near-wall region. The phenomenon of thicker viscous sublayer was reported by few scientist including Wilson and Thomas whose contribution is essential, (Wilson and Thomas, 1985). In fine-dispersive slurry a mathematical model should include an apparent viscosity concept with the support of an adequate rheological model. When using the turbulence model in order to calculate turbulent stress tensor, a properly defined wall damping function is also required.

There are several turbulence models dedicated to Newtonian slurry flows, for instance: one-equation turbulence models of Danon et al., (1977), Mishra et al., (1998) or two-equation models of Launder and Sharma, (1974), k-ε-Ap model of Yulin, (1996), Ling et al., (2003). Danon et al., (1977) built one-equation 'k-l' turbulence model using the empirical turbulence length scale. Two-equation k-ε-Ap turbulence model of Yulin, (1996) is built using kinetic energy of turbulence and its dissipation rate the same as in the standard turbulence model for a single phase flow. The 'Ap' is an algebraic equation describing the solid phase. This mathematical model has been successfully examined, however, only for low values of solids concentration.

Stainsby and Chilton, (1996) developed a hybrid model for non-Newtonian slurries in which the apparent viscosity was calculated by the Herschel-Bulkley rheological model at a low strain rate and by the Bingham model at a high strain rate. Using the time-averaged momentum equation and the k-ε turbulence model of Launder and Sharma, (1974), recommended previously by Bartosik and Shook, (1991), they were able to predict frictional head loss and velocity distributions in fine-dispersive slurry flow. They did not include any changes in the k-ε turbulence model. Their hybrid model has been successfully examined only for low solids concentrations and low yield stresses, and for maximum slurry density equal to 1105 kg/m^3.

Sundaresan et al., (2003) outlined that new experiments and/or analyses are needed to cast light on the important phenomena that cause turbulence damping or generation. It has a special importance in case of slurry flows. The authors suggested that the experiments should be conducted in simple turbulent flows such as grid turbulence, fully developed pipe or channel flow, or simple axisymmetrical flows. Regardless of geometry, experiments must include a wide range of particle parameters in a single fixed facility.

The paper deals with non-isothermal solid-liquid turbulent flow in horizontal pipeline. The slurry contains fine solid particles of averaged diameters below 30 μm surrounded by water as a carrier liquid. As mentioned above, it is quite common that such slurries exhibit non-Newtonian behaviour. Mathematical modelling of such turbulent slurry flow requires the momentum and energy equations, an equation or equations to calculate turbulence stress tensor, and a rheological model with the yield stress in order to calculate apparent viscosity. Additionally, the mathematical model requires proper defined turbulence damping function, called also the wall damping function, which is adequate for enhanced turbulence damping which appears in the near-wall region.

The main objective of the paper is to examine the influence of solids concentration on temperature distribution by taking into account the mathematical model with and without additional damping of turbulence. The additional damping of turbulence is included by taking into account the new wall damping function, while the standard wall damping function is used for the other case.

For purpose of this paper the mathematical model for isothermal flow, developed by Bartosik, (2009), was chosen. The model was successfully examined for broad range of yield stresses, plastic viscosity and pipe diameters, (Bartosik, 2009). Such mathematical model requires, however, additional equation, which is energy equation. Final form of the energy equation, in form of equation for temperature, is presented.

The paper shows that numerical predictions of fine dispersive slurry flow exhibit substantial influence of solids concentration on temperature distribution. Results of numerical prediction demonstrate the importance of turbulence damping near a pipe wall.

2. PHYSICAL AND MATHEMATICAL MODEL

The physical model assumes fine–dispersive slurry which exhibits a yield stress. The slurry consists of water and solid particles with density of 2500 kg/m^3. The solids concentration by volume varies from C_V=0% to C_V=45%. It is assumed that slurry viscosity is described by apparent viscosity, which can be assigned by the Bingham rheological model. The apparent viscosity and slurry density are constant across the pipe for isothermal flow and dependent on temperature for non-isothermal flow. The flow in horizontal pipe is fully developed and turbulent. In order to develop a mathematical model for slurry flow with heat exchange, it is assumed that slurry flow is homogeneous and axially symmetrical.

Taking into account the aforementioned physical model, the time-averaged momentum equation in cylindrical co-ordinates can be described as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\mu_{ap}\frac{\partial \bar{U}}{\partial r} - \bar{\rho}\,\overline{u'v'}\right)\right] = \frac{\partial \bar{p}}{\partial x} \tag{1}$$

The turbulent stress component in equation (1) is designated by the Boussinesque hypothesis, as follows:

$$-\rho\overline{u'v'} = \mu_t \frac{\partial \bar{U}}{\partial r} \tag{2}$$

The turbulent viscosity (μ_t), stated in equation (2), is designated with the support of dimensionless analysis, as follows, (Launder and Sharma, 1974):

$$\mu_t = f_\mu \frac{\bar{\rho} k^2}{\varepsilon} \tag{3}$$

The kinetic energy of turbulence (k) and its dissipation rate (ε), which appears in equation (3), are delivered from the Navier-Stokes equations. Earlier research proved that the Launder and Sharma turbulence model has a potential to predict a slurry flow, (Bartosik and Shook, 1991), therefore this turbulence model was chosen for further development, (Launder and Sharma, 1974). The final forms of k and ε equations for the aforementioned assumptions are the following:

- equation for kinetic energy of turbulence:

$$\frac{1}{r}\left[r\left(\mu_{ap} + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial r}\right] + \mu_t\left(\frac{\partial \bar{U}}{\partial r}\right)^2 = \bar{\rho}\varepsilon + 2\mu_{ap}\left(\frac{\partial k^{1/2}}{\partial r}\right)^2 \tag{4}$$

- equation for dissipation rate of kinetic energy of turbulence:

$$\frac{1}{r}\left[r\left(\mu_{ap} + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial r}\right] + C_1\frac{\varepsilon}{k}\mu_t\left(\frac{\partial \bar{U}}{\partial r}\right)^2 = C_2\left[1 - 0.3\exp\left(-Re_t^2\right)\right]\frac{\bar{\rho}\varepsilon^2}{k} - 2\frac{\mu_{ap}}{\bar{\rho}}\mu_t\left(\frac{\partial^2 \bar{U}}{\partial r^2}\right)^2 \tag{5}$$

The turbulent Reynolds number, which appears in equation (5) was defined using dimensionless analysis, as follows, (Launder and Sharma, 1974):

$$Re_t = \frac{\bar{\rho} k^2}{\mu_{ap} \varepsilon} \tag{6}$$

The crucial point of the mathematical model is proper determination of turbulence damping function (f_μ), which appeared in equation (3). Wilson and Thomas, (1985) suggested that in fine-dispersive slurry flow a region close to the wall exhibits increased viscous sub-layer. Therefore the turbulence damping function (f_μ), which is an empirical function, was redesigned in order to predict enhanced damping of turbulence in the near-wall region. The new turbulence damping function, which includes dimensionless yield stress, is described by the following equation, (Bartosik, 1997, 2009):

$$f_\mu = 0,09 \exp\left[\frac{-3,4\left(1+\frac{\tau_o}{\tau_w}\right)}{\left(1+\frac{Re_t}{50}\right)^2}\right] \tag{7}$$

while the standard turbulence damping function at the pipe wall (f_μ), proposed by Launder and Sharma, (1974), is the following:

$$f_\mu = 0,09 \exp\left[\frac{-3,4}{\left(1+\frac{Re_t}{50}\right)^2}\right] \tag{8}$$

The new turbulence damping function (7), compared to the standard function (8), demonstrate enhanced turbulence damping. The new turbulence damping function includes dimensionless yield stress and has been successfully examined in a comprehensive range of rheological parameters and flow conditions, (Bartosik, 2009, 2010a).

In accordance with the physical model, the apparent viscosity in the Bingham slurry flow can be defined as follows, (Bartosik, 2009):

$$\mu_{ap} = \frac{\mu_{PL}}{\left(1-\frac{\tau_o}{\tau_w}\right)} \tag{9}$$

The wall shear stress, which appeared in equation (9), is designated from a balance of forces acting on unit pipe length, so the wall shear stress can be calculated as follows:

$$\tau_w = \frac{dp}{dx}\frac{D}{4} \tag{10}$$

Finally, for isothermal fine-dispersive slurry flow, the mathematical model comprises three partial differential equations, namely (1), (4) and (5), together with the complimentary equations (2), (3), (6), (7), (9), (10).

In order to examine the influence of solids concentration on the heat transfer process, the mathematical model is extended by the following energy equation written for temperature:

$$\bar{\rho}\frac{\partial \bar{T}}{\partial x} + \bar{\rho}\bar{U}\frac{\partial \bar{T}}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\mu_{ap}}{Pr} + \frac{\mu_t}{Pr_t}\right)\frac{\partial \bar{T}}{\partial r}\right] \tag{11}$$

The turbulent Prandtl number, which appeared in equation (11), was intensively examined by several researchers. Their studies indicate that for the flow on a plate, the turbulent Prandtl number is about $Pr_t=0.5$, while for the boundary layer $Pr_t\approx0.9$, (Blom, 1970).

The axial temperature gradient for thermally fully developed flow, which appears in equation (11), is determined from the energy balance acting on the unit pipe length ($\Delta x=1m$), assuming that temperature in ox direction varies linearly. The final form of the temperature gradient in ox direction, which appeared in equation (11), is the following:

$$\frac{\partial \overline{T}}{\partial x} = \frac{2\, q}{\rho_b\, \overline{U}_b \left(c_p\right)_b R^2} \tag{12}$$

while the Prandtl number is calculated using apparent viscosity:

$$Pr = \frac{\mu_{ap} c_P}{\lambda} \tag{13}$$

Finally, for non-isothermal slurry flow, the mathematical model comprises four partial differential equations, namely momentum and energy equations, and equations for kinetic energy of turbulence and its dissipation rate. Partial differential equations, namely (1), (4), (5) and (11), together with complimentary equations (2), (3), (6), (7), (9), (10), (12) and (13), were solved by finite difference scheme using own computer code. The mathematical model is suitable to predict velocity distribution, frictional head loss, and temperature distribution of fine-dispersive slurry with a yield stress in horizontal pipeline.

Numerical calculations were performed for known dp/dx. The turbulence constants in the turbulence model are the same as those in the turbulence model of Launder and Sharma, (1974), and equal: $C_1=1.44$; $C_2=1.92$; $\sigma_k=1.0$; $\sigma_\varepsilon=1.3$, $Pr_t=0.9$. The mathematical model assumes non slip velocity at the pipe wall, i.e. U=0, and k=0, ε=0. Axially symmetrical conditions were applied at the pipe centre, therefore dU/dr=0, dT/dr=0, dk/dr=0 and dε/dr=0. The mathematical model was solved by finite difference scheme. A differential grid of 80 nodal points distributed on the radius of the pipe was used. The majority of the nodal points were localized in close vicinity of the pipe wall to ensure the convergence process. The number of nodal points was set up experimentally to ensure nodally independent computations.

3. NUMERICAL PREDICTIONS

As mentioned above the crucial point of the mathematical model is the turbulence damping function. In order to illustrate the importance of turbulence damping function, Fig. 1 shows the standard wall damping function (solid line) proposed by Launder and Sharma, (1974), and the new wall damping function, proposed by Bartosik, (1997, 2009), for two arbitrary chosen dimensional yield stresses, i.e. $\tau_o/\tau_w = 0.25$ and $\tau_o/\tau_w = 0.50$ (dashed lines with points). The turbulent Reynolds number is defined by equation (6). It is seen in Fig. 1 that for the turbulent Reynolds number in range from zero up to about 100, the new wall damping function gives lower values compared to the standard one while for $Re_t>100$ the difference between the standard and the new wall damping function does not exist. Lower values of turbulence damping function cause lower turbulent stresses. Lower turbulent stresses mean that damping of turbulence exists.

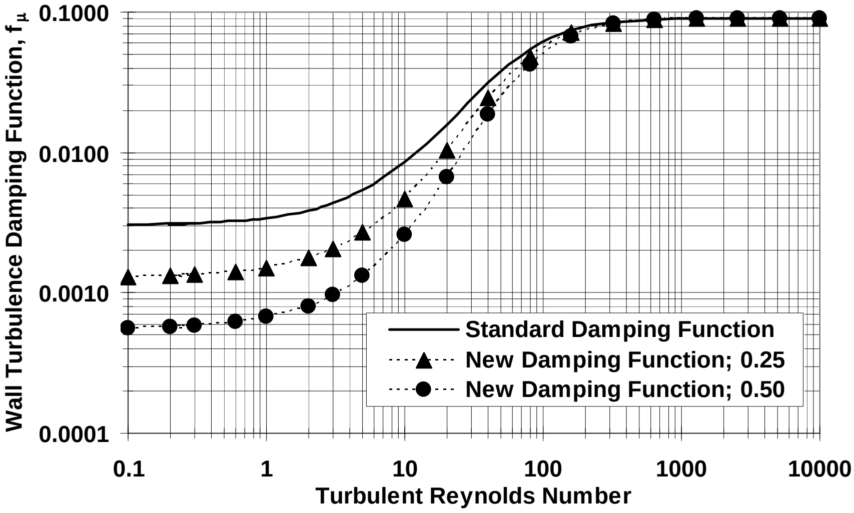

Fig. 1. Importance of turbulence damping function at the pipe wall

In order to emphasize the importance of turbulence damping function on heat exchange in slurry flow, the term "new damping function", which means that calculations were done using equation (7), and the term "standard damping function", which means that calculations were done using equation (8), will be used.

In order to make numerical prediction of the solids concentration influence on temperature distribution in turbulent slurry flow, it was essential to find empirical relations of $\tau_o=f(C_V)$ and $\mu_{PL}=f(C_V)$. Such empirical relations were established by Bartosik, (2011) using experimental data of Slatter, (1994), and Shook and Roco, (1991). It was assumed in numerical predictions that the influence of temperature on slurry properties, like slurry density and slurry apparent viscosity, is qualitatively the same as for carrier liquid, however, in the case of specific heat at constant pressure it was assumed that it is the same as for carrier liquid (c_P=4178 J/(kg K). This is not quite right. However, such assumptions are reasonable when one examines the qualitative influence of the solids concentration on slurry temperature distribution.

Numerical simulations of non-isothermal turbulent flow of fine–dispersive slurry with mean particle diameter below 30 μm were made for the pipe with inner diameter D=0,075 m. Solid particles density was 2500 kg/m^3 and solids concentration by volume varied from 0% to 45%. In order to demonstrate the importance of the chosen turbulence damping function in the model, all predictions were made for constant bulk velocity. The Reynolds number was defined in accordance with the apparent viscosity concept as follows:

$$\mathbf{Re}_{ap} = \frac{\rho_m \left(\mathbf{U}_b\right)_m \mathbf{D}}{\mu_{ap}} \tag{14}$$

It was already demonstrated that the solids concentration influences the slurry velocity profile, (Bartosik, 2010b). Fig. 2 demonstrates predictions of slurry velocity profiles for solids concentration equal to 10%, 20%, 30%, 40%, and 45% by volume for isothermal flow. If the solids concentration reaches 40%, there is a substantial qualitative and quantitative difference between slurry and water velocity profiles. The decrease of local slurry velocity at the pipe wall is compensated by the increase of local velocity in the core region. Such significant differences of velocity shape have to affect the heat transfer process in slurry flow.

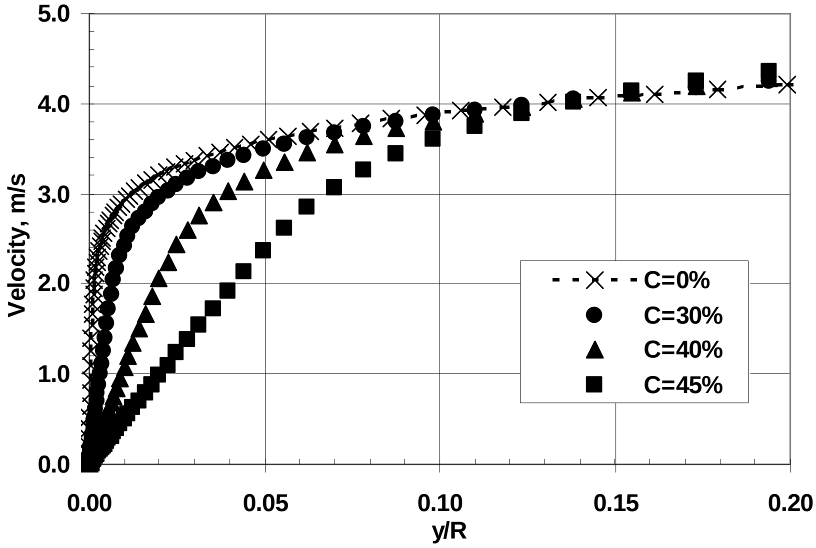

Fig. 2. Dependence of the solids concentration on velocity distribution at the pipe wall for water and Bingham slurry at constant bulk velocity $(U_b)_m$=4.30 m/s, (isothermal flow)

Taking into account slurry flow with heat exchange it was assumed that the wall temperature is constant and equal to 293.15 K. The heat flux, acting on unit length of pipe was applied and equals to Q = -200 W/m. Numerical prediction were made for fine-dispersive slurry with solids concentration equal C_V=20%, 30% and 40%. Density and rheological properties of such Bingham slurry are stated in Table 1.

Table 1. Rheological properties of fine-dispersive Bingham slurry.

C_V %	ρ_m kg/m^3	τ_o N/m^2	μ_{PL} Pa s
20%	1298.56	6.292	$7.63\ 10^{-3}$
30%	1448.74	7.791	$12.39\ 10^{-3}$
40%	1598.92	15.056	$50.51\ 10^{-3}$

Numerical predictions confirmed substantial influence of solids concentration on heat exchange in slurry flow. Fig. 3 shows temperature distribution in slurry and water flow in pipe with inner diameter D=0.075 m. It is demonstrated in Fig. 3 that increasing solids concentration causes the increase in temperature difference $\Delta T=T_b-T_w$. Predictions confirm that even small changes in velocity distribution significantly affect temperature distribution, as shown in Fig. 2 and Fig. 3.

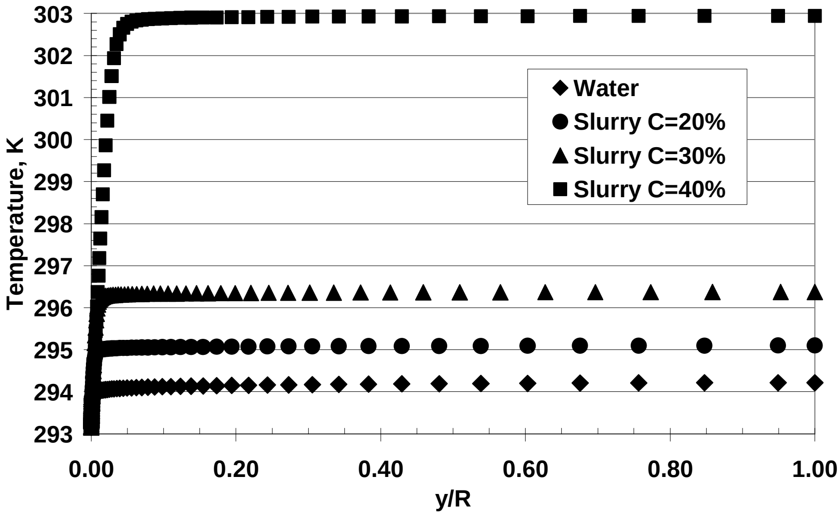

Fig. 3. Temperature distribution in Bingham slurry flow at constant bulk velocity $(U_b)_m$=3.7 m/s, D=0.075 m.

In order to demonstrate the influence of the turbulence damping function at a pipe wall on prediction of temperature distribution across a slurry flow, the standard and the new turbulence damping functions were used. Numerical predictions of temperature distribution in Bingham slurry flow, using two different turbulence damping functions, named the new (7) and the standard (8), are presented in Fig. 4. When the new turbulence damping function is used increased resistance of heat transfer appears. This is coherent with our expectation as level of turbulence is related to the level of heat exchange. This is even more pronounced for higher solids concentration which is shown in Fig. 5. It is worth to mention, however, that in this particular case of calculations, shown in Fig. 5, the apparent viscosity is very high and the Reynolds number is Re=6752.

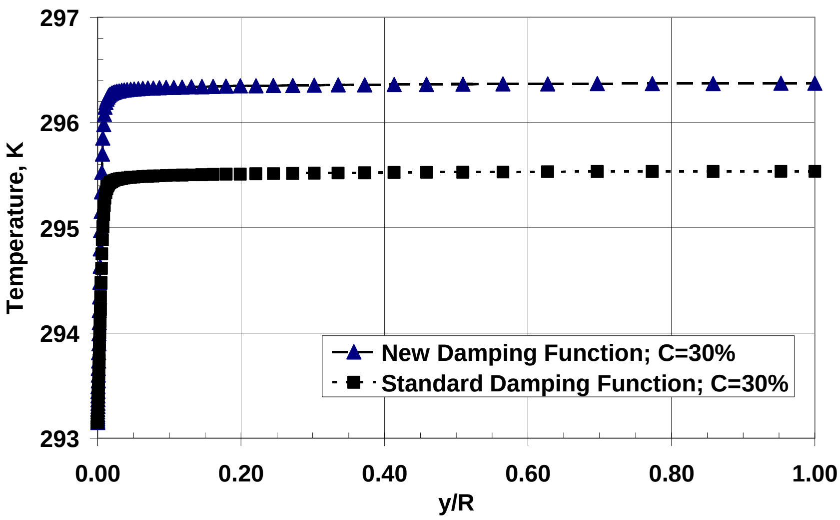

Fig. 4. Temperature distribution in Bingham slurry flow predicted using standard and new turbulence damping functions at constant bulk velocity $(U_b)_m$=3.7 m/s, D=0.075 m, C_V=30%

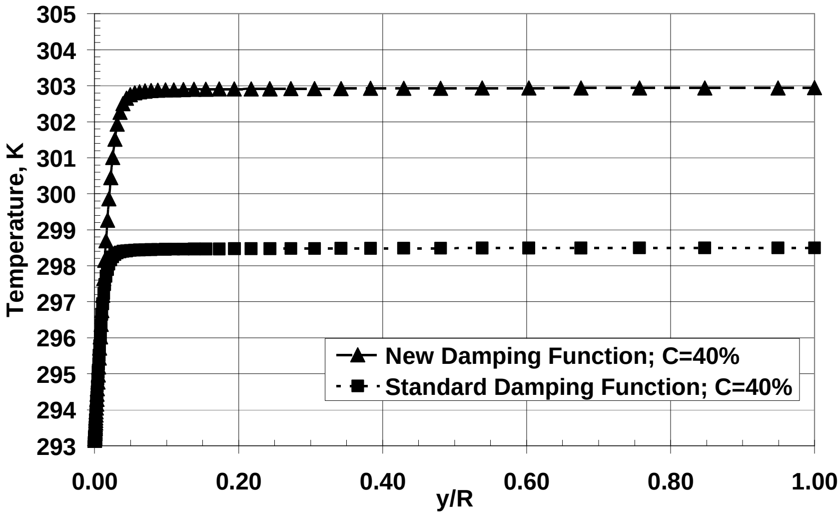

Fig. 5. Temperature distribution in Bingham slurry flow predicted using standard and new turbulence damping functions at constant bulk velocity $(U_b)_m$=3.7 m/s, D=0.075 m, C_V=40%

Increased damping of turbulence, which takes place at the pipe wall, causes reductions of heat transfer coefficient (α). Lower heat transfer coefficient means that for the same heat flux acting on unit pipe length of radius R, and for the same boundary conditions, there is a higher difference of $\Delta T=T_b-T_w$, which is demonstrated in Fig. 4 and Fig. 5. Both Figures 4 and 5 show that using the new turbulence damping function, described by equation (7), which enhances turbulence damping, compared to the standard damping function (8), gives lower heat transfer coefficient.

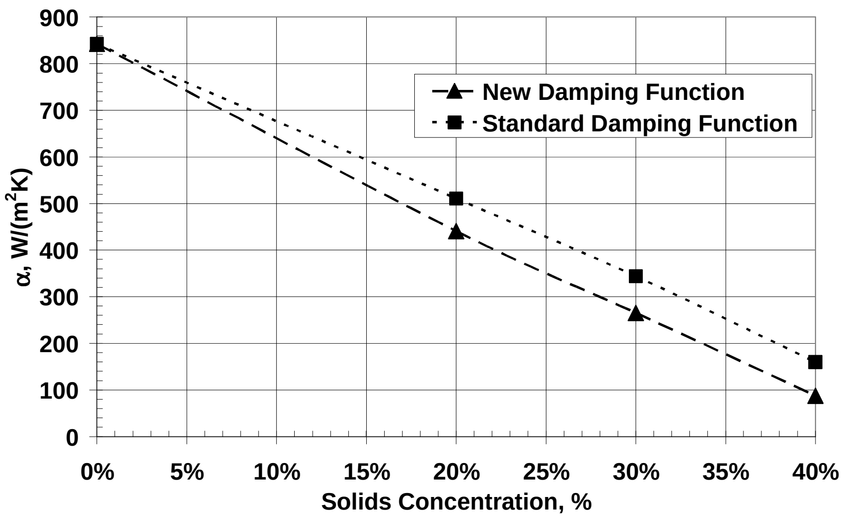

Fig. 6. Dependence of heat transfer coefficient on solids concentration if applying two different turbulence damping functions, D=0.075 m, Q=-200 W/m.

Taking into account numerical predictions of heat exchange in Bingham slurry flow, it was shown in Fig. 6 that the heat transfer coefficient decreases with solids concentration increase. Such conclusion can be made assuming, however, that the wall temperature and the heat flux are constant.

4. CONCLUSIONS

Numerical prediction of Bingham slurry flow exhibits the substantial influence of solids concentration on quality and quantity of the heat exchange process. The paper demonstrates substantial effect of turbulence damping function on velocity profiles and, in result, on temperature distributions.

Numerical simulation of dependence of solids concentration on heat exchange process in turbulent flow of Bingham slurry allows formulating the following conclusions:

1. Solids concentration influence strongly temperature distribution and as consequence heat transfer process. As a result, the heat transfer coefficient decreases with solids concentration increase.
2. Both turbulence damping functions, the standard and the new one, give different velocity profiles at the pipe wall.
3. Assuming that the new turbulence damping function, described by equation (7), is proper, the velocity profile at the pipe wall becomes less steep compared to calculation using the standard turbulence damping function. This is due to viscous forces, which depend on the apparent viscosity and the damping of turbulence, which appears at the pipe wall.
4. Changes of slurry velocity profiles at the pipe wall evoke significant changes in the heat transfer process across the pipe.
5. Less steep slurry velocity profile at the pipe wall results in a decreased heat transfer coefficient (α).
6. It is crucial to use an appropriate turbulence damping function when the heat transfer process in a slurry flow is considered.

Possible cause of '*damping of turbulence*" could be the influence of the solid particles on decreasing time interval of 'bursting phenomena' as particles reduces of higher order fluctuations.

Additional possible reason of existence of damping of turbulence could be the '*lift forces*'. As a result of lift forces large particles are pushed away from the pipe wall and are replaced by fine particles, enhancing the viscous forces of the slurry in vicinity of the pipe wall. If the viscous forces are increasing the '*laminarisation*' of the flow takes place, which was reported by Bartosik, (2008).

References

Bartosik, A., (2011): Solids concentration influence on velocity profile near a pipe wall in Bingham slurry flow, Int. Conf. Transport and Sedimentation of Solid Particles, Wrocław, 6-9 September 2011, pp. 35-46.

Bartosik, A., (2010a): Application of rheological models in prediction of turbulent slurry flow, Flow, Turbulence and Combustion, vol. 84, pp. 277-293.

Bartosik, A., (2010b): Simulation of turbulent flow of a fine dispersive slurry, Chemical and Process Engineering, vol. 31, Nr 1, pp. 67–80.

Bartosik, A., (2009): Simulation and experiments of axially-symmetrical flow of fine- and coarse-dispersive slurry in delivery pipes, Monograph M-11, Kielce University of Technology.

Bartosik, A., (2008): Laminarisation effect in fine-dispersive slurry flow, Archives of Thermodynamics, vol. 29, No. 3, pp. 69-82.

Bartosik, A., (1997): Modification of k-ε model for slurry flow with the yield stress, 10th Int. Conf. Numerical Methods in Laminar and Turbulent Flow, Pineridge Press, vol. 10, pp. 265-274.

Bartosik, A. and Shook, C., (1991): Prediction of slurry flow with non-uniform concentration distribution - comparison of the performance of four turbulence models, Proc. 7th Int. Conf. on numerical Methods in Laminar and Turbulent Flow, Stanford, CA, Part 1, vol. 7, pp. 277-287.

Blom, J., (1970): Experimental determination of the turbulent Prandtl number in a developing temperature boundary layer, Proceed. 4th Int. Conf. on Heat Transfer, vol. VII, Elsevier Amsterdam, Paper FC 2.2.

Danon, H., Wolfshtein, M., Hetsroni, G., (1977): Numerical calculations of two phase turbulent round jet, Int. J. Multiphase Flow, vol. 3, pp. 223-234.

Doron, P., Granica, D., Barnea, D., (1987): Slurry flow in horizontal pipes experimental and modelling, Int. J. Multiphase Flow, vol. 13, No. 4, pp. 535-547.

Launder, B.E. and Sharma, B.I., (1974): Application of the energy-dissipation model of turbulence to the calculation of flow near a spinning disc, Letters in Heat and Mass Transfer, No. 1, pp. 131-138.

Ling, J., Skudarnov, P.V., Lin, C.X., Ebadian, M.A., (2003): Numerical investigations of liquid–solid slurry flows in a fully developed turbulent flow region, Int. J. Heat and Fluid Flow, vol.24, pp.389–398.

Mishra, R., Singh, S.N., Seshadri, V., (1998): Improved model for the prediction of pressure drop and velocity field in multi-sized particulate slurry flow through horizontal pipes, Powder Handling Processing, vol. 10, No. 3, pp. 279-287.

Rozenblit et al., (2000): Heat transfer in horizontal solid-liquid pipe flow, Inter. J. Multiphase Flow, vol. 26, pp. 1235-1246.

Shook, C.A. and Roco, M.C., (1991): Slurry Flow: Principles and Practice, Butterworth-Heinemann, Boston, 1991.

Slatter, P.T., (1994): Transitional and turbulent flow of non-Newtonian slurries in pipes, PhD Thesis, University of Cape Town.

Stainsby, R. and Chilton, R.A., (1996): Prediction of pressure losses in turbulent non-Newtonian flows: development and application of a hybrid rheological model, Proceed. BHR Group, *Hydrotransport-13*, pp. 21-39.

Sundaresan, S., Eaton, J., Koch, D.L., Ottino, J.M., (2003): Appendix 2: Report of study Group on disperse flow, Int. J. Multiphase Flow, vol. 29, pp. 1069-1087.

Wilson, K.C. and Thomas, A.D., (1985): A new analysis of the turbulent flow of non-Newtonian fluids, Can. J. Chem. Eng., vol. 63, pp. 539-546.

Yulin, W.U., (1996): Computation of turbulent dilute liquid-particle flows through a centrifugal impeller by using the k-ε-Ap turbulence model, Fluid Engineering Division Conference, FED, vol. 236, pp. 265-270, vol. 1, ASME 1996.