XX Fluid Mechanics Conference KKMP2012,
Gliwice, 17-20 September 2012

IMPROVED EFFICIENCY OF PRESSURE-VELOCITY COUPLING ALGORITHM WITH WEAK-COMPRESSIBILITY TERMS

Tomasz DYSARZ
Poznan University of Life Sciences, Dep. of Hydraulic and Sanitary Eng., Poznań, Poland;
E-mail: tdysarz@gmail.com, dysarz@up.poznan.pl

Key words: Navier-Stokes equations, pressure-velocity coupling, weakly-compressible flow

The main purpose of the presented research is to incorporate the idea of weak-compressibility into pressure-velocity coupling algorithm to improve the convergence and efficiency of flow solver. The flow solver is based on numerical solution of incompressible Navier-Stokes equations. Two computational strategies are the most popular in this area namely artificial compressibility method and pressure-velocity coupling. The presented approach is some kind of linkage between them. The weakly-compressible flow equations are derived applying a relationship between pressure, density and sound speed with assumption of low Mach number flow. To solve the resulting system of equations some classical time stepping may be used (Bajantri et al., 2007), what is similar to artificial compressibility method. The pressure-velocity coupling algorithm may also be implemented (Munz et al., 2003). In second case the more general algorithm is obtained.

For the purpose of the presented research, the author focus on model described by Song & Yuan (1988) and implemented for the problem of spillway flow simulation by Bajantri et al. (2007). The additional coefficient describing level of weak-compressibility is introduced to the mass balance equation. The mass balance and momentum balance equations written in the Einstein' notation are shown below

$$\beta\dot{p} + a^2\rho u_{i,i} = 0 \qquad \rho\dot{u}_i + \rho\left(u_i u_j - \nu u_{i,j}\right)_{,j} = -p_{,i} \tag{1}$$

In (1), p is pressure and u_i denotes velocity components. ρ and ν are fluid density and kinematic viscosity coefficient, respectively. a is speed of sound. β is additional coefficient describing the level of weak-compressibility. Its range of variability is [0, 1]. The Robin's boundary conditions are used for each variable. The equations (1) are approximated by means of finite volume method in the staggered mesh adopted from Tu & Aliabadi (2007). The velocity components are determined in cell centers, while pressure is approximated in nodes. Multilevel pressure-velocity coupling algorithm based on PISO concept is used. The MGMRES algorithm (Burkardt, 2008) is adopted to solve all linear systems in the problem.

The main idea of the presented method is to use multiple restarts during single time step like in the SIMPLE scheme. During this process coefficient of weak-compressibility β is gradually 'vanishing', what means decreasing from from 1 to 0. The developed algorithm is tested with two well known cases: (1) lid-driven cavity flow, (2) backward facing step. The values of basic physical parameters are as follows: density $\rho = 1000$ kg/m^3, viscosity $\nu = 1$ m^2/s, speed of sound $a = 1500$ m/s. The results are presented for the single set of computational parameters including time step $\Delta t = 1\text{x}10^{-4}$ s and cell size h^2 = 0.2x0.2 m^2. The assessment of efficiency is done by comparison with classical approach based on incompressible flow equations and PISO scheme. Two measures for comparisons are used. An accuracy measure is relative difference in obtained results, namely pressure and velocity components, between analyzed and reference algorithm. Because the volume of this abstract is limited, only the results for one point are presented. The convergence speed is measured as the number of MGMRES iterations to solve Poisson equation for pressure correction systems.

The results are presented in fig. 1. The graph (a) represents the results for lid-driven cavity flow. The graph (b) is prepared on the basis of backward facing step simulation. The abbreviations used means: WCF – weakly-compressible model with PISO scheme, VC 2x1 – vanishing compressibility model with 2 restarts and 1 correction, VC 2x2 – as previous one with 2 restarts and 2 corrections, ICF – incompressible flow model with PISO scheme. The bars represent the MGMRES iteration, the points show relative differences between the results of tested model and reference model (ICF).

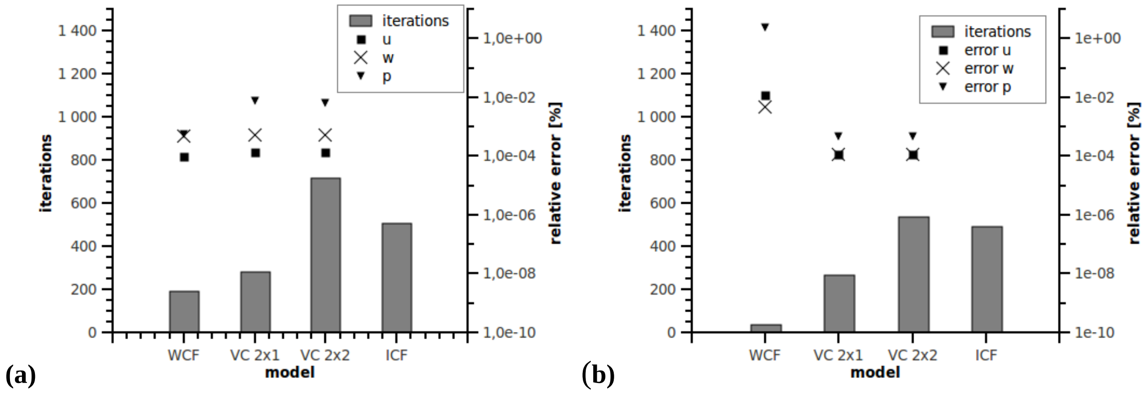

Fig. 1a,b Obtained results: (a) lid-driven cavity flow, (b) flow over backward facing step

VC 2x2 and VC 2x1 models show the same or better compatibility with ICF model in comparison with WCF one. The relative differences are in the range of required MGMRES tolerance in case of lid-driven cavity flow. In the second case, VC 2x2 and VC 2x1 models give differences of few order lower than WCF. The number of necessary iterations may be decreased by smaller number of corrections level. This is well seen in comparison of VC 2x2 and VC 2x1 results in both cases. Hence, the total efficiency of incompressible flow computations may be improved be introduction of weak-compressibility terms.

References

Bajantri, M.R., Eldho, T.I., Deolalikar, P.B. (2007): *Modeling hydrodynamic flow over spillway using weakly compressible flow equations* , J. Hydra. Res., 45 (6), 844–852

Burkardt, J. (2008): *MGMRES, Restarted GMRES solver for sparse linear systems,* *http://people.sc.fsu.edu/~jburkardt/f_src/mgmres/mgmres.html*

Munz, C.-D., Roller, S., Klein, R. , Geratz , K.J. (2003): *The extension of incompressible flow solvers to the weakly compressible regime*, Comp. & Fluids, 32, 173–196

Song, C.C.S., and Yuan, M., (1988): A *weakly compressible flow model and rapid convergence method*, J. Fluid Engrg., 110, 441–445

Tu, S., Aliabadi , S. (2007): *Development of a hybrid finite volume/element solver for incompressible flows*, Int. J. Numer. Meth. Fluids, 55, 177–203