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Simplified adaptivity indicator for edge based mesh refinement.

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Adaptive algorithms require some driving mechanism to trigger computational mesh modification. Residual driven adaptation intends on modifying the computational mesh, so some measure of solution error is decreased. Re-meshing based techniques work on the cell spacing defined by the metric tensor field, constructed from the estimated solution error. This metric field is used by the mesh generator to create a computational mesh. The metric $\mathcal{M}$ is constructed from the solution error as a function of position and estimated solution error $\mathcal{E}(\mathbf{x})$.

$$\mathcal{M} = \mathcal{M}(\mathcal{E}(\mathbf{x})) \tag{1}$$

As, in general there is no direct way of calculating the error of numerical simulation, some estimation must be used. Following the techniques developed in [1], [2], [3], [4], [5], [7] this work employs Hessian of the solution for the purpose of measuring the interpolation error.

Details of the Hessian based approach can be found in [5] or [1]. The main concept of the approach is to assume interpolation error as proporional to the Hessian of the solution.

$$\mathcal{E} \sim \mathbf{h}^{\intercal}\mathcal{H}\mathbf{h} \tag{2}$$

The aim of the current work is to propose a simplified Hessian based formulation of error indicator $\mathcal{I}$, for edge based mesh refinement adaptivity. Such an approach is especially profitable in case of a parallel flow simulation [6], as there is no need for re-meshing during the adaptive step.

Adaptive mechanisms based on the mesh refinement, (and not on mesh re-generation), are restricted to only modifying the existing mesh. The mechanism of creating new elements is limited to splitting the existing ones edges. Therefore, such an adaptive mechanism might employ an adaptive indicator, instead of a full metric field. Such an indicator serves rather as a trigger for the adaptive algorithm, than as a source of spacing information. In case of residual based adaptation $\mathcal{I}$ is based on the measured interpolation error $\mathcal{E}$.

$$\mathcal{I} = \mathcal{I}(\mathcal{E}(\mathbf{x})) \tag{3}$$

We present the derivation of the simplified error estimation. The pros and cons of the proposed method are examined through a computational test case. Finally we present results of mesh adaptation with the use of a metric based re-meshing and refinement based on the error indicator under discussion. Comparison of simulation results is presented as well. Figure 1 shows an example result from a parallel adaptive simulation. On on top shown are contour lines of the Mach field at different stages of the adaptive process. At the bottom the isolines of the corresponding error indicator are shown. Error indicator used was calculated using the Hessian of the Mach field. Simulation was performed in parallel using 16 computational cores of a sheared memory cluster.

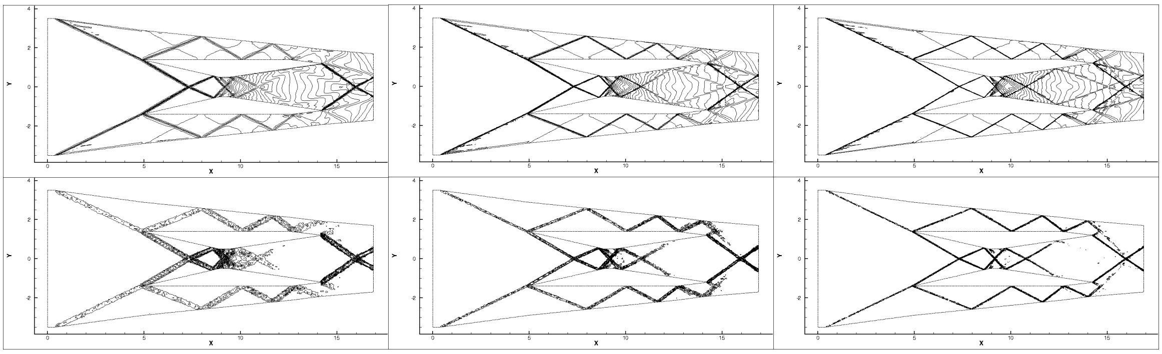

Figure 1: SCRAMJET test case. Isolines of the Mach number field (top). Isolines of the error indicator $\mathcal{I}$ (bottom) calculated using Hessian of the corresponding Mach field. Starting from left: solution on the initial mesh, solution after a single adaptation (middle), solution after a second adaptation (right). Ma=3.0 at inlet. Simulation performed in parallel using 16 computational nodes.

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