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METHODS OF SPATIO-TEMPORAL DATA ANALYSIS

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Abstract

An overview of methods convenient for spatio-temporal data analysis coming from either mathematical modeling (DNS) or advanced optical measurements (TR-PIV) is to be presented. The methods are based on decomposition of multivariate data in both space and time. Energetic methods include Proper Orthogonal Decomposition (POD) and Bi-Orthogonal Decomposition (BOD). Special attention is paid to stability analysis using Oscillating Pattern Decomposition (OPD) method, which is a good candidate for noise and vibration sources identification in flow-structure interaction.

Key words: Spatio-temporal data, turbulence, PIV, POD, BOD, POPs, OPD

INTRODUCTION

Recently the spatio-temporal data is at disposal thanks to advanced mathematical modeling or experimental techniques. Mathematical modeling offers spatio-temporal data form Direct Numerical Simulations (DNS) and from other methods suitable for study dynamics of flow behavior as Large Eddy Simulation (LES), Unsteady RANS or others. In the last decade powerful instrumentation allows to get similar data from experiments using optical methods based on Particle Image Velocimetry (PIV) technique. The time-resolved versions of PIV are now available not only in classical planar version, but also in stereo and volumetric versions.

The time-resolved version means that the data acquisition is performed in accordance with the general rules covering a reasonable part of the fluid system response spectrum. The rules to be met include the Nyquist criterion and the autocorrelation functions of the time series, which should be resolved properly, at least in context of the largest structures characterized by the turbulence integral scale.

In practice this corresponds to the acquisition frequency of order of a few kilohertz for common air turbulence laboratory conditions, for liquids the frequency could be considerably lower. The resolution in space (i.e. size of an interrogation area) and in time (i.e. acquisition period) should be in equilibrium. The same size of structures should be resolved in both domains. The structures of subgrid scales, if present, produce data noise, which could not be used for analysis. However it is not necessary to resolved all scales down to Kolmogorov scale, in general.

DECOMPOSITION METHODS

The spatio-temporal data consists of spatial velocity distributions – snapshots, acquired in time-series. The decomposition methods are based on idea of the Hilbert space, which is defined by all snapshots forming the natural basis of the Hilbert space.

The goal of the decomposition methods is to find another appropriate base with a distinct physical meaning. The new base consists of so-called modes. Clear and relatively simple modes can be derived from complex and extensive data sets, representing even highly turbulent flow, and capturing key features of the underlying dynamical system. This knowledge helps to understand the substance of undergoing physical processes. Moreover, suitably selected modes can be used for low-order modeling and representation of important features of the complex turbulent dynamic system. They also can be used for prediction of the system behavior.

The POD and BOD methods are looking for orthonormal basis corresponding to non-correlated modes maximizing the dynamic data variance, i.e. kinetic energy for velocity data. The OPD method evaluates the basis representing oscillating modes, each of which is characterized by a single frequency and damping.

Energetic Methods

The existence of so-called "coherent structures" in turbulent flows is now well accepted. Lumley (1967) introduced the concept of "building blocks" (i.e. basis of non-specified functions) based on the concept of "energetic modes" on which the velocity field is projected. Extraction of deterministic features from a random, fine grained turbulent flow has been a challenging problem. Lumley proposed the Proper Orthogonal Decomposition (POD) method, an unbiased technique for identifying such structures. The method consists in extracting the candidate which is the best correlated, in statistical sense, with the background velocity field. The different structures are identified with the orthogonal eigenfunctions of the decomposition theorem of probability theory. This is thus a systematic way to find organized motions in a given set of realizations of a random field.

Kinetic energy of spatio-temporal data is defined as half sum of velocity components variances. This means that the highest energy patterns are those with a big amplitude and frequent occurrence. Typical high-energy modes are periodical patterns. In this case the two modes are related to each periodical pattern very often, corresponding to the situations shifted by a quarter of period. For analysis of periodical aspect of such flows the flow-field reconstruction using those two modes is adequate.

The Bi-Orthogonal Decomposition (BOD) represents itself an extension of the POD suggested by Aubry et al. (1991). While POD analyses data in spatial domain only, the BOD performs spatiotemporal decomposition.

The spatial modes are, in general, linear combinations of all snapshots. Temporal modes represent time evolution of the given mode appearance, could be interpreted as projection of a given spatial mode to the snapshots series.

Each mode consists of the energy contents (sum of energy of all local velocity components), the spatial mode (topos) and the temporal mode (chronos). The modes are ordered according to decreasing energy content very often. The original series of snapshots could be fully reconstructed using entire set of modes. Neglecting the high order modes the low-energy random noise could be filtered. The noise could arise in consequence of the process randomness, measurement/evaluation errors or in connection with unresolved subgrid structures in flow.

Both toposes and chronoses form orthonormal bases. To study the embedded system dynamics, the toposes multiplied by square root of energy (i.e. amplitude) could be used to characterize the system evolution in time.

BOD method analyses a deterministic space-time signal (e.g. velocity) $\mathbf{u}(\mathbf{x},t)$, which is decomposed in the following way:

$$\mathbf{u}(\mathbf{x},t)=\sum_k \lambda_k\ \boldsymbol{\varphi}_k(\mathbf{x})\boldsymbol{\psi}_k(t), \qquad (1)$$

the $\boldsymbol{\varphi}_k(\mathbf{x})$ are spatial eigenfunctions (toposes), $\boldsymbol{\psi}_k(t)$ are temporal eigenfunctions (chronoses), λ_k^2 are the common eigenvalues (variances, double kinetic energies). Both toposes and chronoses are normalized to form the orthonormal bases.

Essentially, the BOD and POD methods are based on fundamentally different principles. In fact, BOD can be seen as a time–space symmetric version of the POD. However, the main difference seems to be the assumptions on the analyzed signal, which has to be square integrable only for the BOD, instead of square integrable, ergodic and stationary for the POD. The BOD is a more general method and the POD method should be considered as a particular case. Moreover, the BOD is not derived from an optimization problem of the mean-square projection of the signal as in POD, although the method of calculation of BOD leads also to an eigenvalue problem of a correlation operator. The geometrical interpretation in state space, especially the principal axes of the ellipsoid vanishes in the case of BOD.

Stability Methods

Stability methods are based on modal structures representing temporal or spatial linear evolution dynamics of the flow-field. The methods were introduced in climatology to model temporal and spatial evolution of meteorological data. Oscillating Pattern Decomposition (OPD) method is based on PIPs and POPs approaches introduced by Hasselmann (1988) in the field of climatology. A few attempts of the methods application in general fluid dynamics has been made (see e.g. Garcia & Penland, 1991).

Each stability mode is characterized by a complex frequency involving information on frequency, phase and growth/decay. There are several modifications of the method involving complex or cyclostationary variants.

The OPD method can be applied on time–resolved data only. It is based on stability analysis of the mean flow. Any fluctuation of the flow is considered to be a kind of perturbation, which can either grow exponentially in time (if the flow is unstable), or decay (if the flow is stable). This concept complies with Lyapunov stability theory applied on the mean flow, however finite-time concept is considered instead. As the method is based on the long-time statistics, the mean growth of any pattern should be negative only (i.e. the mean flow is stable), otherwise the structure exceeds the flow-field boundaries in a finite time.

An OPD mode consists of a unique eigenmode and eigenvalue, both represented generally by complex numbers. The complex eigenmode represents a structure or pattern moving in space in a cyclic manner. The structure can be typically a wave or travelling vortex propagating in space. Complex eigenvalue contains information on the cyclical phenomenon frequency and decay in time. This knowledge has a unique feature: it allows the prediction of system development, connected with the given mode. For typical flows with a convective velocity, structures in the form of propagating waves are fairly common. Each spatial pattern of OPD stability mode could be observed in the snapshots series, however it could be hidden in other modes occurring simultaneously.

The OPD method has several advantages. The unambiguous definition of a single frequency connected with a given mode allows assessment of aerodynamic forces and noise and identification of their sources. The knowledge of exact frequency and location of the fluctuating pressure is very important for the definition of the interaction between the flow and a body, as responses including resonance can be predicted.

Knowledge of the convective structures, appearing quasi-periodically in a flow, and their behavior provides important information on a flow-field as a dynamical system. The sensitivity of the flow to a given perturbation detected in the flow can be studied by the OPD method. This feature is directly related to the problem of flow receptivity and allows prediction of the system response to a perturbation appearing in a given flow-field.

Note that both propagating waves and stationary monotonously decaying structures fixed in space are captured by OPD modes. Fixed structures are represented by real modes with vanishing imaginary part of eigenvalue and spatial mode. This means that the system has zero frequency (imaginary part of eigenvalue) and thus does not oscillate. The mode dynamics is represented solely by exponential decay in time.

In this context, the OPD method is advantageous as a unique tool for identification of wavy structures in the flow, and for prediction of their behavior in statistical sense. It provides valuable information on the flow dynamic behavior.

For all these features, OPD is far superior to POD and BOD, provided that applicable time–resolved data is available (meaning that the sample rate is high enough to resolve the highest relevant frequencies in the flow and that the duration of the experiment is long enough to cover several cycles of the lowest relevant frequencies).

In the OPD approach the fluctuating part of Navier-Stokes equation is modeled by Langevin equation for the linear Markov process:

$$\frac{d\mathbf{u}(t)}{dt} = \mathbf{B} \cdot \mathbf{u}(t) + \xi(t) \qquad (2)$$

where $\mathbf{u}(t)$ is vector of velocity fluctuations, $\mathbf{B}$ is the deterministic feedback matrix, $\xi(t)$ is noise driving the system which can be interpreted as influence of smaller, unresolved scales. The OPD modes characterize the deterministic feedback matrix of the system.

The OPD method is suitable for analysis of non-stationary dynamical phenomena in which several processes with different frequencies are involved. OPD is able to decompose and separate such processes, they could be studied separately afterwards. The processes can be of travelling nature; this is the typical case of flowing fluid, while pulsating structures fixed in space are really rare. In Fig. 1 there are 3 types of spatial OPD modes shown schematically. For simplicity only scalar quantity is considered, e.g. vorticity (see also von Storch et al., 1995). The real and imaginary parts of an OPD spatial mode are denoted as P_r and P_i respectively.

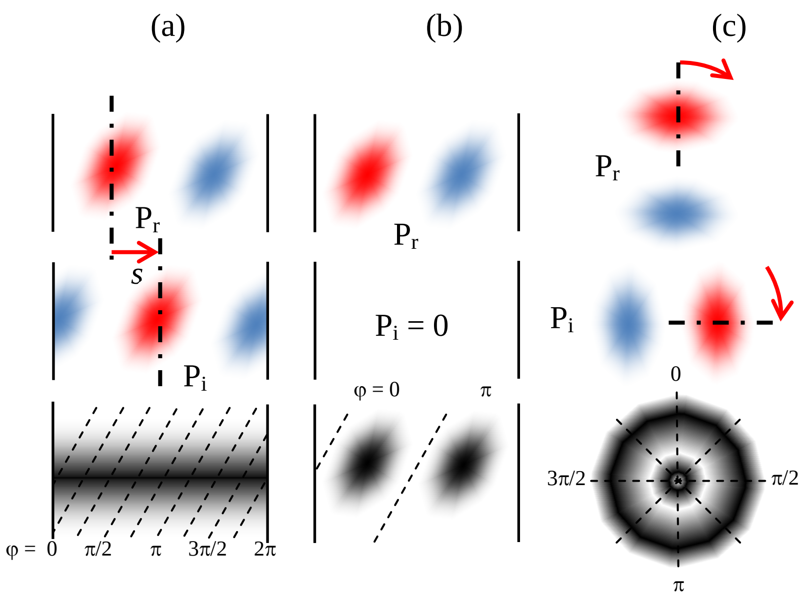

Figure 1 – The 3 types of spatial OPD modes

The upper two rows show the representation in terms of real and imaginary parts P_r and P_i. Bottom row shows representation by phases (dashed curve) and amplitudes (solid curve).

The 3 types correspond to (a) linearly propagating wave, (b) standing wave, and (c) purely rotary wave. For standing wave (b) the imaginary part P_i vanishes.

The OPD modes in (a) and (c) have the amplitudes shown only if they are generated by a uniform phase forcing function. The amplitude distribution in (c) has minima at the origin and outside the outer circle. The maximum is shown by the light curve. The red arrows indicate the structures movement during a quarter of period.

In (a) the structures propagation rate U_p could be calculated from a structure displacement s during the half-period $T/2$ or a mode frequency f :

$$U_p = \frac{2s}{T} = 2sf \ . \quad (3)$$

The standing wave case in (b) corresponds to uniformly decaying non-oscillating mode, because corresponding eigenvalue is real and thus the mode frequency is 0.

The rotation frequency in (c) is given by mode frequency, of course.

However, distinguishing oscillatory and non-oscillatory modes in practical cases is not straightforward (although pure real modes are typically observed). The oscillation of rapidly decaying modes is not very explicit. Really oscillating modes can be defined e.g. as those with decaying amplitude by one order (10-times) during one oscillating period. That means that the ratio $n = \tau_e / T = \tau_e f$ of e-fold time τ_e and oscillating period $T = 1/f$ should be bigger than 0.43. The modes with smaller n can be termed pseudoperiodical or nearly aperiodically decaying modes.

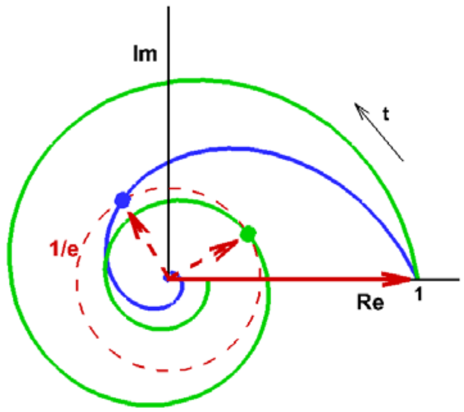

Figure 2 – Time evolution of the POP signal

In Fig. 2 typical evolution the POPs signal amplitude is shown for time 0 represented by the red vector, real part 1 and imaginary 0. In this demonstration two cases are shown with the e-fold time smaller (blue line, $\tau_e = 0.33T$) and bigger (green line, $\tau_e = 1.08T$) than the oscillation period T . The decay amplitude to the value $1/e$ is indicated by the dashed red ring and vectors.

CONCLUSIONS

An overview of methods suitable for analysis of spatiotemporal data is presented. The suggested methods are based on the data decomposition into modes with a distinct features and physical meaning.

The energetic principle reveals the most energetic structures in the flow-field. It allows effective modeling of the dynamical system, reducing the number of degrees of freedom, capturing the maximal energy fraction in the same time. The POD method is shown as well as its generalization BOD.

The stability principle is applied to disclose the modes characterized by a single frequency forming the OPD spectrum. The OPD analysis provides unique information on quasiperiodical phenomena in the flow-field playing a key role in the identification of dynamical pressures. They are important in structure-flow force interaction. The results have direct consequence to evaluation flow-induced vibrations of elastic bodies and generation of aerodynamic noise.

The propagating waves could be studied using the OPD method applied on velocity or pressure spatio-temporal data. As a result of such an analysis, sources of aerodynamic noise and underlying physical mechanisms could be identified. The OPD method has been introduced by Uruba (2011).

Recently the OPD, BOD and POD methods are implemented into the latest DANTEC Dynamic Studio software, version 3.30.

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