METHODS OF SPATIO-TEMPORAL DATA ANALYSIS

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Nowadays we have dynamical velocity vector fields of turbulent flow at our disposal thanks of either mathematical simulations (DNS) or of experiments (time-resolved PIV). Unfortunately, there is no standard method for analysis of such data describing complicated extended dynamical systems, which is characterized by excessive number of degrees of freedom. An overview of candidate methods convenient to spatio-temporal analysis for such data is to be presented. Special attention will be paid to energetic methods including Proper Orthogonal Decomposition (hereinafter POD) and its extension the Bi-Orthogonal Decomposition (hereinafter BOD) for joint space-time analysis. The stability analysis is represented by the Oscillation Patterns Decomposition (hereinafter OPD).

In the presented paper we consider the data acquired in multiple points simultaneously, covering a given measuring zone. The data is resolved in time as well, meaning that the data acquisition is performed in accordance with the general rules covering a reasonable part of the fluid system response spectrum. The rules to be met include the Nyquist criterion and the autocorrelation functions of the time series, which should be resolved properly, at least in connection with the largest structures characterized by the turbulence integral scale. In practice this means that the acquisition frequency is of order of kilohertz for common laboratory conditions in air turbulence, for liquids the frequency could be considerably lower.

The resolution in space (i.e. size of the interrogation area) and in time (i.e. acquisition period) should be in equilibrium. The same size of structures should be resolved in both domains. The structures of subgrid scales, if present, will produce the data noise, which could not be used for analysis.

The spatio-temporal data could be scalars (temperature, concentration, vorticity component) or vectors (velocity vectors with 2 or 3 components). The analysis could be carried out on a spatio-temporal data representing distribution of any physical quantity. The velocity vectors are considered very often, and then the sum variances could be interpreted as a fluctuating system kinetic energy (to be precise twice of it).

The data size representing distribution of a physical quantity in space defines the number of degrees of freedom of the underlying dynamical system. That is number of points for scalars, possibly multiplied by number of components for vectors. Number of snapshots should be smaller or equal to number of degrees of freedom to justify assumption of linear independency of the snapshots.

The measuring zone is typically in shape of a rectangle in plane or a rectangular block in the 3D space.

Turbulent fluid flows behave in characteristic patterns, known as modes. In a recirculating flow, for example, a hierarchy of vortices can be imagined, a big main vortex driving smaller secondary vortices, and so on. Most of the motion of such a system can be realistically described using only a few of those patterns. From a purely mathematical point of view, similar modes can be extracted from the governing equations using eigenvalue

decomposition. But in many cases the mathematical model is very complicated or not available at all. In an experiment, the mathematical description is not available at all and it is necessary to rely on measured data only.

Mathematical implementation of the decomposition methods is based on idea of the Hilbert space, which is defined by all snapshots forming the natural basis of it. The goal of the decomposition methods is to find another appropriate base with a distinct physical meaning. The POD and BOD methods are looking for orthonormal basis corresponding to non-correlated modes maximizing the dynamic data variance. The OPD method evaluates the basis representing oscillating modes, which are characterized by a single frequency and damping.

The energetic methods rely on energetic contents of the modes, which are decorrelated. Unfortunately decorrelation does not mean necessarily independence of the modes. In reality the structures are not independent and they could be deterministic and possibly periodical. The energetic modes are pulsating by definition. They are able to represent only Eulerian structures correctly, however the typical structures are Lagrangian, they are convected by the mean stream.

The stability approach offers more physical definition of structures entrained by mean flow and forming waves. The traveling modes are characterized by periodical topology with decaying amplitude and oscillation period. Those modes are really present in flow and could interact with the flow-field and the boundaries. Moreover, the sufficient set of the modes could be used for to build the model of the dynamical system and it could be used for a short-time forecasting.

References

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