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2D MODEL OF GUIDE VANE FOR LOW HEAD WATER TURBINE ANALYTICAL AND NUMERICAL SOLUTION OF INVERSE PROBLEM

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Abstract

Low-head hydraulic turbines are the subjects to individual treatment of design. This comes from the fact that hydrological environment is not of a standard character. Therefore the method of design of the water turbine stage has a great importance for those who are interested in such an investment. As a first task in a design procedure the guide vane is considered. The proposed method is based on the solution of the inverse problem within the frame of 2D model. By the inverse problem authors mean a design of the blade shapes for given flow conditions. In the paper analytical solution for the simple cylindrical shape of a guide vane is presented. For the more realistic cases numerical solutions according to the axis-symmetrical model of the flow are also presented. The influence of such parameters as the inclination of trailing edge, the blockage factor due to blade thickness, the influence of loss due to dissipation are shown for the chosen simple geometrical example.

Key words: hydraulic turbine, inverse problem in turbomachinery, guide vanes.

INTRODUCTION

The proposed method of the blade vane design is based on the solution of inverse problem within the frame of 2D model. The following algorithm has been presented by (e.g.: R. Puzyrewski, 1998). The starting point is the assumed picture of axis symmetrical flow surfaces *f* bordered by the boundary *AB*, *AC*, *BD*, *CD* as shown in Fig. 1.

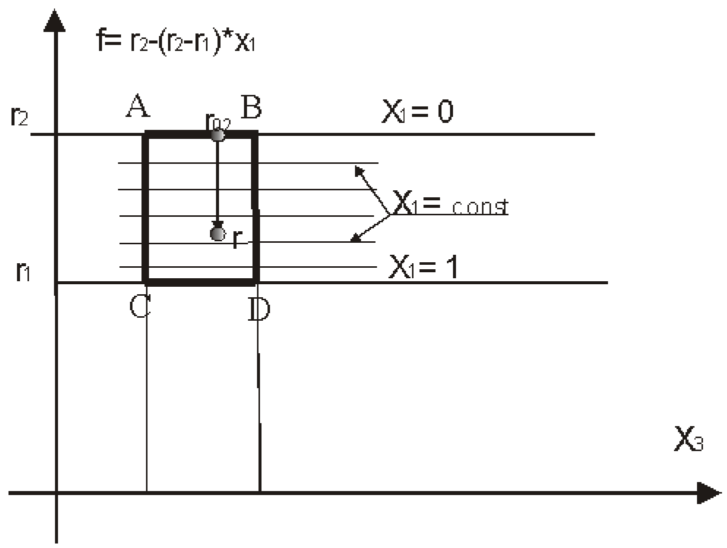

Fig. 1. Meridional shape of the analyzed guide vane.

If one assumes the flow surfaces coordinate as: $x^{(1)}$ = const, the angular coordinate as $x^{(2)}$ = const, and axis coordinate as: $x^{(3)}$ = const, then the set of governing equations in a non-orthogonal (in a general case) coordinate system can be derived as follows (R. Puzyrewski, 1998):

- **Mass flow rate conservation equation:**

$$\left(1-\tau\left(x^{(1)},x^{(3)}\right)\right)\rho U_{x^{(3)}}\frac{f\left|\frac{\partial f}{\partial x^{(1)}}\right|}{\sqrt{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}}=m\left(x^{(1)}\right) \tag{1}$$

where: $f = f(x^{(1)}, x^{(3)})$ – assumed flow surface, ρ – density, $m(x^{(1)})$ – mass flow rate function given at inlet, $\tau(x^{(1)}, x^{(3)})$ – blockage factor due to blade thickness.

- **Momentum conservation eq. in $x^{(1)}$ direction:**

$$sgn\left(\frac{\partial f}{\partial x^{(1)}}\right)\rho\left(-\frac{1}{f}U_{x^{(2)}}^2+\frac{\frac{\partial^2 f}{\partial x^{(3)2}}}{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}U_{x^{(3)}}^2\right)$$
$$=-\left|\frac{\partial f}{\partial x^{(1)}}\right|\left(\frac{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}{\left(\frac{\partial f}{\partial x^{(1)}}\right)^2}\frac{\partial p}{\partial x^{(1)}}-\frac{\frac{\partial f}{\partial x^{(3)}}}{\frac{\partial f}{\partial x^{(1)}}}\frac{\partial p}{\partial x^{(3)}}\right)-\rho\left|\frac{\partial f}{\partial x^{(1)}}\right|\left(\frac{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}{\left(\frac{\partial f}{\partial x^{(1)}}\right)^2}\frac{\partial \Pi}{\partial x^{(1)}}-\frac{\frac{\partial f}{\partial x^{(3)}}}{\frac{\partial f}{\partial x^{(1)}}}\frac{\partial \Pi}{\partial x^{(3)}}\right)+\rho F_{x^{(1)}} \tag{2}$$

- **Momentum conservation eq. in $x^{(2)}$ direction:**

$$\rho\frac{U_{x^{(3)}}}{\sqrt{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}}\left(f\left(\frac{\partial U_{x^{(2)}}}{\partial x^{(3)}}\right)+U_{x^{(2)}}\left(\frac{\partial f}{\partial x^{(3)}}\right)\right)=\rho F_{x^{(2)}} \tag{3}$$

- **Momentum conservation eq. in $x^{(3)}$ direction:**

$$\rho\frac{\partial U_{x^{(3)}}}{\partial x^{(3)}}\left(\frac{\partial U_{x^{(3)}}}{\partial x^{(3)}}-\frac{U_{x^{(3)}}}{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}\frac{\partial f}{\partial x^{(3)}}\frac{\partial^2 f}{\partial x^{(3)2}}\right)=$$
$$=\sqrt{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}\left(\frac{\frac{\partial f}{\partial x^{(3)}}}{\frac{\partial f}{\partial x^{(1)}}}\frac{\partial p}{\partial x^{(1)}}-\frac{\partial p}{\partial x^{(3)}}\right)-\rho\sqrt{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}\left(\frac{\frac{\partial f}{\partial x^{(3)}}}{\frac{\partial f}{\partial x^{(1)}}}\frac{\partial \Pi}{\partial x^{(1)}}-\frac{\partial \Pi}{\partial x^{(3)}}\right)+\rho F_{x^{(3)}} \tag{4}$$

where: $p = p(x^{(1)},x^{(3)})$ – pressure, $U_x^{(2)}$, $U_x^{(3)}$ – tangential and meridional velocity components, $\Pi=\pm g x^{(3)}$ – gravity potential, $F_{x^{(1)}}, F_{x^{(2)}}, F_{x^{(3)}}$ – components of cascade forces acting upon the flow.

- **Energy conservation equation:**

$$\frac{U_{x^{(2)}}^2+U_{x^{(3)}}^2}{2}-U_{x^{(2)}}U_{\mathrm{w}}+cT+\frac{p}{\rho}+\Pi=e_c(x^{(1)}) \tag{5}$$

where: U_w – rotor circumferential velocity (r.p.m), cT – internal energy, changing due to losses, Π – gravity potential. If one assumes T = const it means that no losses in the flow domain exist.

- **Guide vane model condition**

$$\vec{U}\cdot\vec{F}=0 \tag{6}$$

This means that in the flow domain there is no energy extraction from a fluid particle.

The additional simplifying assumptions concerning $F_{x1} = 0$ and behavior of T in the flow domain allow closing the system of 5 equations for the unknowns: U_{x2}, U_{x3}, p, F_{x2}, F_{x3} and seek for the analytical and numerical solutions.

It can be shown that the set of equations, with simplifying additional assumption, can be reduced to the hyperbolic type where two families of characteristics exist i.e.:

$$x^{(1)} = \text{const} \qquad \text{I family} \tag{7}$$

and

$$\frac{dx^{(3)}}{dx^{(1)}} = -\frac{\frac{\partial f}{\partial x^{(1)}}\frac{\partial f}{\partial x^{(3)}}}{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2} \qquad \text{II family} \tag{8}$$

or

$$\frac{dr}{dx^{(1)}} = \frac{\frac{\partial f}{\partial x^{(1)}}}{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2} \qquad \text{II family} \tag{9}$$

Along the second family the pressure satisfies the equation:

$$\frac{dp}{dx^{(3)}} = -\frac{\rho U_{x2}^2}{f\frac{\partial f}{\partial x^{(3)}}} + \rho U_{x3}^2 \frac{\frac{\partial^2 f}{\partial x^{(3)2}}}{\frac{\partial f}{\partial x^{(3)}}\left(1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2\right)} \tag{10}$$

and in the another form:

$$\frac{dp}{dx^{(1)}} = \frac{\rho U_{x2}^2}{f}\frac{\frac{\partial f}{\partial x^{(1)}}}{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2} - \rho U_{x3}^2 \frac{\frac{\partial f}{\partial x^{(1)}}\frac{\partial^2 f}{\partial x^{(3),2}}}{\left(1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2\right)^2} \tag{10a}$$

or

$$\frac{dp}{dr} = \frac{\rho U_{x2}^2}{f} - \rho U_{x3}^2 \frac{\frac{\partial^2 f}{\partial x^{(3),2}}}{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2} \tag{10b}$$

ANALYTICAL SOLUTION FOR CYLYDRICAL CASE

For the simple case of cylindrical vane, the flow surfaces one can assume as follows:

$$f = r_2 - (r_2 - r_1)x^{(1)} \tag{11}$$

where: $r_2 > r_1$, and $x^{(1)} \in [0,1]$ is constant along cylindrical flow surfaces shown in Fig. 2. This is the first family of characteristics according to relation (7). The second family of characteristics, following relation (9) has the form:

$$\frac{dr}{dx^{(1)}} = -(r_2 - r_1) \tag{12}$$

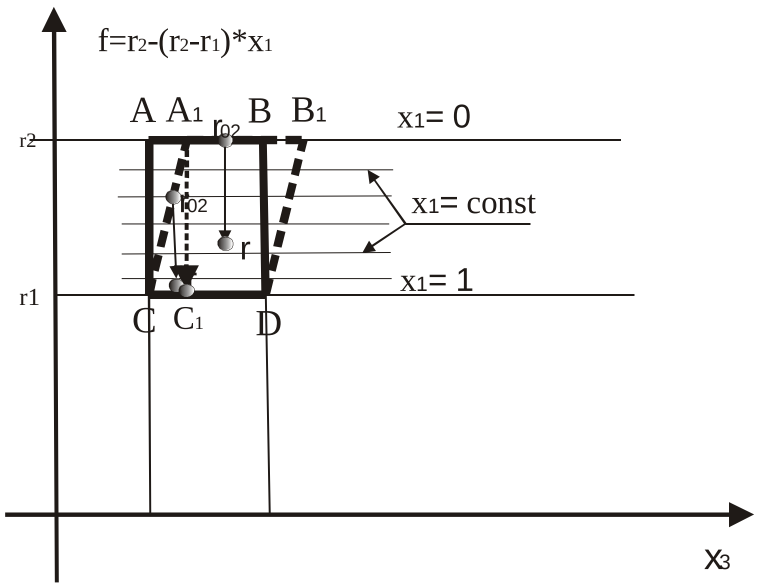

Fig. 2. The sketch of streamlines and characteristics.

The characteristics in Fig. 2 are shown as perpendicular lines starting from the borders *AB* or A_1C. The integral of eq. (12) has the form:

$$r = r_2 - (r_2 - r_1)x^{(1)} \tag{13}$$

Along the characteristics II the equation (10b) has the form:

$$\frac{dp}{dr} = \frac{\rho U_{x2}^2}{r} \tag{14}$$

One can integrate the above equation with additional simplifying assumptions, namely:

1. no losses in the flow cT = const,
2. no blockage due to profile thickness $\tau(x^{(1)}, x^{(3)}) = 0$,
3. uniform distribution of parameters along the leading edge $e_{0c}(x^{(1)})$ = const, and U_{0x3} = const.

From the energy conservation equation (5) we get:

$$U_{x2}^2 = 2(e_{0c} - \frac{U_{0x3}^2}{2}) - 2\frac{p}{\rho} \tag{15}$$

The assumption 3 allows stating that:

$$E_0 = e_{0c} - \frac{U_{0x3}^2}{2} = const \tag{16}$$

Then eq. (14) leads to the solution:

$$p = \rho E_0 - (\rho E_0 - p_{02})\left(\frac{r_{02}}{r}\right)^2 \tag{17}$$

where: p_{02} – pressure at the starting point for $r = r_{02}$ of characteristics II, as in Fig. 2.
The components of velocity vector are:

$$U_{x2} = \frac{r_{02}}{r}\sqrt{2*(e_{0c} - \frac{U_{0x3}^2}{2} - \frac{p_{02}}{\rho})} \tag{18}$$

$$U_{x3} = U_{0x3} = const \tag{19}$$

The above solution depends on boundary condition given as function $p_{02}(x^{(3)})$ along the border AB or A_1C as shown in Fig. 2.
For the case of linear change of pressure along the order AB:

$$p_{02}(x^{(3)}) = p_{00} - \Delta p_{AB}\left(\frac{x^{(3)} - x^{(3A)}}{x^{(3B)} - x^{(3A)}}\right) = p_{00} - \Delta p_{AB}\overline{x}^{(3)} \tag{20}$$

the mean value of pressure drop across the blade guide vane is given by:

$$\overline{p_{BD}} \overset{def}{=} \frac{1}{r_2 - r_1}\int_{r_1}^{r_2} p_{BD}\,dr \tag{21}$$

$$\overline{\Delta p_{BD}} = p_{00} - \overline{p_{BD}} = \Delta p_{AB}\frac{r_2}{r_1} \tag{22}$$

which allows estimating the outlet angle as:

$$\overline{\alpha_1} = ArcTan\left(\frac{m}{\pi}\frac{1}{\sqrt{2\rho\overline{\Delta p_{BD}}\frac{r_1}{r_2}}}\frac{1}{(r_2 + r_1)r_2\ln\left(\frac{r_2}{r_1}\right)}\right) \tag{23}$$

where: m – the mass flow rate in [kg/s] units. Moreover one can find the shape of stream surface representing the skeleton of designed blade in the form of analytical function:

$$\varphi = \frac{2\sqrt{2}}{3}\frac{r_2}{U_{x3}}\sqrt{\frac{\Delta p_{AB}}{\rho}}\frac{z}{r^2}\sqrt{\frac{z}{x_{(3B)}}} = C(U_{x3}, r_2, \Delta p_{AB})\frac{z^{\frac{3}{2}}}{r^2} \tag{24}$$

where: r, φ, z are the cylindrical coordinates.
The example of such a shape is shown in Fig. 3.

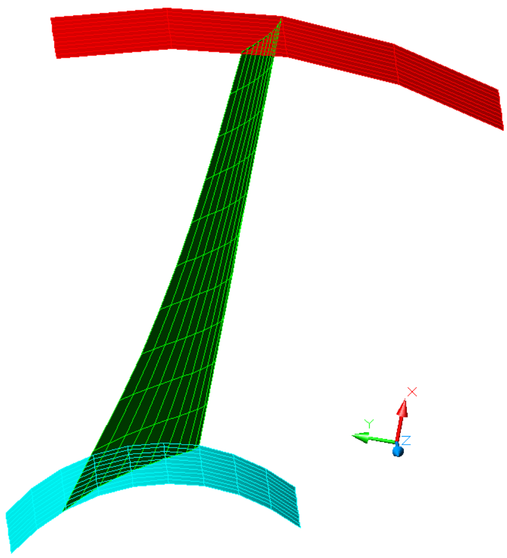

Fig. 3. The example of the skeleton shape according to formula (24).

It is worth mentioning that the outlet angle α_1 changes to lower values at inner diameter r_1 comparing to the value α_1 at outer diameter r_2.

NUMERICAL SOLUTIONS

Analytical solution may be used as the check for numerical solutions according to the algorithm presented in the introduction. Among very large possible number of parameters defining the geometry of designed blade, only a few factors were chosen to show their influence on the blade shape. They are listed in the table below.

Table 1. The chosen factors to investigate the influence on the blade shape.

Factor	**Parameters kept constant**	**Influenced parameter**
Trailing edge inclination	Pressure drop at outer radius, no losses	Outlet angle
Blockage factor (blade thickness)	Pressure drop at outer radius, inclined trailing edge, no losses.	Existence of solution
Loss coefficient distribution in the flow domain	Pressure drop at outer radius, inclined trailing edge.	Existence of solution

Trailing edge inclination

In the Fig. 4 below it is shown the example of cylindrical flow through the vane with different inclined trailing edge marked by distance a_1 at outer radius

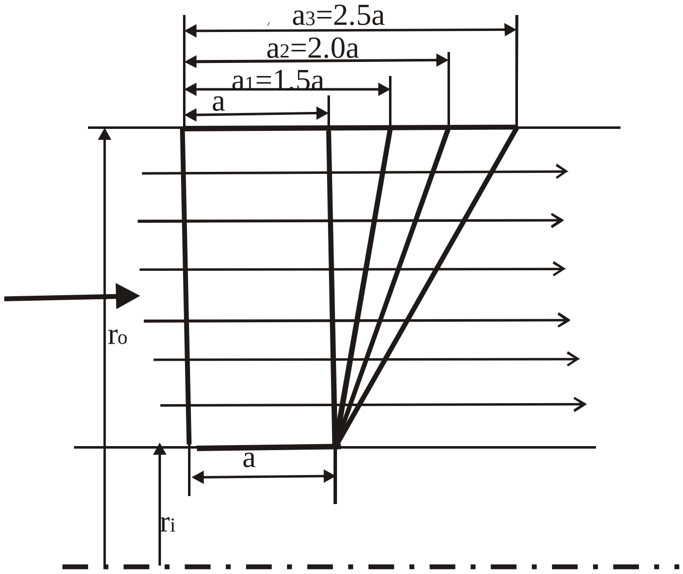

Fig. 4. The sketch of the inclined trailing edge.

In Fig. 5 the distribution of the outlet angle along the trailing edge is shown. It is noteworthy how the inclination uniforms the distribution of outlet angle.

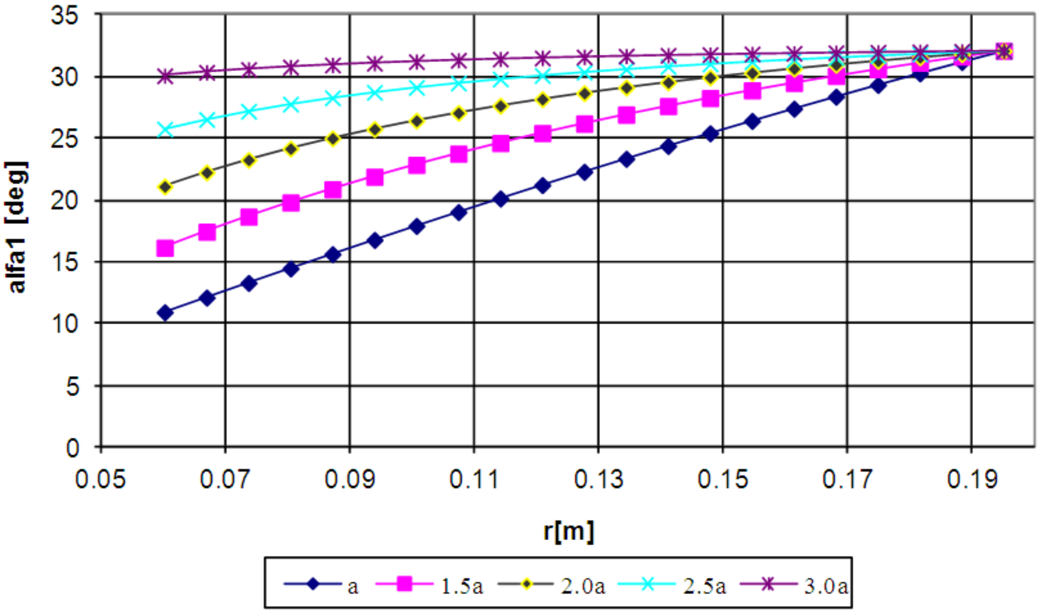

Fig. 5. Behavior of outlet angle α_1 for different inclination of the trailing edge.

The mean values of outlet angle for different α_1 width are shown in Fig. 6.

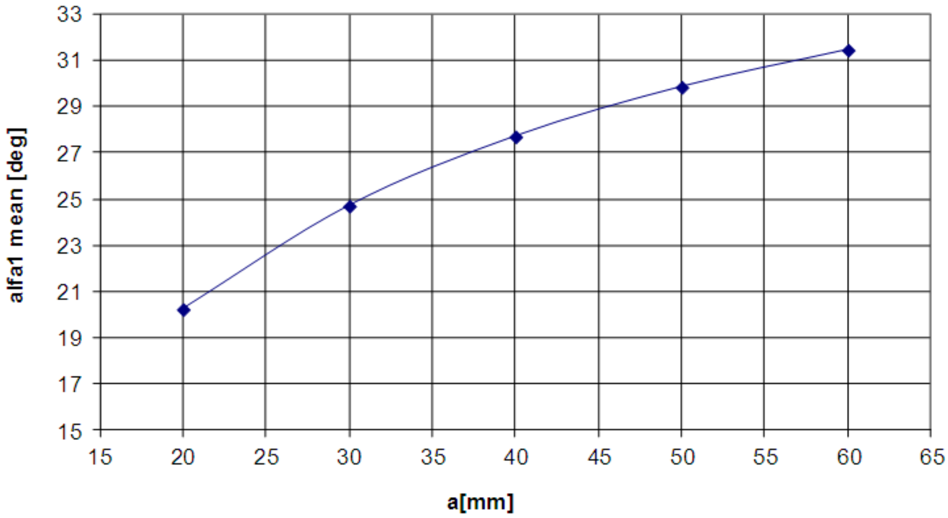

Fig. 6. The mean value of outlet angle α_1.

The mean pressure drop also changes as it is shown in Fig. 7 when inclination increases. The relative pressure drop falls down as outlet angle increases due to trailing edge inclination.

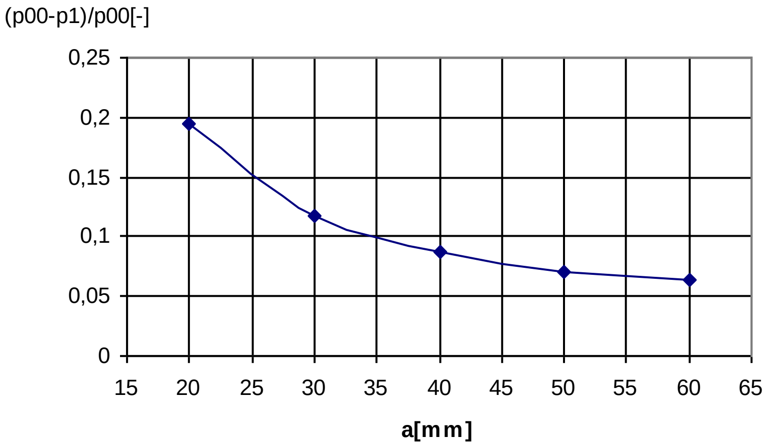

Fig.7. The mean relative pressure drop across a blade.

Blockage factor influence

The influence of blade thickness is introduced by blockage factor $\tau > 0$, which appears in eq. of mass flow rate conservation (1). Factor $1-\tau < 1$ causes the increase of velocity component $U_{x(3)}$ what results in lower pressure and $U_{x(2)}$ component in energy conservation equation. This may lead to the lack of solution.

The blockage factor τ may be introduced in the form of function:

$$\tau(x^{(1)}, x^{(3)}) = t_1 \left(\bar{x}^{(3)}\right)^{t_2} \left(1 - \bar{x}^{(3)}\right)^{t_3} \left(1 + t_4 x^{(1)}\right) \qquad (25)$$

where: t_1, t_2, t_3, t_4 – the coefficients and $x^{(3)}$ $(0 \le x^{(3)} \le 1)$ dimensionless coordinate in the domain of the blade. Such a function gives sufficient freedom to predict the blade distribution thickness of designed blade. The question how to distribute the blade thickness along the skeleton line remains open.

Let us consider two examples. In the first example the parameters were introduced as follows:

$$t_1 = 0.5; \quad t_2 = 2; \quad t_3 = 1.5; \quad t_4 = -0.2;$$

The shape of blockage factor function in coordinates $(x_{(1)}, x_{(3)})$ is shown below. The maximum value of τ is 0.04.

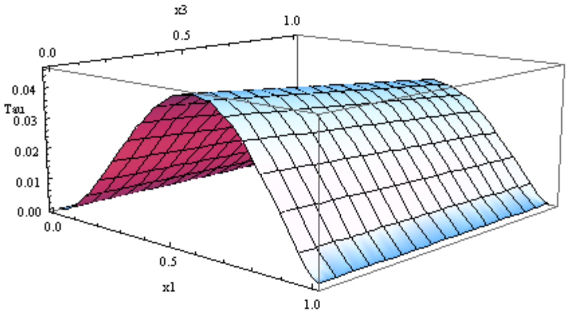

Fig. 8. The first distribution of blockage factor according to formula (25).

The blade shape in two projections are shown in Fig. 9.

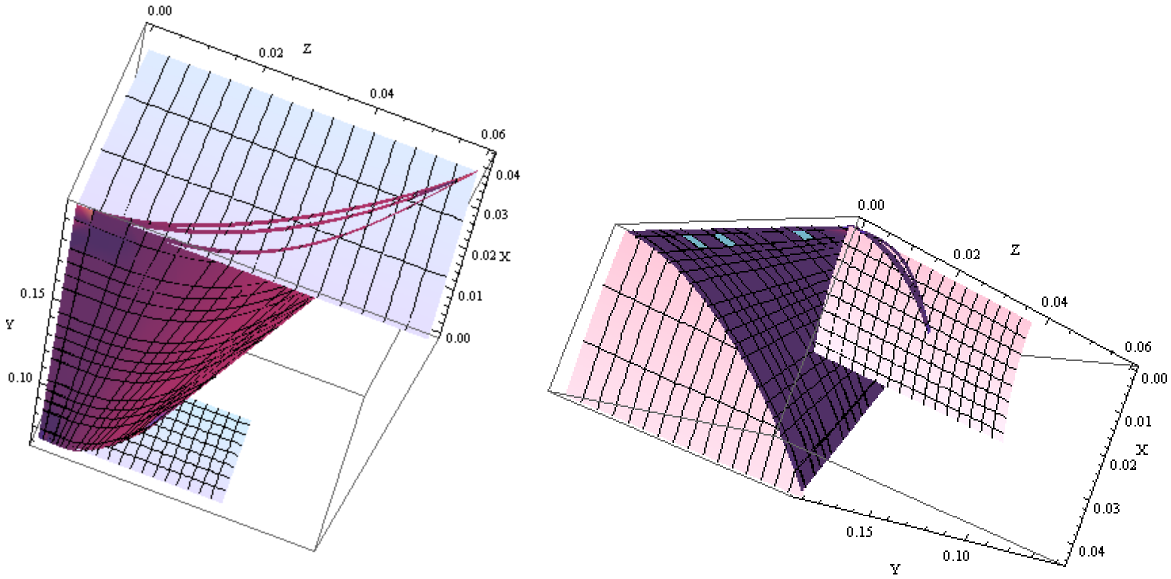

Fig. 9. Two projections of the obtained blade shape.

In the second example we choose the following set of numbers as follows:

$$t_1 = 0.5;\;\; t_2 = 1;\;\; t_3 = 1.5;\;\; t_4 = -0.2;$$

The shape of blockage factor has the higher value of maximum $\tau = 0.09$ as it can be seen in Fig. 10.

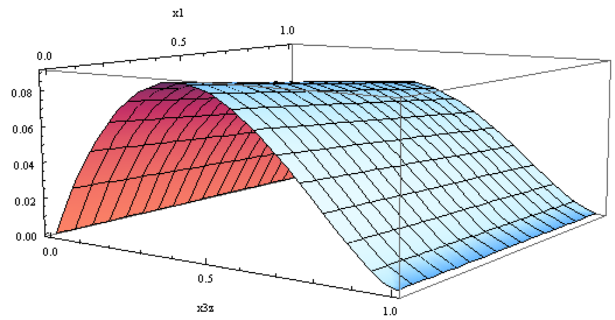

Fig. 10. The second distribution of blockage factor according to formula (25).

Too high thickness causes a lack of solution in the area close to the blade inlet. It is shown in Fig. 11.

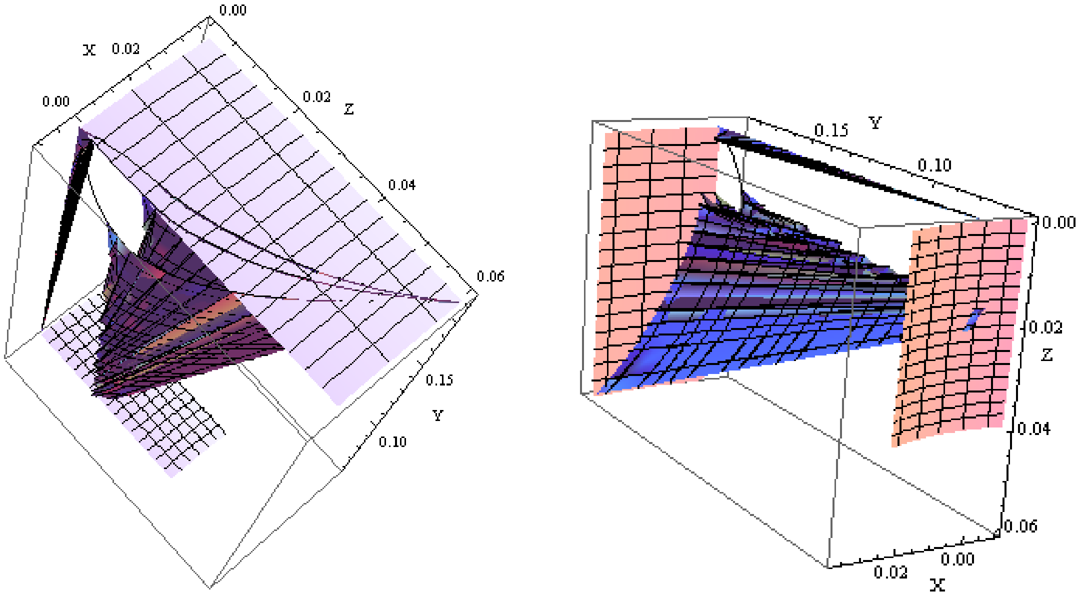

Fig. 11. Two projections of the blade with “white spots” close to leading edge.

The lack of solution due too high blockage factor was reported in (R. Puzyrewski & K. Namieśnik, 1996) for the case of gas turbine inverse problem treated within the frame of 2D model.

Loss coefficient distribution

The dissipation is the factor, which has to be introduced into the model. Let us assume that the losses are defined as a part of inlet kinetic energy, and they are distributed in the flow domain according to the coefficient:

$$\varsigma(x^{(1)}, x^{(3)}) = \left(\mathrm{w}_0 + \mathrm{w}_1 x^{(1)}\right)^{nl} \left(\mathrm{w}_2 + \mathrm{w}_3 \bar{x}^{(3)}\right)^{ns} \qquad (26)$$

As an example we can consider the set of coefficients as follows:

$w_0 = 0.3$; $w_1 = -0.15$; $w_2 = 0.0$; $w_3 = 1.5$; $n_l = 2.0$; $n_s = 3.0$

The above coefficients gave the behavior of function ζ as it is shown in Fig. 12:

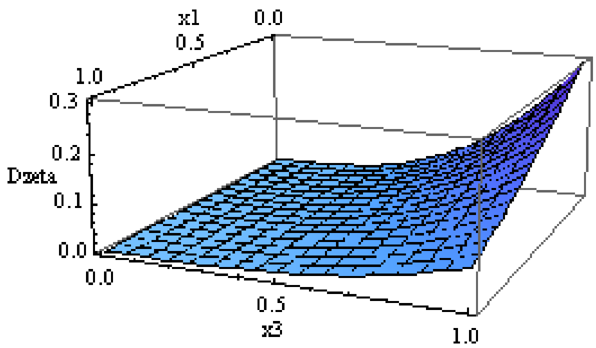

Fig. 12. The distribution of loss coefficient in blade domain according to (26).

If we repeat the computation for the conditions represented in Fig. 9 the presented above dissipation coefficients lead to the blade shape as in Fig. 13. The mean loss coefficient related to isentropic velocity of pressure drop is about 0.05 (~5 %).

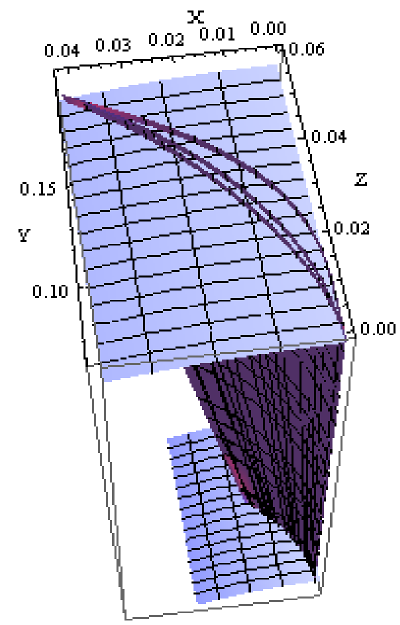

Fig. 13. The shape of the blade as in Fig. 9 but corrected due to losses.

Let us now increase the losses according to function ζ shown below:

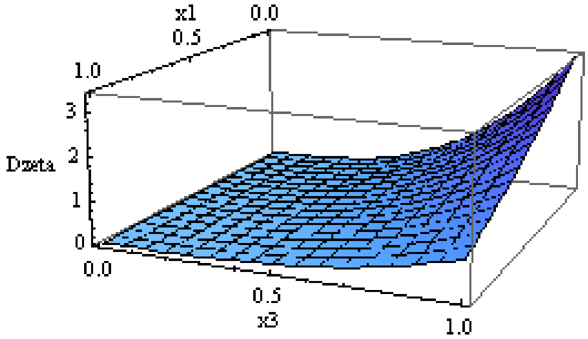

Fig. 14. The example of the increased losses coefficient distribution.

The isentropic loss coefficient in this case has the high value of level 0.48 (48 %). It causes the lack of solution at outlet part of blade as it shown in Fig. 15.

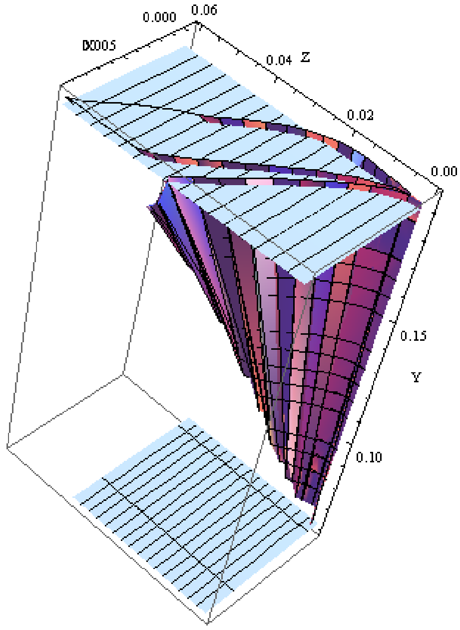

Fig. 15. The shape of blade with broken exit part due to lack of solution.

Too high losses do not allow organizing the flow in the shape of cylindrical surfaces.

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