2D MODEL OF GUIDE VANE FOR LOW HEAD WATER TURBINE ANALYTICAL AND NUMERICAL SOLUTION OF INVERSE PROBLEM

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Low-head water turbines are the subjects to individual design treatment. It comes from the fact that hydrological environment is not of a standard character. Therefore the method of design of the water turbine stage is of a great importance for those who are interested in such an investment. As a first task the guide vane design was undertaken. The proposed method is based on the solution of inverse problem within the frame of 2D model. The starting point is the assumed picture of axis symmetrical flow surfaces *f* bordered by the boundary as in Fig. 1.

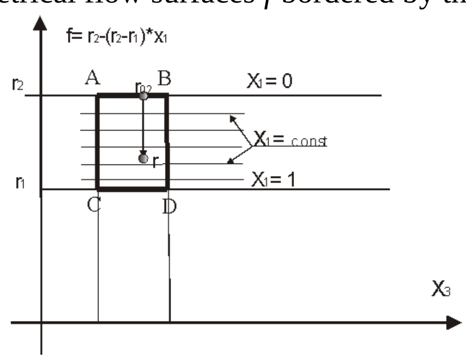

Fig. 1. Meridional shape of the analyzed guide vane.

If one assumes the flow surfaces as coordinate $x^{(1)} = \text{const}$, the angle coordinate as $x^{(2)} = \text{const}$, and axis coordinate as $x^{(3)} = \text{const}$, then the set of governing equations in such a non-orthogonal coordinate system can be derived as follows [1]:

- **Mass flow rate conservation equation:**

$$\left(1-\tau\left(x^{(1)},x^{(3)}\right)\right)\rho U_{x^{(3)}}\frac{f\left|\frac{\partial f}{\partial x^{(1)}}\right|}{\sqrt{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}}=m\left(x^{(1)}\right) \tag{1}$$

where: $f = f(x^{(1)}, x^{(3)})$ - assumed flow surface, ρ – density, $m(x^{(1)})$ – mass flow rate function given at inlet, $\tau(x^{(1)}, x^{(3)})$ – blockage factor due to blade thickness.

- **Momentum conservation eq. in $x^{(1)}$ direction:**

$$sgn\left(\frac{\partial f}{\partial x^{(1)}}\right)\rho\left(-\frac{1}{f}U^2_{x^{(2)}}+\frac{\frac{\partial^2 f}{\partial x^{(3)2}}}{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}U^2_{x^{(3)}}\right)$$
$$=-\left|\frac{\partial f}{\partial x^{(1)}}\right|\left(\frac{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}{\left(\frac{\partial f}{\partial x^{(1)}}\right)^2}\frac{\partial p}{\partial x^{(1)}}-\frac{\frac{\partial f}{\partial x^{(3)}}}{\frac{\partial f}{\partial x^{(1)}}}\frac{\partial p}{\partial x^{(3)}}\right)-\rho\left|\frac{\partial f}{\partial x^{(1)}}\right|\left(\frac{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}{\left(\frac{\partial f}{\partial x^{(1)}}\right)^2}\frac{\partial \Pi}{\partial x^{(1)}}-\frac{\frac{\partial f}{\partial x^{(3)}}}{\frac{\partial f}{\partial x^{(1)}}}\frac{\partial \Pi}{\partial x^{(3)}}\right)+\rho F_{x^{(1)}} \tag{2}$$

- **Momentum conservation eq. in $x^{(2)}$ direction:**

$$\rho \frac{U_{x^{(3)}}}{\sqrt{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}}\left(f\left(\frac{\partial U_{x^{(2)}}}{\partial x^{(3)}}\right)+U_{x^{(2)}}\left(\frac{\partial f}{\partial x^{(3)}}\right)\right)=\rho F_{x^{(2)}} \tag{3}$$

- **Momentum conservation eq. in $x^{(3)}$ direction:**

$$\rho \frac{\partial U_{x^{(3)}}}{\partial x^{(3)}}\left(\frac{\partial U_{x^{(3)}}}{\partial x^{(3)}}-\frac{U_{x^{(3)}}}{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}\frac{\partial f}{\partial x^{(3)}}\frac{\partial^2 f}{\partial x^{(3)2}}\right)= \\ =\sqrt{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}\left(\frac{\frac{\partial f}{\partial x^{(3)}}}{\frac{\partial f}{\partial x^{(1)}}}\frac{\partial p}{\partial x^{(1)}}-\frac{\partial p}{\partial x^{(3)}}\right)-\rho\sqrt{1+\left(\frac{\partial f}{\partial x^{(3)}}\right)^2}\left(\frac{\frac{\partial f}{\partial x^{(3)}}}{\frac{\partial f}{\partial x^{(1)}}}\frac{\partial \Pi}{\partial x^{(1)}}-\frac{\partial \Pi}{\partial x^{(3)}}\right)+\rho F_{x^{(3)}} \tag{4}$$

where: $p = p(x^{(1)},x^{(3)})$ – static pressure, $\Pi = \pm g x^{(3)}$ – gravity potential, $F_{x^{(1)}}, F_{x^{(2)}}, F_{x^{(3)}}$ – components of cascade forces acting upon the flow.

- **Energy conservation equation:**

$$\frac{U_{x^{(2)}}^2+U_{x^{(3)}}^2}{2}-U_{x^{(2)}}U_W+cT+\frac{p}{\rho}+\Pi=e_c\left(x^{(1)}\right) \tag{5}$$

An additional assumptions concerning the $F_{x1} = 0$ allows to close the system of equations and seek for analytical and numerical solutions.

For the simplest case of cylindrical boundary (r, φ, z) the above set of equations for isentropic flow leads to the analytical solution which allows designing the shape of guide vane as the function of

$$\varphi(r,z)=A\frac{z^{1+\frac{n}{2}}}{r^2}=\frac{2\sqrt{2}}{2+n}\frac{r_2}{U_{x3}}\sqrt{\frac{\Delta p_{AB}}{\rho}}\frac{z}{r^2}\sqrt{\left(\frac{z}{x^{(3B)}}\right)^n} \tag{6}$$

where A, n, $U_x^{(3)}$, ... are parameters of the problem.

The shape of skeleton guide vane is shown in Fig. 2.

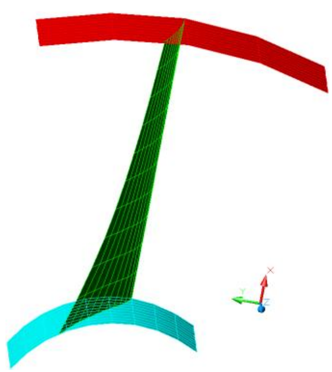

Fig. 2. Result of the analytical solution of the inverse problem for guide vane.

The more realistic solutions included the influence of loss coefficient, blockage factor, the inclination of exit line is the matter of numerical solutions presented in the paper.

References

1. R. Puzyrewski (1998): *14 Lectures on Theory of Turbomachinery Stage-Two dimensional Model(2D)*, edited by Technical University of Gdańsk (in polish).