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# INVESTIGATION OF THE 2D FREE TURBULENT JET

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*Abstract*

The experimental, analytical and numerical investigations of two-dimensional free jet is given as a first step to study the Coanda effect and its hysteresis. Analytical solution of free turbulent jet based on the assumption of shear layer has been carried out. What is more, the comparison of spreading ratio of jet for analytical, numerical and experimental investigations is presented apart from others features of free jet like velocity profile across and along the stream. Also, the location of the virtual origin is analyzed.

*Key words: free turbulent jet, Computational Fluid Dynamics*

## INTRODUCTION

The purpose of the study was to examine characteristics of two-dimensional free jet by using experimental, analytical and numerical methods with FloVent software simulations. They were performed for the Reynolds number 10000-38000.

Correctness of analytical calculation effects, thanks to the large resource of literature data, were the easiest for verification, It provided baseline information, towards which we checked the accuracy of own experimental and numerical results. Unfortunately, we did not have any information about other, converging to our study with FloVent software use. However FloVent is a powerful Computational Fluid Dynamics (CFD) software that predicts 3D airflow, heat transfer, contamination distribution and thermal comfort. It is mostly used for typically engineering calculations, especially in ventilations issues [1].

The research discussed in this article, is the first phase of the more capacious examinations. All following experimental investigations were performed at University of Warmia and Mazury in Olsztyn, according to research projects PB Nr 512 030 31/3280 “Use of the hysteresis of the Coanda effect to increase the efficiency of the mixing ventilation”. What is more, it should be remembered, that the scope of our analysis was dependent on the main purpose - improving the effectiveness of comfort ventilation systems, by understanding, controlling and using the hysteresis’s of the Coanda effect potential.

## SURVEY OF THEORIES ON 2D FREE TURBULENT JETS

Considering free, isothermal jet, with increase of the distance from the slot results in central core of uniform velocity gradually diminishes thanks to the fact, that edge of the jet mixes with the surroundings. Four zones of jet expansions are being observed:

- zone 1 – a short core zone when the maximum velocity of the airstream remains practically unchanged;
- zone 2 – a transient zone;
- zone 3 - a zone of fully developed turbulent flow;
- zone 4- a zone of diffuser jet degradation, where maximum air velocity decrease rapidly [2].

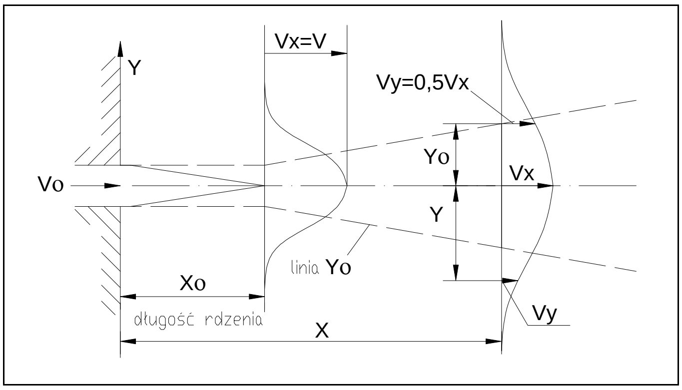

Fig. 1: Two-dimensional free turbulent jet [2].

In the case of the plane free jet, the decrease is much smaller in axial velocity than the circular jet, which results in an increased range of stream.

According to the assumption, that the pressure throughout the jet everywhere equals the surrounding pressure $p_\infty$, the jet momentum is constant downstream [3]. Considering the stream at a certain distance $x$ downstream the nozzle, which primarily came from a slot of with $b$ and with core velocity $U$, could as well been produced by a larger slot of width $b'$ with core velocity $U' < U$.

$$\rho U'^2 b' = J = \rho U^2 b \tag{1}$$

Where appropriately:

$\rho$ - density of the fluid,

$U, U'$ - assumed uniform velocity at the slot,

$b, b'$ - width of the slot,

$J$ - jet momentum per unit span of slot.

To determine the mean velocity $u$ at distance $y$ perpendicular to the jet axis the variable $J$, the fluid density $\rho$ and location $x$, $y$ in local reference system are required.

The locations $y_{m/2}$, along the x axis, in which the average velocity is equal to the half of maximum speed in the transverse section of velocity profile $u = \frac{1}{2} u_m$ can be assumed as a measure of the width of the jet. For all $x$ values in the assumed local reference system the mean velocity profiles are similar according to the formula (2):

$$\frac{u}{u_m} = F\left(\frac{y}{y_{m/2}}\right) \tag{2}$$

Furthermore, according to Görtler it may be assumed, that the eddy viscosity $\varepsilon$ is constant across the flow at each $x$ position and proportional to $u_m y_{m/2}$, and then velocity profile is given as (3).

$$u = u_m \sec h^2 \frac{0.88y}{y_{m/2}} = \left\{\frac{3J\sigma}{4\rho x}\right\}^{\frac{1}{2}} \sec h^2 \frac{\sigma y}{x} \tag{3}$$

where:

$\sigma$ – parameter occurring in the analysis of the free jet,

$u_m$ - maximum value of $u$ at given station.

According to Newman, this solution gives good results for $y < 1{,}3 y_{m/2}$. In other cases, this formula gives values slightly too high. Until $x/b$ exceeds about 25, the flow is not independent of the slot width $b$. Results from Bourque present a value of $\sigma = 12$ near the slot and $\sigma = 7{,}5$ for large $x/b$. The measurements of Reichardt and Förthmann give the same dependence and assume $\sigma = 7{,}7$. According to that:

$$\begin{aligned} & y_{m/2}/x = 0.88/\sigma = 0.114 \\ & \rho u_m^2 x/J = 3\sigma/4 = 5.78 \\ & \varepsilon/u_m y_{m/2} = 1/3.52\sigma = 0.037 \end{aligned} \tag{4}$$

where:

$\varepsilon$ - eddy viscosity for free jet.

In literature the following formula for the free jet width $y_{m/2}$ can be found, eg. Kotsovinos [4]:

$$\frac{y_{m/2}}{b} = K_1\left(\frac{x}{b} + K_2\right) \tag{5}$$

where:

$K_1$; $K_2$ - coefficients.

The $K_1$ coefficient is considered to be a measure of the spreading rate of the jet [4]. From the relation (5) the location of the virtual origin $x_0$ of the jet can be obtained:

$$x_0 = -K_2 b \tag{6}$$

The above relations are correct for turbulent jet, and will not be matched for core zone, which takes about $6b$ [5]. In our case, for slot weight $b = 20mm$, it distance about.

## EXPERIMENTAL INVESTIGATIONS

The measurements were made in the laboratory of the Chair of Environmental Engineering at the University of Warmia and Mazury in Olsztyn. The examinations took place in the special test chamber. Its dimensions were 3850x2255x2009 mm (Fig.2). The air on the test stand was derived by air intake with throttling orifice, thanks to that the change of the quantity of the air in the whole system was possible. On the pressure side of the fan the 2D Witoszynski's nozzle was placed, which was used to generate a homogenous flat air stream. At the end of the nozzle on one of their sides the flat plate was located. The plate suspended in the bearing could be manually turned in the angle range of $\alpha = 0° - 90°$. In the investigation of the free jet the plate was not used.

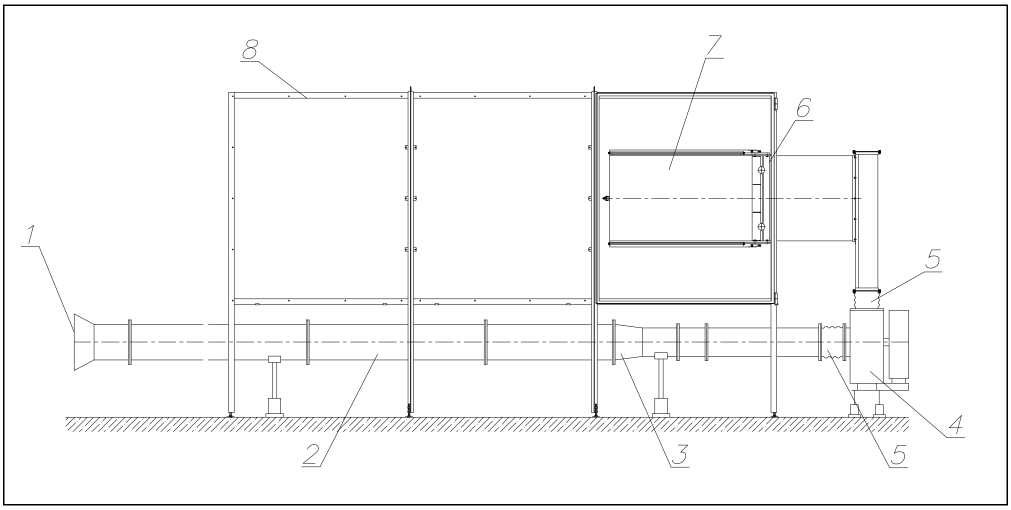

Fig 2: Test stand: 1- air intake with throttling orifice; 2- the suction conduit; 3- the surveys of static pressure; 4- fan; 5- flexible connector; 6- the Witoszyński nozzle; 7- removable flat plate; 8- plexiglas test stand housing.

The air stream was coming from the two-dimensional Witoszynski nozzle of dimensions h*b=0.6*0.02 m*m, so the nondimensional parameter λ=h/b=30, Fig.3. The velocity range used during the investigation was can be changed in the range 8 m/s to 28 m/s, so the Reynolds numbers based on the nozzle's width was about Re= 10000 till almost 38000.

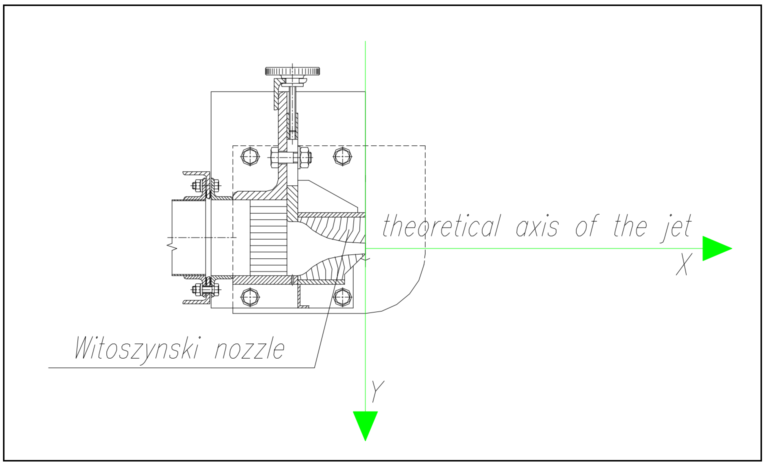

Fig. 3: The Witoszyński nozzle and the local reference system.

The measurement of velocity profiles and turbulence level was made by means of thermoanemometr ATU 2001. In this measuring system the variable measurement grid was used. Assuming optimizing the number of measurement points has been adjusted for differential velocity distribution in the air stream. The measurements were performed in 11 parallel section perpendicular to the stream axis. The first section was located immediately after the nozzle, in the core area, at the distance X = 0.01 m. The next measurement sections were located evenly downstream the nozzle at the distance X=0.10 m, up to the limit X = 1.00 m. Each measurement section has a specific width, until the velocity reaches a value lower than 1m /s.

Table 1 contains the results of fan air volume, initial air velocity at the nozzle (nominated by measurement on orifice and by using thermoanemometr) and Reynolds number for six measurements series.

| **No.** | **Air volume of fan** | **Air velocity at the slot determined** | | **Reynolds number** |
|---|---|---|---|---|
| | | **as air volume/nozzle area** | **by hot-wire measurements** | |
| | [m$^3$/s] | [m/s] | | [-] |
| **1** | 0.096 | 7.98 | 7.98 | 10570 |
| **2** | 0.107 | 8.88 | 8.63 | 11768 |
| **3** | 0.127 | 10.56 | 10.18 | 13983 |
| **4** | 0.152 | 12.67 | 12.86 | 16784 |
| **5** | 0.250 | 20.90 | 20.78 | 27685 |
| **6** | 0.338 | 28.19 | 28.02 | 37343 |

Table 1: Fan air volume, initial air velocity and Reynolds number for six measurements series.

The Reynolds number determined in accordance to the formula:

$$\mathrm{Re} = \frac{U \cdot b}{\nu} \tag{7}$$

where U, b and υ are appropriately: uniform velocity at the slot, width of the slot and kinematic viscosity of the fluid.

The test stand was localized in closed area of laboratory room, from the air was drawn, the whole measuring system can be treated as isothermal.

## NUMERICAL SIMULATIONS

Numerical investigations to compare experiment with numerical results were done by using FloVent software. Figure 4 illustrates the test stand modeled in FloVent software where the coordination system is also marked. Dark blue color lines shows the plexiglas test stand walls. Red color rectangle presents the nozzle. Block marked with purple, is the more dense area of the measuring grid, which were subjected to analysis of test parameters on the free jet.

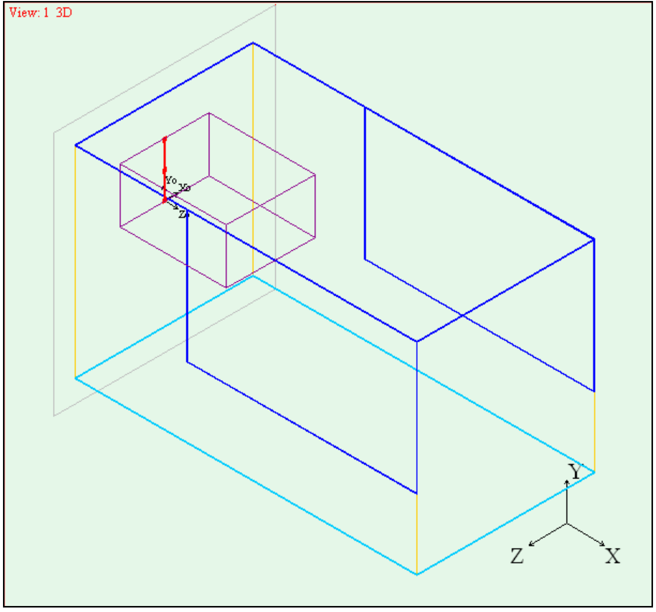

Fig. 4: Scheme of test stand in Flovent software.

During the simulations a grid system with about 500 000 cells was used. In the area where the jet properties were analyzed in more detail, the single grid cell size was of dimensions X=5mm, Y=20mm, Z=30mm. The total analyzed area was of dimensions X=1.20m; Y=0.62m; Z=1.00m. So in the rest of the test bench, the cell dimensions should not exceed 75mm in all directions. The figure 5 illustrates the grid system.

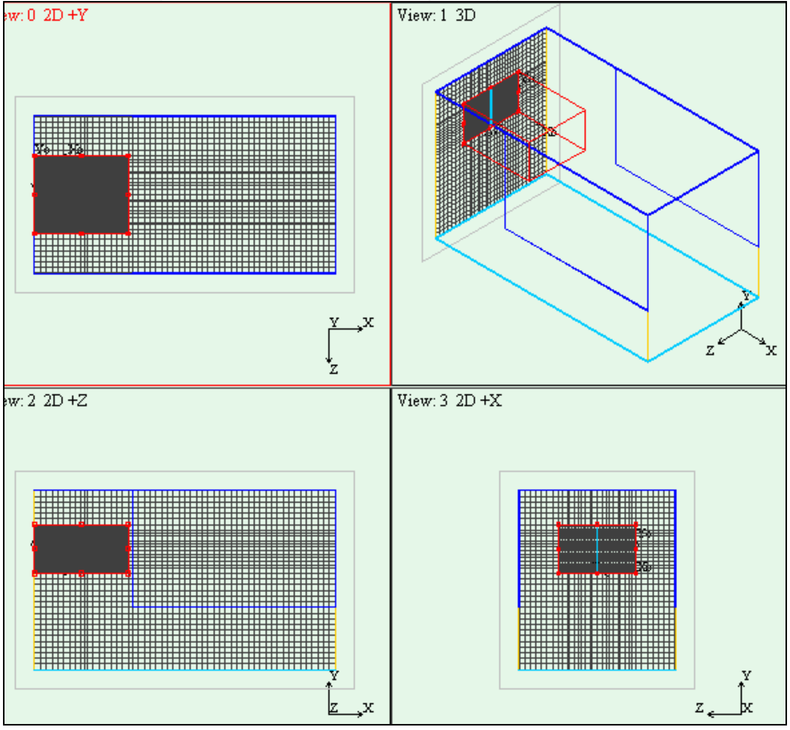

Fig. 5: The grid system in FloVent software.

LVEL K-Epsilon turbulence model was used during the simulation, which is tested in the largest number of engineering calculations and very often had given quite satisfactory results. This model is characterized by the simplicity and stability. The FloVent software has got also two others turbulence models: Capped LVEL and LVEL Algebraic.

## RESULTS

Analysis of experimental results has shown, that maximum velocity value $u_m$ at given station is located in air jet axis, where $y = 0{,}00m$. Using the formula (3) the comparison of theoretical and experimental maximum velocity values was done. The used calculation method, allows good agreement with experiment only for fully turbulent jet. As the result of that, for the first two measurement sections (near the nozzle) calculations are insufficient. Because of that, for further consideration, the first two measurement sections (nearest the nozzle), will be omitted. The jet momentum value, according with formula (1), and determined $\sigma$ parameter value are showed in table 2.

| x $[m]$ | $\sigma$ [-] for series no. | | | | | |
|---|---|---|---|---|---|---|
| | 1<br>Re=10570 | 2<br>Re=11768 | 3<br>Re=13983 | 4<br>Re=16784 | 5<br>Re=27685 | 6<br>Re=37343 |
| 0,20÷1,00 | 7,58 | 7,38 | 8,07 | 8,78 | 8,30 | 8,47 |
| | J [kg/s$^2$] for series no. | | | | | |
| - | 1,49 | 1,86 | 2,61 | 3,73 | 10,42 | 18,60 |

Table 2: Parameter σ and jet momentum $J$ per unit span for slot height $b = 20mm$.

Experimental and analytical results of maximum velocity value $u_m$, for different measurements in x directions, are characterized by a good convergence. Average disparity is about 0,2%. Inclusion in the analysis also numerical results gives worse results (maximal local divergence 10,6%). Despite, the average discrepancy between experimental and numerical results is at acceptable level - 3,6%. Selected results are compare in Table 3.

| x $[m]$ | Maximum velocity value $u_m$ [m/s] for series no. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2<br>Re=11768 | | | 5<br>Re=27685 | | | 6<br>Re=37343 | | |
| | experimental | analytical | numerical | experimental | analytical | numerical | experimental | analytical | numerical |
| 0,01 | 8,64 | 8,64 | 8,85 | 20,78 | 20,81 | 20,68 | 28,02 | 28,05 | 27,88 |
| 0,10 | 8,16 | 8,14 | 8,65 | 20,60 | 20,65 | 20,21 | 27,57 | 27,62 | 27,25 |
| 0,20 | 6,63 | 6,66 | 7,00 | 17,07 | 17,05 | 16,37 | 22,99 | 23,03 | 22,07 |
| 0,30 | 5,48 | 5,48 | 5,58 | 13,94 | 14,00 | 13,03 | 18,55 | 18,60 | 17,57 |
| 0,40 | 4,73 | 4,74 | 4,73 | 11,82 | 11,79 | 11,06 | 15,72 | 15,73 | 14,91 |
| 0,50 | 4,20 | 4,19 | 4,24 | 10,46 | 10,48 | 9,90 | 14,01 | 14,07 | 13,35 |
| 0,60 | 3,80 | 3,80 | 3,92 | 9,32 | 9,34 | 9,15 | 12,95 | 13,00 | 12,33 |
| 0,70 | 3,59 | 3,56 | 3,68 | 8,59 | 8,59 | 8,59 | 12,08 | 12,10 | 11,59 |
| 0,80 | 3,26 | 3,26 | 3,49 | 8,06 | 8,04 | 8,15 | 11,19 | 11,19 | 10,98 |
| 0,90 | 3,06 | 3,06 | 3,33 | 7,37 | 7,34 | 7,77 | 10,61 | 10,61 | 10,48 |
| 1,00 | 2,90 | 2,92 | 3,19 | 7,15 | 7,14 | 7,45 | 9,76 | 9,77 | 10,05 |

Table 3: Maximum velocity value $u_m$ [m/s] at a given station for selected series - experimental, analytical and numerical results comparison.

The nondimension analyzed result $U_m/U_0$ are plotted in Fig.6. Quite reasonable agreement between experiment and analysis given by Görtler can be observed. The discrepancies are greater if the numerical result are joined. For tests with the smallest Reynolds numbers the overstatement of the numerical results can be seen in the initial zone of the flow.

Experimental data for higher Reynolds number are higher from FloVent results in the middle part of jet.

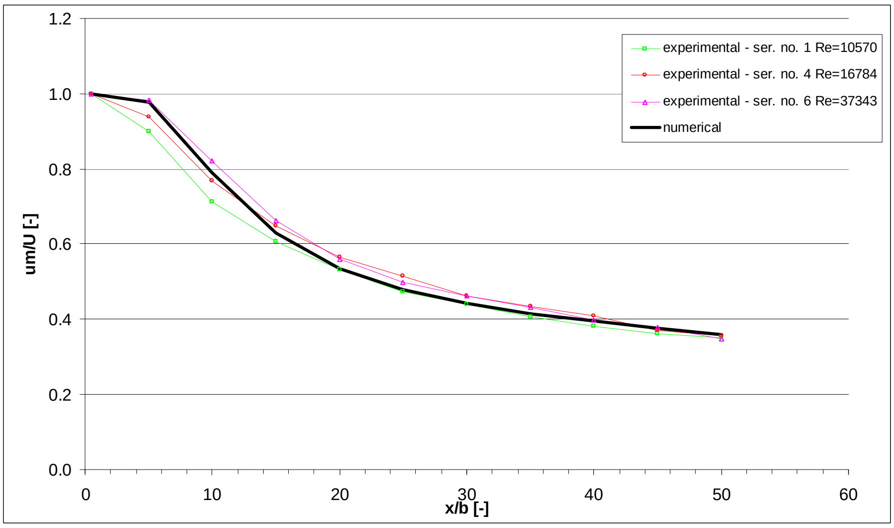

Fig. 6: Maximum velocity value $u_m$ at a given station to velocity at the slot $U$ for selected data.

Calculations of axial velocity decrease along the axis were made according to the formula [2]:

$$\frac{u_m}{U} = \sqrt{\frac{b}{m \cdot x}} \tag{8}$$

where:

$m$ - mixing parameter.

The designated size of mixing parameter was $m = 0{,}16$. The relationship with formula (3) should be noted and thus:

$$m\sigma = \frac{4}{3} \tag{9}$$

The mean velocity $u$ profiles at all locations in $x$ direction have been determined in accordance with formula (3). Figures 7 and 8 represent some results of investigations with Re=27 685. In the first case the investigations have been made for the nozzle closest to the station ($x = 0{,}01m$). Second graph represents the analysis for location $x = 0{,}50m$. For the initial area of jet, where flow was not turbulent the correlation between the results from different computational methods is acceptable.

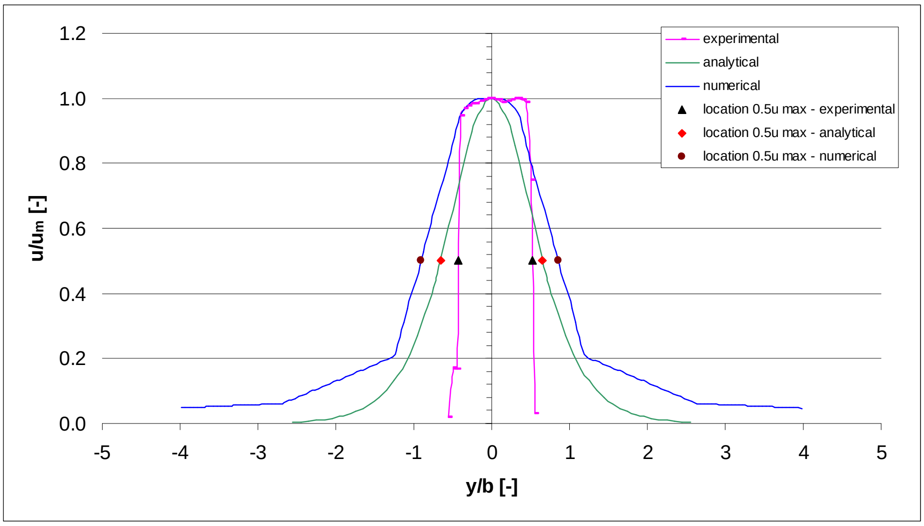

Fig. 7: Mean velocity $u$ profiles and location $u=\frac{1}{2}u_m$ value for experimental, analytical and numerical results; Re=27 685; $x=0{,}01m$.

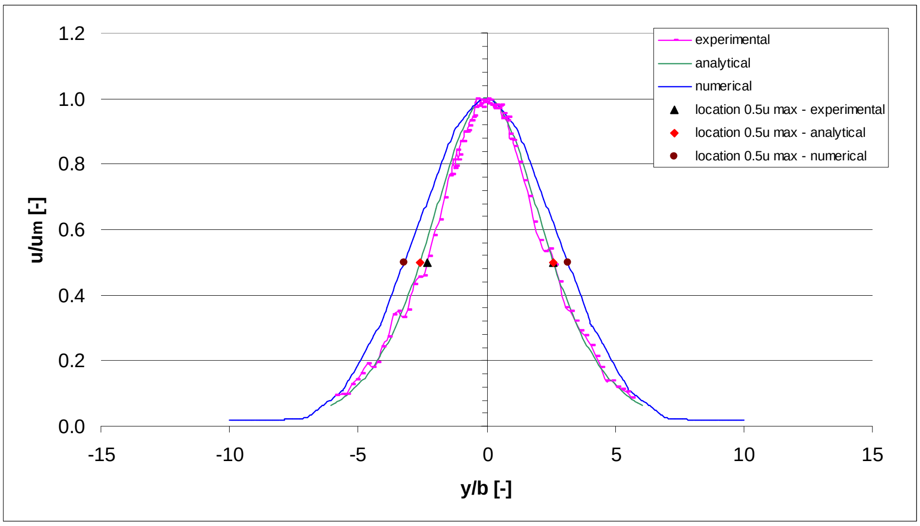

Fig. 8: Mean velocity $u$ profiles and location $u=\frac{1}{2}u_m$ value for experimental, analytical and numerical results; Re=27 685; $x=0{,}50m$.

The results discrepancy in location $y_{m/2}$ for value $u=\frac{1}{2}u_m$, regardless of the investigation method, does not exceed value ±3,5cm. A better compatibility was noticed between analytical and experimental results. The biggest deflection was in that case ±1,2cm. Table 4 contain selected results of location $y_{m/2}$ for value $u=\frac{1}{2}u_m$ for one measurements series (Re=37 343).

| x<br>$[m]$ | location $y_{m/2}$ for value $u=\frac{1}{2}u_m$<br>series no. 6 Re=37 343 | | | | | |
|---|---|---|---|---|---|---|
| | experimental | | analytical | | numerical | |
| | 1/2vmax [m/s] | Y [m] | 1/2vmax [m/s] | Y [m] | 1/2vmax [m/s] | Y [m] |

|  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|
| 0,01 | 14,0121 | ±0,0095 | 14,0264 | ±0,0134 | 13,9383 | ±0,0177 |
| 0,10 | 13,7858 | ±0,0123 | 13,8122 | ±0,0138 | 13,6233 | ±0,0180 |
| 0,20 | 11,4944 | ±0,0179 | 11,5174 | ±0,0198 | 11,0355 | ±0,0259 |
| 0,30 | 9,2731 | ±0,0278 | 9,2976 | ±0,0304 | 8,7854 | ±0,0392 |
| 0,40 | 7,8609 | ±0,0409 | 7,8647 | ±0,0425 | 7,4562 | ±0,0515 |
| 0,50 | 7,0026 | ±0,0504 | 7,0344 | ±0,0531 | 6,6749 | ±0,0637 |
| 0,60 | 6,4729 | ±0,0622 | 6,4984 | ±0,0622 | 6,1672 | ±0,0746 |
| 0,70 | 6,0408 | ±0,0696 | 6,0517 | ±0,0717 | 5,7925 | ±0,0854 |
| 0,80 | 5,5934 | ±0,0809 | 5,5946 | ±0,0839 | 5,4919 | ±0,0963 |
| 0,90 | 5,3027 | ±0,0914 | 5,3059 | ±0,0933 | 5,2396 | ±0,1070 |
| 1,00 | 4,8824 | ±0,1024 | 4,8834 | ±0,1102 | 5,0226 | ±0,1182 |

Table 4: Location $y_{m/2}$ for value $u = \frac{1}{2}u_m$; Re=37 343.

Based on formulas (5) and (6) the analysis of the virtual origin were determined. Also in this case, the results for the first two sections were omitted as before. According to our experimental results the virtual origin was always located behind the nozzle ($K_2 < 0$), while the analytical investigations were not so interchangeable. According to them the virtual origin was located in three cases behind the nozzle and in three other cases in front of the nozzle. In turn, numerical results pointed the location of virtual origin always upstream from the jet orifice.

Correlation of spreading rate $K_1$ and non-dimensional virtual origin $K_2$ for our own investigations and literature data [4] representing Fig. 9.

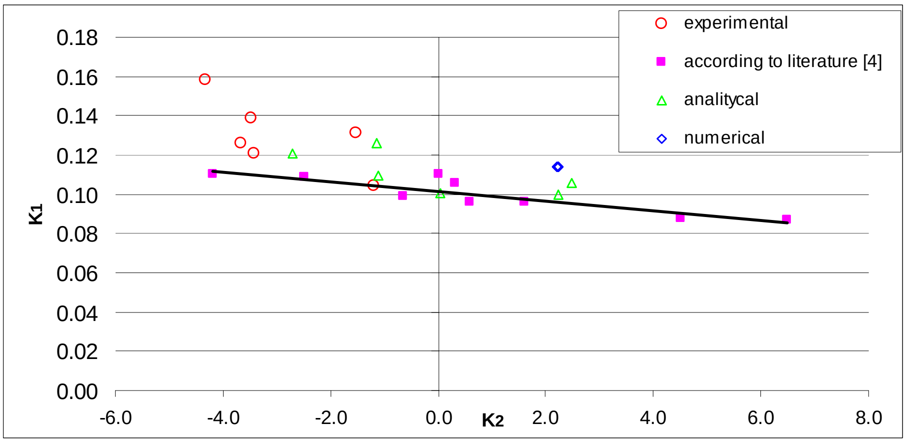

Fig. 9: Coefficients of spreading rate $K_1$ and non-dimensional virtual origin $K_2$.

In paper [4] argument has been made that the growth of the jet is not linear on a large scale. It was confirmed, to some extent, by our results, as shown in Fig. 10.

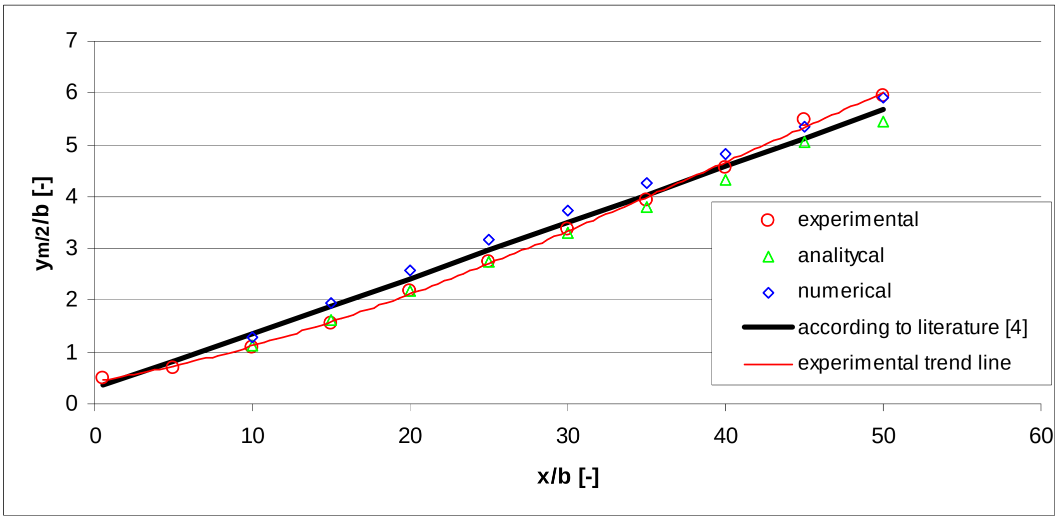

Fig. 10: Nonlinear spreading of half-width of a plane jet for $x < 50b$.

According to paper [4] the black line in Fig. 10 represents nonlinear spreading of half-width of a plane jet along the below given relation:

$$\frac{y_{m/2}}{b} = 0{,}228 + 0{,}0913\frac{x}{b} + 0{,}00005101\left(\frac{x}{b}\right)^2 + 0{,}000000331\left(\frac{x}{b}\right)^3 \qquad (10)$$

While by means of our results the following trend line can be written:

$$\frac{y_{m/2}}{b} = 0{,}4317 + 0{,}0489\frac{x}{b} + 0{,}0021\left(\frac{x}{b}\right)^2 + 0{,}00002\left(\frac{x}{b}\right)^3 \qquad (11)$$

However, some attention should be paid, because our investigations are made in a smaller range of $x < 50b$, while the data given in [4] are determined for range $x < 200b$, which may provide a notice disagreement.

## CONCLUSIONS

It was shown that the average value of parameter $\sigma = 8{,}1$ and so than the value is slightly higher than the value $\sigma = 7{,}7$ know from literature [3].

The determined additionally value of mixing parameter $m = 0{,}16$ confirms actually the relationship (9).

Furthermore, the similar relations as relation (4) was determined for our jet, which are given below (12):

$$\begin{aligned} y_{m/2} / x &= 0.88/\sigma = 0.109 \\ \rho u_m^2 x / J &= 3\sigma / 4 = 6.08 \\ \varepsilon / u_m y_{m/2} &= 1/3.52\sigma = 0.035 \end{aligned} \qquad (12)$$

We were able to confirm, that the growth of the half-width of the plane turbulent jet is not exactly linear on a large scale.

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