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LOW DIMENSIONAL ANALYSIS IN MODEL IDENTIFICATION OF AEROELASTIC STRUCTURES

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Abstract

In this paper, the design of low dimensional model of the flow around aeroelastic structure is presented. The data from high-dimensional aeroelastic analysis is decomposed onto modes, that are used to approximate the flow. Reduced order model, obtained in Galerkin projection is adjusted to the input data using genetic algorithm-based calibration. The same procedure might be used to enforce the same response of the model and a real structure tested in wind tunnel.

Key words: : CFD, aeroelasticity, POD, DMD, reduced order models, Galerkin projection

INTRODUCTION

Wind tunnel experiments play an important role in testing of new aircraft designs. Such experiments are very expensive both in the preparatory phase - the construction of scale model and during the measurements themselves. Therefore the numerical simulations, allowing to carry out the research without the risk of object damage, are used to predict the presence of undesirable phenomena like aerodynamic flutter (Roszak et al., 2009).

The design of the numerical model of the real object always involves some simplifying assumptions, arising both from the spatial discretization and used mathematical formulations, leading to the discrepancies between the numerical aeroelastic model and the real object, that are very hard to overcome due to huge number of degrees of freedom of numerical models.

The approach presented in this paper is based on the reduction of the aeroelastic model. Low dimensional models, with a very small number of degrees of freedom, enable the adjustment of the model's response to the measurements of real object, done in wind-tunnel experiments or in the flight tests.

The reduced order model of fluid flow is based on Galerkin method (Rempfer, 2000, Noack et al., 2003) using empirical expansion modes like POD (Sirovich, 1987, Holmes et al., 1998) or DMD (Schmid and Sesterhenn, 2010, Frederich and Luchtenburg, 2011). To include the influence of moving boundary onto flow physics, Arbitrary Lagrangian-Eulerian approach (Serrate et al., 2001, Stankiewicz et al., 2010) is used.

As the mode basis is truncated to a limited number of the most energetic modes, high frequencies are filtered and small scales are neglected in the reduced order model. The truncation, as well as possible inconsistency of data set and the reduced-order formulation (Couplet et al., 2005), result in additional discrepancies between reduced order Galerkin model and high-fidelity data (numerical or experimental).

To correct the behavior and improve the accuracy of Reduced Order Galerkin Model, the coefficients of the Galerkin system of ODE are adjusted (Couplet et al., 2005). Such a calibration results in the modification of linear/quadratic terms of Galerkin system.

FLUID-STRUCTURE INTERACTION ALGORITHM

Computational Aeroelasticity (Kamakoti and Shyy, 2004) examines the interactions between a fluid and a deformable structure using the methods of Computational Fluid Dynamics (CFD) and Computational Structural Mechanics (CSM) (Roszak et al., 2009).

The high-dimensional approach used in this work relies on the use of independent solvers for solid and fluid mechanics, that might use different discretization methods (like Finite Volume and Finite Element Methods) and mesh densities. Non-conforming grids are the reason of using additional interpolation tools to exchange the information on the coupling interface. The computational FSI algorithm used in this work is shown in fig. 1.

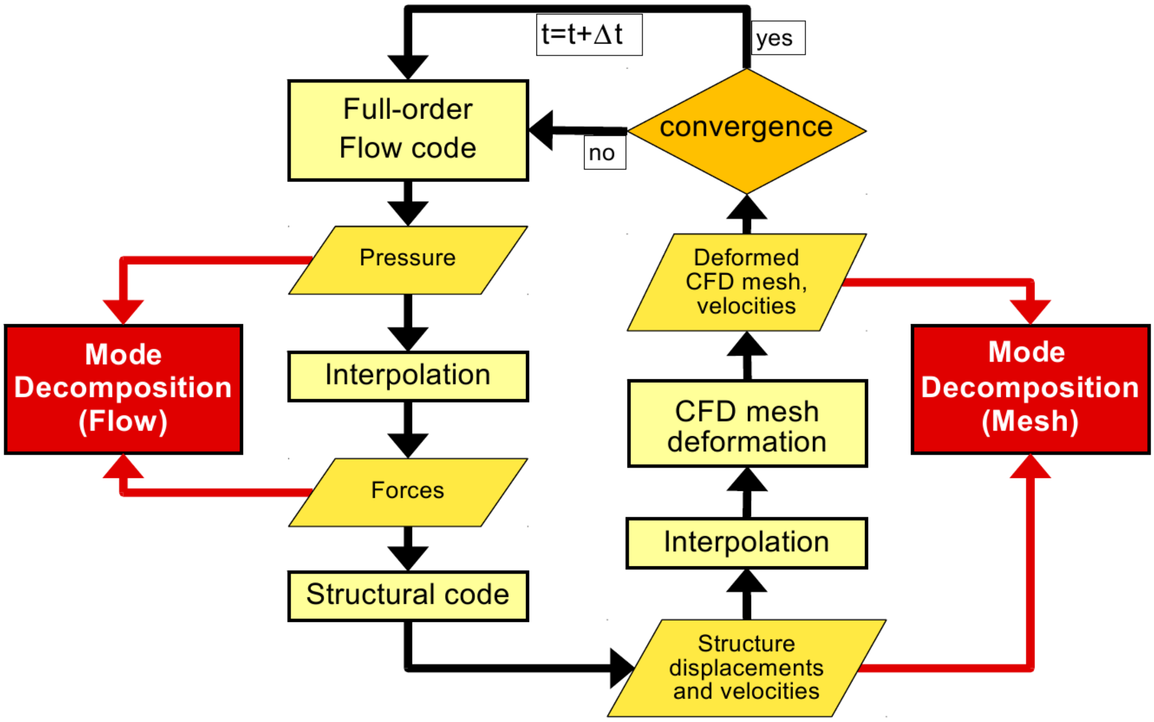

Fig. 1: The block diagram of aeroelastic simulation. Red blocks are used in the design of Reduced Order Model

The velocity and pressure fields are calculated by efficient and parallel CFD solver DLR TAU-code (Schwamborn et al., 2006). The pressures are interpolated onto the structure using modules based of finite-element meshes, as well as bucket and oct-tree neighbor search algorithms.

Under applied aerodynamic load in-house CSM solver (Morzyński, 1987) calculates the deformations of the structure. The nodal displacements on the boundary of the structure are interpolated onto CFD mesh. Then the displacements and velocities in the interior of CFD mesh are calculated using deformation tools based on elastic or spring analogy (Farhat, 1998).

The deformation of the flow domain affects the velocity and pressure fields, which in turn causes the change in structure's strain and stress. The loop presented above runs until the convergence in a given time step is reached.

The numerical models, both of fluid flow and the structure, use many simplifying assumptions, like the use of Euler or RANS equations instead of full Navier-Stokes ones and the assumption of linear relationship between the stress and strain in the structure (Landau and Lifshitz, 1986). The aforementioned simplifications lead to the discrepancies between the numerical aeroelastic model and the real object.

Therefore, recent trends in the process of aircraft structure design verification consist of the coupling of numerical methods and experimental studies. Due to the large number of degrees of freedom of numerical models the coupling mentioned above is usually limited to the verification and validation of numerical model (Roszak et al., 2009).

The solution of the aforementioned problem might be the reduction of the model describing the interesting aeroelastic phenomena. Limitation of the number of degrees of freedom of the aeroelastic system to a few-several hundred allows to adjust the response of Reduced Order Model to the measured signal of the real object, identified using techniques like Eigenvalue Realization Algorithm (ERA) (Caicedo et al, 2004).

REDUCED ORDER MODELLING OF FLUID FLOW

Galerkin method

In this paper the reduction of model's dimension is based on Galerkin method (Rempfer, 2000, Noack et al., 2003). The variables in governing equation (like incompressible Navier-Stokes or Euler equations) are approximated using some base solution and a weighted sum of modes (1):

$$u^{[N]} = u_0 + \sum_{j=1}^{N} a_j u_j = \sum_{j=0}^{N} a_j u_j, \qquad a_0 \equiv 1 \tag{1}$$

The weights are the mode amplitudes in consecutive time steps, and the modes are time-invariant. Using a truncated mode basis, with only the most energetic modes, one can define a residual o approximated equation (2):

$$R^{[N]} = \dot{u^{[N]}} + \nabla \cdot (u^{[N]} \otimes u^{[N]}) + \nabla p^{[N]} - \frac{1}{Re} \triangle u^{[N]} \tag{2}$$

The projection of this residual onto space spanned by the modes is called Galerkin projection (3):

$$\left(u_i, R^{[N]}\right)_\Omega = \int_\Omega u_i R^{[N]} d\Omega = 0 \tag{3}$$

where Ω represents whole computational domain. This projection leads to a system of ordinary differential equations, called Galerkin System (4):

$$\dot{a}_i = \frac{1}{Re} \sum_{j=0}^{N} a_j l_{ij} + \sum_{j=0}^{N} \sum_{k=0}^{N} a_j a_k q_{ijk} \tag{4}$$

with linear and quadratic terms as shown below (5):

$$l_{ij} = \left(\mathbf{u}_i, \triangle \mathbf{u}_j\right)_\Omega \quad \text{and} \quad q_{ijk} = -\left(\mathbf{u}_i, \nabla \cdot (\mathbf{u}_j \otimes \mathbf{u}_k)\right)_\Omega \tag{5}$$

Depending on the analyzed problem, some terms (like pressure or modified convective term) might be required (Noack et al., 2005). In the case of Euler equations Galerkin projection results in no linear terms.

Arbitrary Lagrangian-Eulerian Approach

To model the fluid flow with a changing boundaries, Arbitrary Lagrangian-Eulerian (ALE) Approach (Sarrate et al., 2001, Donea et al., 1982) is used. In ALE formulation, the velocity of the boundary and the fluid mesh $\boldsymbol{u}_{grid}$ is included in the modified convective term (6):

$$\dot{\mathbf{u}} + \nabla \cdot ((\mathbf{u} - \mathbf{u}_{grid}) \otimes \mathbf{u}) + \nabla \mathbf{p} - \frac{1}{Re} \triangle \mathbf{u} = 0 \tag{6}$$

It can be assumed, that mesh velocity might be decomposed in a similar manner as the fluid velocity fields (7):

$$\mathbf{u}_{grid} = \sum_{j=1}^{N_G} a_j^G \mathbf{u}_j^G \tag{7}$$

where modal mesh deformations are time-invariant. The projection of modified convective term leads to additional term in Galerkin system (8):

$$-\left(\mathbf{u}_i, \nabla \cdot ((\mathbf{u} - \mathbf{u}_{grid}) \otimes \mathbf{u})\right)_\Omega = -\left(\mathbf{u}_i, \nabla \cdot (\mathbf{u} \otimes \mathbf{u})\right)_\Omega + \left(\mathbf{u}_i, \nabla \cdot (\mathbf{u}_{grid} \otimes \mathbf{u})\right)_\Omega = \\ = \sum_{j=0}^{N} \sum_{k=0}^{N} q_{ijk} a_j a_k - \sum_{j=1}^{N_G} \sum_{k=0}^{N} q_{ijk}^G a_j^G a_k \tag{8}$$

where:

$$q_{ijk}^G = -\left(\mathbf{u}_i, \nabla \cdot (\mathbf{u}_j^G \otimes \mathbf{u}_k)\right)_\Omega \tag{9}$$

As the mesh deformation modes are time-invariant, q_{ijk}^G term is only affected by the integration over elements of deforming mesh.

Modal Decomposition

The mode bases used in the model reduction might be classified in terms of mathematical, physical and empirical approaches, as discussed in (Noack et al., 2003). In the case of empirical approach, the modes are determined in the decomposition of (previously obtained) experimental or numerical data. One of the most popular modeling approaches in fluid dynamics is Proper Orthogonal Decomposition (POD) (Holmes et al., 1998, Sirovich, 1987) and its modifications like Sequential POD (Jørgensen et al., 2003) and Double POD (Siegel et al., 2008).

POD modes are optimal in energy representation by construction, so they possibly better describe the Navier-Stokes attractor (limit-cycle oscillations of periodic flow), than the same number of modes obtained in any different manner (Noack, 2005). In this method, velocity vectors from M consecutive time steps and centered (by subtracting the mean flow) and form matrix V. POD modes used in model reduction are the eigenvectors u_i of the standard eigenproblem $Cu_i = \lambda_i I\, u_i$ of the autocorrelation matrix C (10):

$$C = \frac{1}{M} \acute{V} \acute{V}^T, \tag{10}$$

related to eigenvalues λ_i of largest magnitude.

While the number of snapshots M is substantially smaller than the number of degrees of freedom, a modification of traditional POD, called Snapshot method (Sirovich, 1987), is used.

In the case where the snapshots are not captured from limit-cycle and the amplitude of oscillations is changing, recently developed Dynamic Mode Decomposition (Schmid and Sesterhenn, 2010, Frederich and Luchtenburg, 2011) might be used.

The time-varying process is approximated by the use of linear operator on the current state vector (11):

$$q(t + \triangle t) \approx e^{\triangle t A} q(t) \tag{11}$$

The right hand side in the equation (11) might be approximated using the multiplication of the sequence of known solutions and the companion matrix S:

$$S = \begin{pmatrix} 0 & 0 & \ldots & 0 & c_0 \\ 1 & 0 & \ldots & 0 & c_1 \\ 0 & 1 & \ldots & 0 & c_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 1 & c_n \end{pmatrix} \tag{12}$$

where the coefficients $c_0 \ldots c_n$ are obtained from the solution of the system of equations (11).

The eigenvectors of matrix S are used to obtain the DMD modes.

CALIBRATION OF REDUCED MODEL

The neglect of the most of the eigenvalue spectrum and using only the most energetic modes result in the filtering of high frequencies (Couplet et al., 2005) and discrepancies between the low- and high-dimensional flow models. The possible inconsistency of data set and reduced order formulation (like the neglect of some of the terms in projected governing equations (Cordier et al., 2009)) and structural instability of Galerkin projection (Noack et al., 2003) are the sources of additional errors.

To minimize the differences between the reduced order model and high-dimensional data, calibration of the Galerkin model might be performed. This process consists in adjusting the values of linear and/or quadratic terms of the model, e.g. by addition of “eddy” viscosities (Podvin, 2001) (13) or addition of correction values to all the linear/quadratic terms (Couplet et al., 2005, Galletti et al., 2004), in order to recover the effects of truncated modes (14):

$$l_{ij}^{+} = \frac{\nu_{T,i}}{\nu} l_{ij}, \qquad i = 1 \cdots N \tag{13}$$

$$\dot{a}_i = \underbrace{\nu \sum_{j=0}^{N} \left(l_{ij} + l_{ij}^{+} \right) a_j + \sum_{j=0}^{N} \sum_{k=0}^{N} \left(q_{ijk} + q_{ijk}^{+} \right) a_j a_k}_{f_i(\mathbf{a})} \tag{14}$$

The values of corrections to the terms of Galerkin system are determined by the optimization procedure, where objective function, related to the prediction error of the model (15), is minimized.

$$\begin{aligned} \chi_0 &:= \sum_{i=1}^{N} \int_0^T \left(a_i^{ROM}(t) - a_i^{DNS}(t) \right)^2 dt = \mathrm{Min} \\ \chi_1 &:= \sum_{i=1}^{N} \int_0^T \left(\dot{a}_i^{ROM}(t) - f_i(\mathbf{a}^{DNS}(t)) \right)^2 dt = \mathrm{Min} \end{aligned} \tag{15}$$

Additionally, error definitions might be based on different parameters, like the agreement of frequencies or growth rates. In this work, the calibration of the model is based on Genetic Algorithm (Stankiewicz et al., 2011).

TEST CASE – AGARD 445.6 WING

As a test case, aeroelastic AGARD 445.6 wing (based on symmetric NACA65A004 airfoil) has been chosen. In this case, the Reynolds number is assumed high enough to neglect boundary layer effects and solve Euler equations. Mach number equals M = 0.32 and the angle of attack is $\alpha = 0.26^o$. The CFD mesh (fig. 2, left) used for high-dimensional simulation consists of 1873636 tetrahedral elements and 319918 nodes. The structure of the wing is modeled with 200 triangular plate elements with varying thickness and 121 nodes (fig. 2, right).

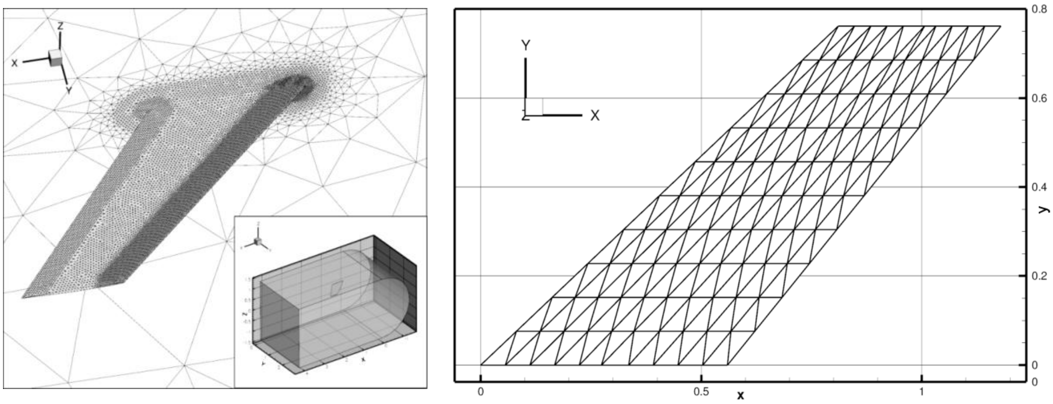

Fig. 2: The AGARD wing meshes used for high dimensional CFD (left) and CSM (right) simulations

The deformations of the structure under aerodynamic load are calculated using FEM solver. For a given configuration, growth of amplitude is observed (fig. 3).

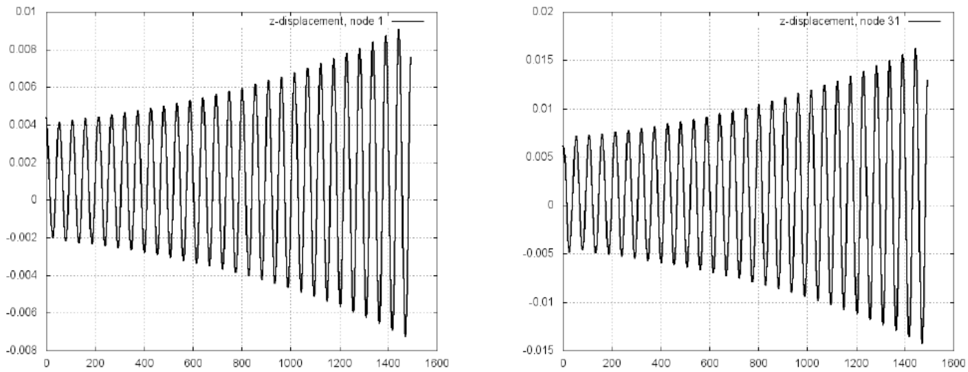

Fig. 3: Z-component of the displacement of a node on the end of a leading (left) and trailing (right) edge of the AGARD wing

Five periods of the oscillations calculated using high-dimensional aeroelastic systems have been selected for further analysis. The flow velocities and pressures, coupled with the aerodynamic loads (interpolated onto CSM mesh) have been decomposed using DMD. The most dominant (complex) mode is depicted in fig. 4, and the mode norms and frequencies for the first 12 modes are presented in fig. 5.

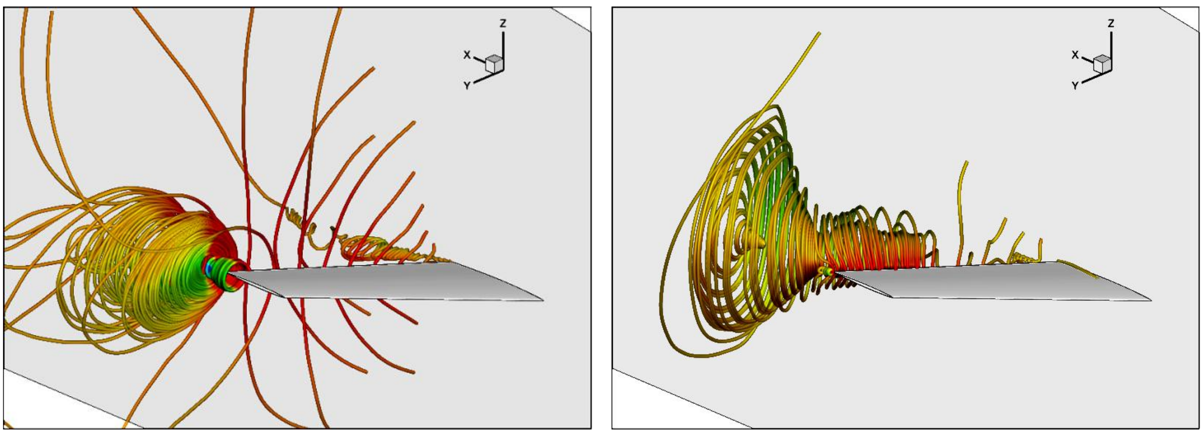

Fig. 4: Real (left) and imaginary (right) part of second DMD mode of the flow around AGARD wing

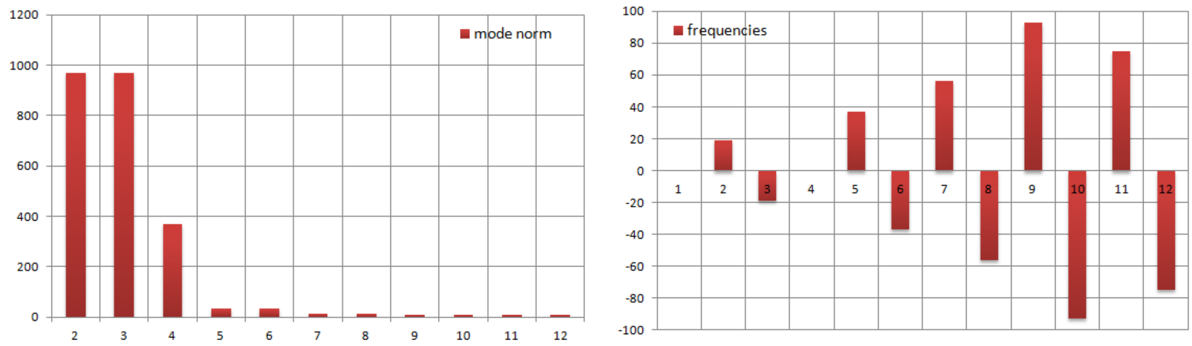

Fig. 5: Norms (left) and frequencies (right) of the first DMD modes of the flow around AGARD wing

It can be seen, that the first 3 modes have a significantly larger norms than the rest of the modes. Additionally, further modes have a frequencies being multiplications of the second mode frequency. While the 4th mode has zero frequency (similarly to the base solution stored in first mode) and modes 2. and 3. are complex conjugate, it has been chosen to use only real and imaginary part of second DMD mode in Galerkin approximation.

The displacements of the structure, coupled with deformed CFD meshes, have been analyzed using POD. Resulting eigenvalues, mode coefficients and modes are depicted in figures 6 and 7, respectively.

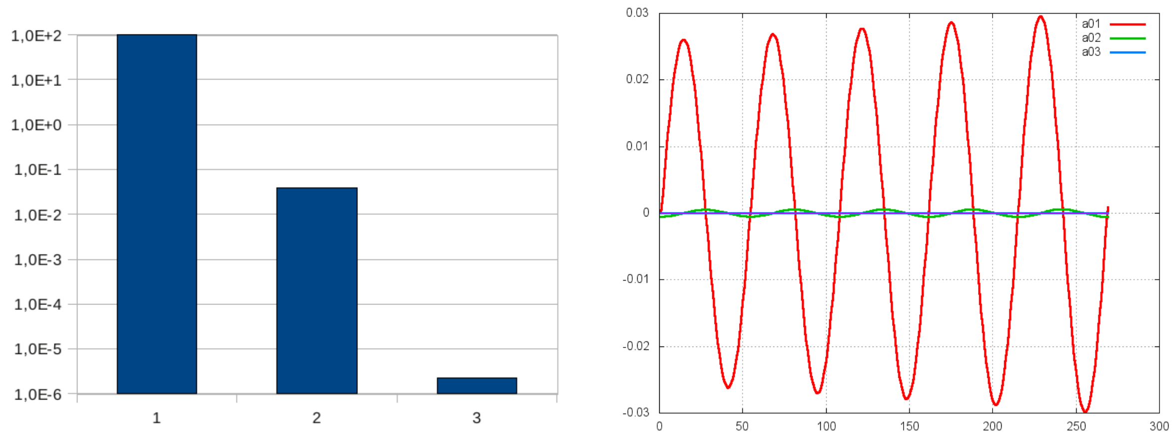

Fig. 6: Normalized eigenvalues corresponding to the first three POD modes of the deformation/velocity of the AGARD wing mesh (left) and the mode coefficients (right)

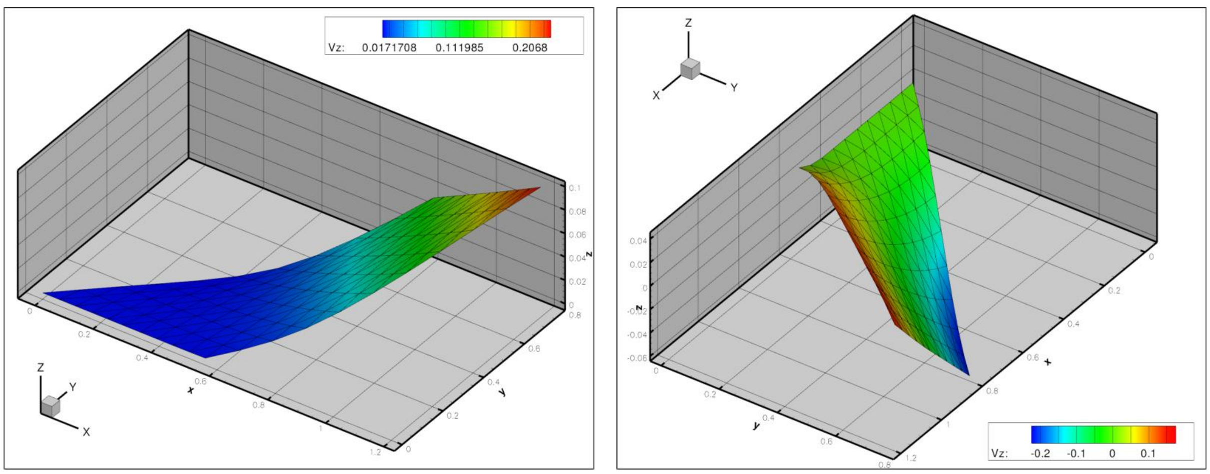

Fig. 7: First two POD modes of the deformation/velocity of the AGARD wing mesh

The Galerkin projection of approximated Euler equation in Arbitrary Lagrangian-Eulerian approach onto space spanned by DMD modes resulted in reduced order model of the flow with moving boundary. This model replaced the high-dimensional CFD solver in aeroelastic system presented before (fig. 1). Due to the coupling of the quantities before and after interpolation (pressure field with aerodynamic load, structure strain with CFD mesh deformation) on the modal decomposition stage, the interpolation between CFD and CSM meshes, as well as the deformation of CFD mesh, is straightforward.

The response of low-dimensional aeroelastic model, compared to reference high-dimensional data, is presented in fig. 8.

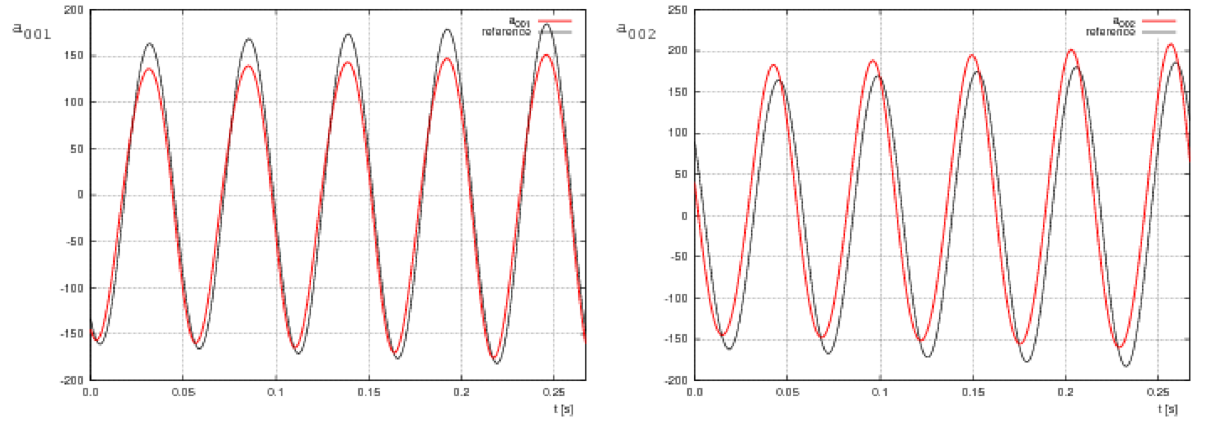

Fig. 8: Mode amplitudes of Galerkin model for the first two modes, compared to the coefficients of mode decomposition

To achieve higher agreement between both models, genetic algorithm-based calibration procedure has been performed. The total number of individuals (chromosomes) in each population has been set to 50. After 8 iterations, the mean-square error of mode coefficients (amplitudes) has been reduced more than 6 times, and the mean square errors of time derivatives of these coefficients and kinetic energy of turbulence - almost 4 times. The calibrated model is shown in fig. 9.

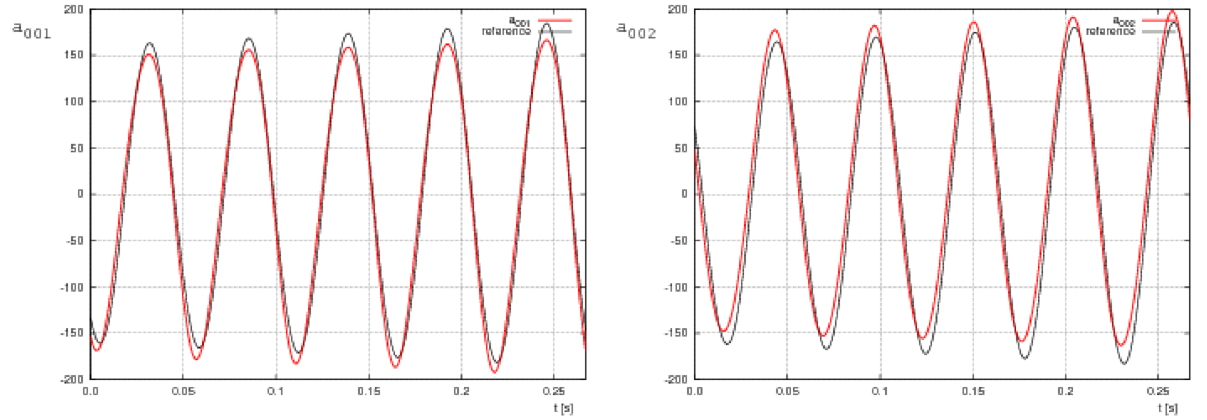

Fig. 9: Mode amplitudes of the calibrated model for the first two modes, compared to the coefficients of mode decomposition

REDUCED ORDER MODELLING IN MODEL IDENTIFICATION

The validation of the aeroelastic model of an aircraft is done via wind-tunnel tests. Model identification techniques like ERA result in parameters describing the real object, like modal frequencies and dampings.

The last stage of the design of low-dimensional model of AGARD wing is the adjustment of model's behavior to some arbitrary chosen parameters, possible to obtain in wind-tunnel experiment. In this paper it is assumed, that logarithmic decrement of the damping of the model presented in previous section, equal to -0.027, is too small, and the decreasing amplitude of oscillation should be observed. The desired value of this parameter has been artificially set to 0.1.

Using the same calibration tool as in previous section, new target function has been chosen, basing on the weighted values of frequency and the logarithmic decrement of damping errors. The optimized solution, found after 9 iterations of the algorithm, is presented in fig. 10.

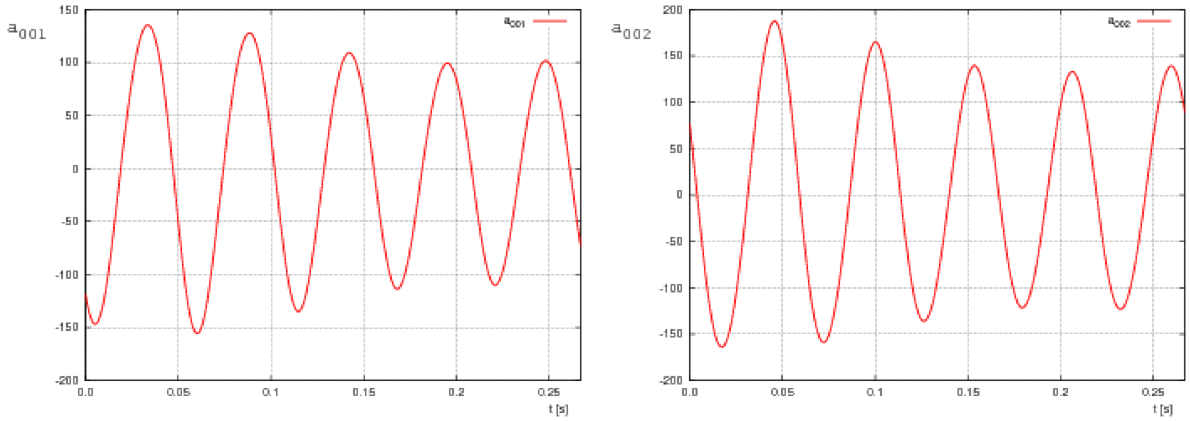

Fig. 10: Mode amplitudes for the first two modes after the adjustment to the given parameters

As can be seen, after the adjustment of calibration terms, the oscillations in Galerkin model are damped. Achieved logarithmic decrement of damping is equal to 0.08, very close to the desired value.

If another frequency should be achieved, the calibration of the parameters of structure model will be also required.

SUMMARY

In this paper the design of low dimensional model of an aeroelastic AGARD 445.6 wing is presented. The data from high-dimensional aeroelastic analysis has been decomposed using Proper Orthogonal Decomposition and Dynamic Mode Decomposition. Resulting mode bases, after truncation to the most energetic modes, has been used to create low dimensional model of a flow around aeroelastic AGARD wing.

The model, constructed using Galerkin Projection, has been calibrated in Genetic Algorithm-based optimization to minimize mean-square errors between the mode amplitudes computed by the model and projected from input data onto the mode space. After the calibration, good agreement with high-dimensional model is achieved in amplitudes, frequencies and damping.

In the final section, low-dimensional model has been adjusted to sample parameters in the form obtainable from wind tunnel tests. The same genetic algorithm-based tool has been used, with objective function based on the measurement of frequency and logarithmic decrement of damping.

The designed low dimensional flutter model is consistent with real object. It gives the opportunity to study flutter for various configurations of aircraft without the wind tunnel experiments, including those that actually would result in the destruction the object.

The use of ROM models can significantly shorten the time needed to estimate the aeroelastic properties of aircraft in flight, allowing to predict the potentially dangerous flow conditions. These factors affect the substantial reduction in time and cost implementation of the new prototype aircraft by reducing the number of wind tunnel tests.

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