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INVERSE METHOD FOR BLADE-TO-BLADE AND AXISYMMETRIC VISCOUS FLOW MODELS IN CYLINDRICAL COORDINATES USING STREAM-FUNCTION TRANSFORMATION

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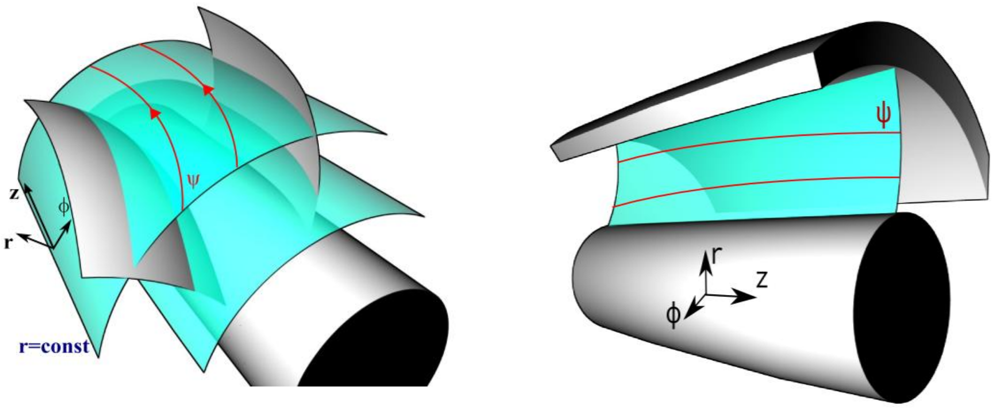
A new, inverse method for viscous 2D, laminar and turbulent duct flows will be presented. The method is based on incompressible Navier-Stokes equations transformed to the stream function coordinate system (von Mises, 1927). For turbulent flows, the Reynolds-averaged Navier-Stokes equations with RANS turbulence model are considered. The method formulations for 2D flow configurations in the cylindrical system: blade-to-blade ($r=const$) and axisymmetric ($d/d\varphi=const$, possibly with swirl) models are of main interest as a design step before applying blade-to-blade solvers (see Fig. 1).

For each model, von Mises coordinates are obtained from the stream function formulation. The transformation has unique properties. The coordinate variables become a functional of flow velocity. The relationship between new coordinates and velocity is delegated to a new independent equation. Therefore, the design problem can be formulated easily. Additionally, the curvilinear flow domain corresponds to a rectangular shape in the new coordinate system. This simplifies the numerical solution of the presented problem.



a) b)

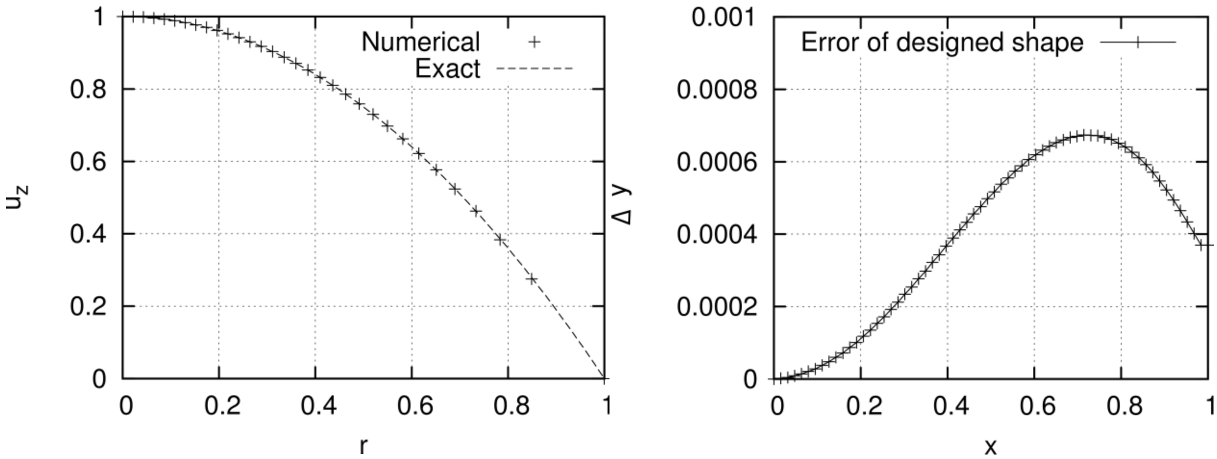
Figure 1. The stream surfaces of two different flow problems in the cylindrical system of coordinates: a) blade-to-blade, b) axisymmetric.

After the coordinate transformation, the continuity and momentum equations are derived. The model is extended with the additional equation from the transformation. The dimensional analysis of typical flow design problems is presented. Depending on it, simplifications of the model are proposed. The appropriate boundary conditions for viscous flow design are discussed. The no-slip velocity condition in the connection with the Jacobian of the transformation leads to the indeterminate problem at the wall (Dulikravich 1995).

A special treatment of those regions to regularise the singularity of discretisation is proposed and its convergence behaviour is analysed.

The design problem is solved numerically in the stream-function coordinates domain. The continuity and momentum equations are coupled with the additional stream function equation. As a result, the pressure and velocity fields as well as shape of streamlines are obtained. The shape of streamline aligned to the wall boundary represents the designed geometry. The numerical implementation of the model in the finite difference approach with the pseudo-time marching technique is shown.

The validation of the inverse method for several analytical test cases is presented and analysed, starting with design of straight pipe with the laminar flow (cf. Fig.2). The scheme discretisation errors and general convergence of the flow solver are discussed. Detailed analysis of the singularity influence on the solution is shown. Next, a real-world flow design problem is solved as an example. The result of inverse problem is validated via comparison with numerical simulation of obtained geometry (the flow analysis task). A commercial Navier-Stokes solver is used for this purpose. The robustness of the method is presented.



a) b)

Figure 2. a) Velocity profile on pipe's cross-section at x=0.5, b) normalised difference between exact and numerically designed shape for grid 64x32 in (x,Ψ).

Potential applications to the design problem of fluid-flow machines are considered. The advantages and drawbacks of the method are discussed. The applicability of the method to 3D flow model is prescribed.

References

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