XX Polish Fluid Mechanics Conference,
Gliwice, 17-20 September 2012

THE DECAY POWER LAW IN TURBULENCE GENERATED BY GRIDS

Joanna JURKOWSKA, Zygmunt WIERCIŃSKI,
Institute of Fluid-Flow Machinery, Gdańsk, Poland;
E-mail: jj@imp.gda.pl, zw@imp.gda.pl

Abstract

It is well known that the turbulence in flow can be characterized by two main parameters: the intensity and scale. The scale can be related to length or time. There are different length scales described in literature (micro, macro, Kolmogorov, dissipation and a few others).

The turbulence in wind tunnels can be generated by a variety of means like passive and active grids, most often grids of round or square wires, perforated plate and so on.

The aim of this investigation is to gain a detailed knowledge like skewness and kurtosis of velocity fluctuations, wave number spectra, etc. on the turbulence generated by wire grids and to apply it to the laminar-turbulent transition investigation on the flat plate.

Additionally, some new results on the influence of the boundary conditions of turbulence generation on the decay law of turbulence are given.

Key words: turbulence generation, turbulence intensity, grid, isotropy, homogeneity

INTRODUCTION

Recently there is a new interest in generation of turbulence of more broader range of characteristics like scales and more specific parameters e.g. zero shear stress. Therefore a new methods of its generation are used. The methods of turbulence generation can be divided into *passive* (regular static grids, fractal grids) and *active* (jet, grid with wings).

In Gad-el-Hak & Corrsin (1973) opinion, turbulence homogeneity improvement can be attained by the active grids that are equipped with controllable nozzles. Compared with the passive case, the downwind-jet active grids has a smaller pressure drop across it and gives a smaller turbulence level. The upwind-jet grid gives a larger static pressure drop, larger turbulence level and scales, which is much like that commonly observed behind passive grids of higher solidities. According to the authors, a coflow injection grid generates turbulence with a greater degree of isotropy and homogeneity than it would be in the case of passive grid or counterflow injection grid.

Mydlarski & Warhaft (1996) used the turbulence generator consisting of grid bars with triangular wings that rotate and flap in a random way. They have explored the evolution of grid turbulence with Reynolds number. Their experiments, made with the usage of such active grid, appeared to be first to bridge the gap between the low-Reynolds-number laboratory studies of grid-generated turbulence and the high-Reynolds-number experiments done in the atmosphere and the oceans. Their results suggests that much can be learned about the behaviour of turbulence at high *Re* using a small wind tunnel.

New experimental results of decaying turbulence were presented by Valente & Vassilicos (2011), to highlight the similarities and differences between turbulent flow behind regular grid and the turbulence generated by fractal square grid and to compare them with measurements made by Mydlarski & Warhaft. Their investigation also examine the homogeneity and isotropy of the decaying turbulence. They noted that the study of freely decaying turbulence requires experiments where a wide range of Reynolds number can be achieved by modifying the initial conditions.

INVESTIGATION RIG

The investigation was carried out in the subsonic wind tunnel of low level of turbulence, Tu<0.08% and velocity up to 100 *m/s*. The enhanced level of turbulence was generated by five wicker grids of following dimensions (named appropriately Grid 1, 2, 3, 4 and 5):

1) d=0.3 *mm*, M=1 *mm*,
2) d=0.6 *mm*, M=3 *mm*,
3) d=1.6 *mm*, M=4 *mm*,
4) d=3.0 *mm*, M=10 *mm*,
5) d=3.0 *mm*, M=30 *mm*.

The coordinates systems for the turbulence intensity and scale measurements is fixed to the grid with x coordinate parallel to the mean velocity of flow.

The investigation consisted of measuring the average velocity profiles and the velocity fluctuations by means of the 55P15 thermoanemometry probe of DANTEC. Data were transferred to the PC via a data acquisition card NI 6040.

Changing parameter during the investigation was also the velocity of the incoming stream. For each grid, it was equal to:

1) for grid 1: U= 10, 15, 20 *m/s*,
2) for grid 2: U= 10, 15, 20 *m/s*,
3) for grid 3: U= 6, 10, 15, 20 *m/s*,
4) for grid 4: U= 6, 10 *m/s*,
5) for grid 5: U= 4, 6 *m/s*.

TURBULENCE INTENSITY BEHIND THE GRID

Isotropy and homogeneity of turbulence

In general, the turbulence intensity is defined as the ratio of standard deviation to the mean flow velocity, $\boldsymbol{U}$. If the velocity field is described by the coordinate system x_i where x_1 is an axis oriented in the direction of the mean flow velocity ($U = U1, U2 = U3 = 0$), a ratio:

$$Tu_i = \frac{\sqrt{\overline{u_i^2}}}{U} = \frac{u_i'}{U} \quad , \quad i=1,2,3 \tag{1}$$

where $i=1$ corresponds to the longitudinal turbulence intensity and the two others to the components of the transverse intensity. In case of isotropic turbulence their characteristics do not depend on the spatial orientation of the coordinate system.

One of the methods to assess the isotropy of turbulence is to determine the skewness factor in the flow velocity distribution (Batchelor, 1953; Mohamed & LaRue, 1990):

$$S(u) = \frac{\overline{u^3}}{\overline{u^2}^{3/2}} \tag{2a}$$

The turbulence is isotropic, if the skewness factor is zero and hence, the probability density function of the variable *u'* has normal distribution.

In a similar way to the concept of skewness, *kurtosis* is a descriptor of the shape of a probability distribution. The measure of kurtosis ia a fourth central moment of mean velocity divided by the standard deviation to the fourth power:

$$K(u) = \frac{\overline{u^4}}{\overline{u^2}^{4/2}} \tag{2b}$$

When $K(u) = 3$, the probability distribution is normal. When $K(u) < 3$ or $K(u) > 3$, kurtosis are called **platykurtic distributions** (flat) or **leptokurtic distributions** (focused) respectively.

There is an initial distance wake region downstream of a grid where the flow is strongly inhomogeneous. This is due to the fact that initially isolated bar wakes increase their size and coalesce to form a truly homogeneous flow. Experiment of Roach (1986) shows that the area where the turbulence is inhomogeneous, depends on the mesh size and the flow can be considered homogeneous by ten mesh lengths downstream of a grid. However, different authors give different assumptions.

Mohamed & LaRue (1990), as a method to investigate the homogeneity of the flow behind the grid, use *transverse variation* of the difference of the root mean square of the downstream velocity, $\overline{u^2}^{1/2}$, and the centreline value normalized by the centreline value, $\overline{u_0^2}^{1/2}$:

$$V(u) = \frac{\overline{u^2}^{1/2} - \overline{u_0^2}^{1/2}}{\overline{u_0^2}^{1/2}} \tag{3}$$

The experiments show, the lower is the Reynolds number, Re_M (4), based on the mesh size, M, the closer to homogeneous is the flow behind the grid.

$$Re_M = U_M M/\nu \tag{4}$$

where $U_M = U_\infty/(1-S)$ is an averaged flow velocity at the grid mesh.

Decay power law

An important parameter describing the grid is the grid fill factor (5) determined from the equation:

$$S = 1 - \beta = 1 - \left(1 - \frac{d}{M}\right)^2 \tag{5}$$

where $\beta = F_1/F_0$ is the ratio of the area unoccupied by the grid rods, of diameter d, and the total grid area; M is the grid mesh size.

Roach (1986) gives a very simple formula for the level of turbulence, depending on the diameter of the grid wire, d:

$$Tu = c\left(\frac{x}{d}\right)^{-n} \tag{6}$$

In accordance with the Roach's experiments, a value of the experimental factor c is usually equal to approximately 0.8, and $n = \frac{5}{7}$.

Another equations used to determine the decay of turbulence behind a grid were given by Mikhailova et al. (2005):

$$Tu = c_1 \left(\frac{x}{M} \right)^{-n1} \tag{7a}$$

$$\frac{Tu}{\sqrt{S}} = c_2 \left(\frac{x}{M} \right)^{-n2} \tag{7b}$$

where constant values c_1, n_1 and c_2, n_2 are determined experimentally. In accordance with the data of different authors, cited by Mikhailova et al. (2005), the exponent n_1 takes values 0.5 to 0.7. In expression (7b), c_2=86, $n_2 \approx 0.95$ in the initial wake region (for $7 \leq x/M \leq 20$), and c_2=41, $n_2 \approx 0.7$ in the main region (for $x/M>20$) in the wake downstream of the wicker grids.

Finally, Comte – Bellot et al. (1965), following for example Batchelor & Townsend (1948), gives the power law for the data on $1/\overline{u^2}$:

$$1/Tu^2 = U^2 / \overline{u^2} = A/((x - x_0)/M)^m \tag{8}$$

where x – the coordinate, positive in the downstream direction with origin at the grid, x_0 – the virtual origin, A and m – respectively the decay coefficient and exponent determined experimentally. One can find different values for the decay exponent, determined in many experiments by different authors, e.g. 1, 10/7, 6/5, 1.43 or $1.16 \leq n \leq 1.37$ (Mohamed & LaRue, 1990).

INVESTIGATION RESULTS

First of all, isotropy and homogeneity of the flow behind the grid were investigated. Figures 1a and 1b present the skewness factor, *S(u)*, for grids 2 and 5, depending on *x/M*. Fig. 1a shows the results for velocities *U*=15 and 20 *m/s*, which correspond to the Reynolds numbers: Re_M = 4750, 6100. Directly behind the grid, for *x*=9 *cm*, skewness factor is different from zero, but does not exceed 13% at the highest point. Then tends to zero, which indicates that the probability density function of the variable *u'* approaches the Gaussian distribution. Near the leading edge it is slightly increasing, but over the plate (from *x/M*=150) we observe the skewness factor not exceeding 5%.

Directly behind the grid 5 (Fig. 1b) the flow is strongly anisotropic. At a distance of 9 *cm* from the grid, *S(u)* = -0.27 (for *U*=6 *m/s*, Re_M=9000) and *S(u)*= -0.09 (for *U*=4 *m/s*, Re_M=15000). The skewness factor is negative because of the bar wakes velocity fluctuations that are smaller than the average velocity flow fluctuations. From the leading edge we observe the skewness factor increase and for *x/M*=30 it is still very high, *S(u)*=16%.

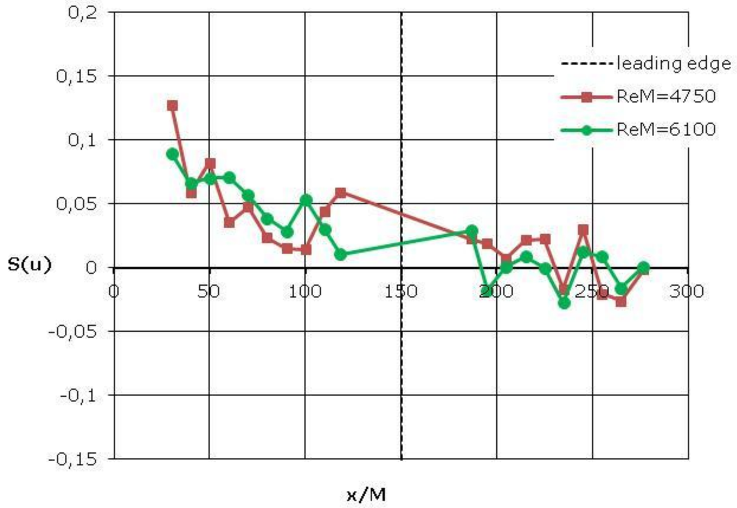

Fig. 1a. Skewness factor for Grid 2 and velocities *U*=15 and 20 *m/s*.

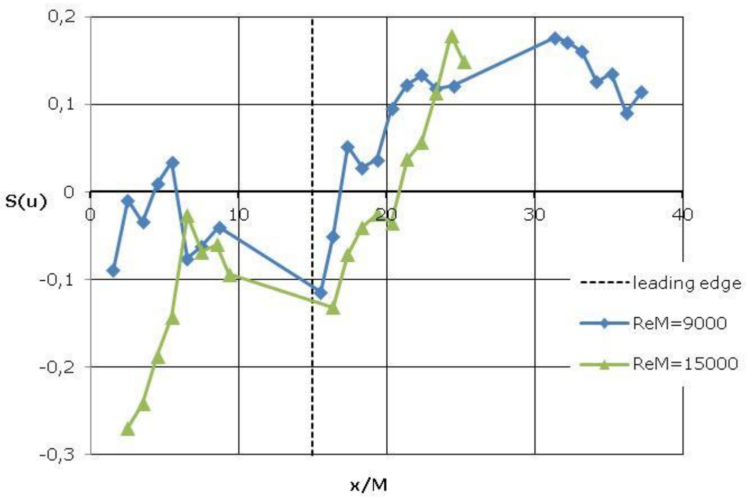

Fig. 1b. Skewness factor for Grid 5 and velocities *U*=4 and 6 *m/s*.

Kurtosis for the Grid 2 oscillates around a value of 3. At the distance of 9 cm from the grid distribution is leptokurtic but *K(u)* does not exceed a value of 3.1. Then it decreases slightly which means a distribution becomes more flat. The smallest value of kurtosis, *K(u)*=2.79, is at distance of *x/M*=255, for *U*=20*m/s*. (According to Batchelor, 1953, the distribution can be considered as normal to the value of *K (u)* = 2.86.)

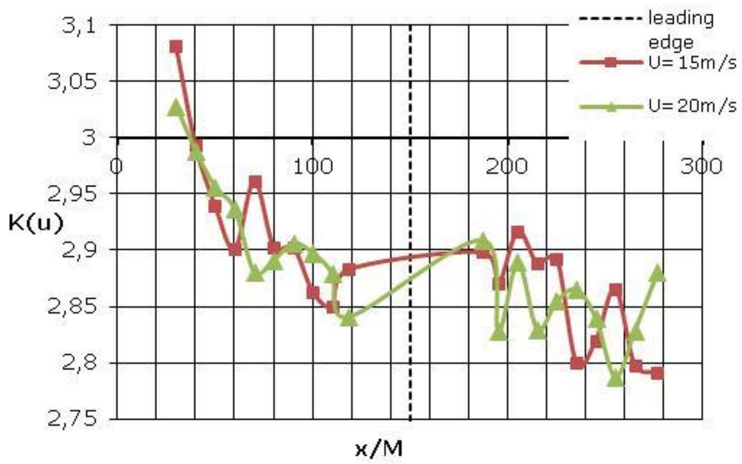

Fig. 2a. Kurtosis for Grid 2 and velocities *U*=15 and 20 *m/s*.

A quite different distribution of kurtosis is given by a flow velocity for the Grid 5. At the beginning *K(u)* is small, about 2.6 for velocity *U*=4m/s. Then increases slightly and gains a value of 2.99 at *x/M*=36 for velocity *U*=6m/s.

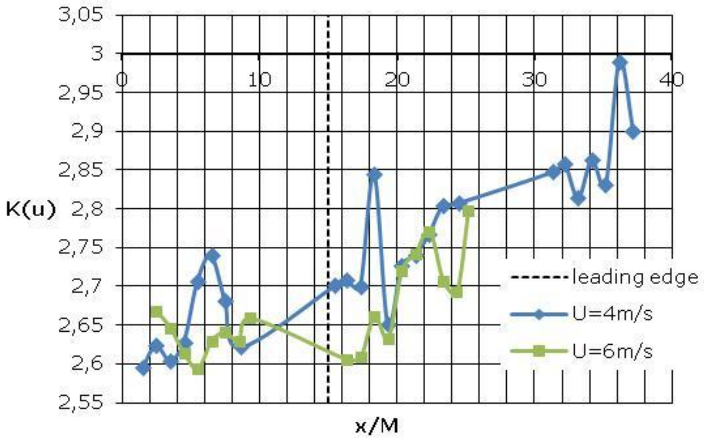

Fig. 2b. Kurtosis for Grid 5 and velocities *U*=4 and 6 *m/s*.

Figures 3 and 4 present transverse variation as a function of *x/M*, where *y* is the coordinate normal to the velocity vector. Fig. 3 relates to the Grid 2 and mean flow velocity *U*=20 *m/s.* Measurements were made at three distances from the grid: 9, 19 and 29 *cm*, i.e. *x/M*=30, 63, 97. For a distance of 29 *cm*, *V(u)* does not exceed 4% (for *x*=9 and 19 *cm* it does not increase much), so it can be assumed that the turbulence is homogeneous at this point.

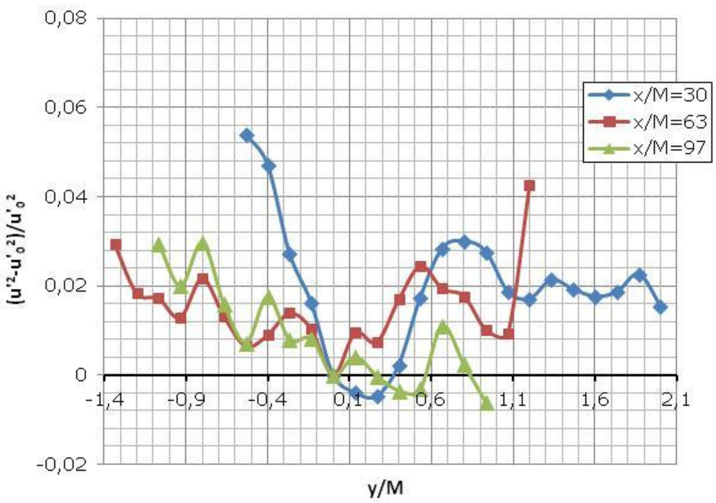

Fig. 3. Transverse variation for grid 2 and velocity *U*=20 *m/s*.

Figure 4 shows the case of Grid 5 and the mean flow velocity U=6 m/s. At a distance of 9 cm (x/M=3) the difference between the flow behind the mesh and the flow tracing the wire is clearly visible. As the distance from the grid increases we can see the transverse variation reduction, but the flow is still far from homogeneous (about 20%). So we can agree with Valente & Vassilicos (2011), who claim that for regular grids the turbulent flow should be considered inhomogeneous in transverse planes for *x/M*<25.

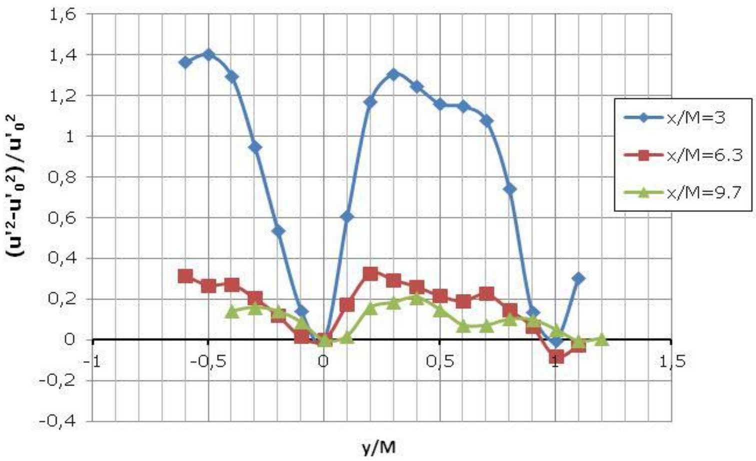

Fig. 4. Transverse variation for Grid 5 and velocity *U*=6 *m/s*.

The best correlation describing the decay of turbulence behind the grid was gained for the formula (8). The results of the investigation for five wicker grids of different dimensions are shown at Figure 5. As was mentioned before, the flow behind the grid 5 is strongly anisotropic and inhomogeneous, which is visible in Table 1. Also results for the Grid 4 differ significantly from the averaged values of *A* and *m*. The virtual origin, x_0, is very small, from about -0.1 to 0.3 *mm* and, in our opinion, can be negligible. Correlation coefficient is equal to 0.99. The decay coefficient for all grids together, determined according to formula (8), *A*=22 and exponent *n*=1.58.

Grid number	A	m
1	48.5	1.417
2	59.3	1.343
3	56.2	1.376
4	7.9	1.818
5	52.3	1.027

Tab. 1. Coefficient *A* and exponent *m* from the formula 8, for different grids.

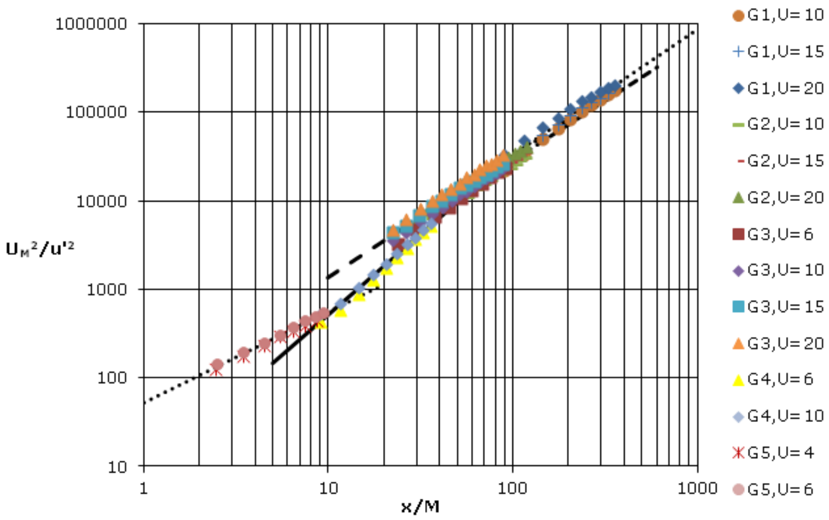

Fig. 5. The decay power law (8) behind grids.

The results for correlations (7a) and (7b) are presented below (Fig. 6 and 7). Correlations at Fig. 6 are made separately for each grid and the coefficients c_1 and n_1 are presented in

Table 2. As we can see, c_1 and n_1 increase consequently with the grid dimensions, except for the Grid 5.

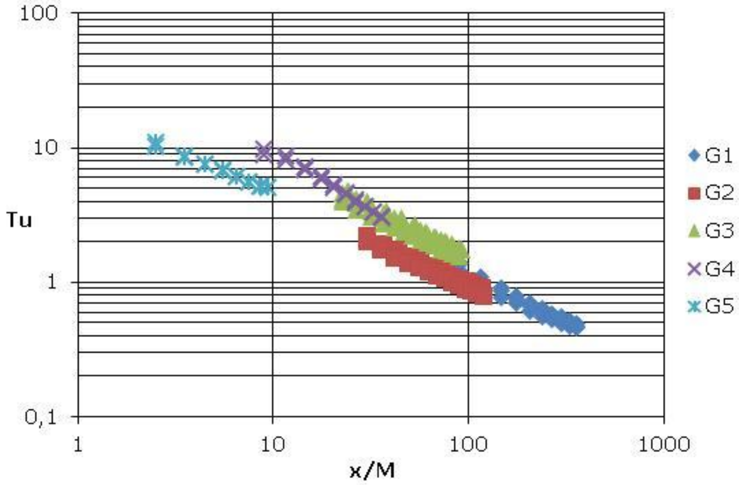

Fig. 6. The decay power law (7a) behind grids.

Grid number	c_1	n_1
1	4.4	0.354
2	19.2	0.656
3	36.6	0.684
4	63.6	0.839
5	17.8	0.572

Tab. 2. Coefficient c_1 and exponent n_1 from the formula 7a, for different grids.

The correlation coefficient for the relation presented at Fig. 7 is 0.988. Coefficients c_2 and n_2 are similar for each grid and they are as follows: c_2 = 63.6, n_2 = 0.8.

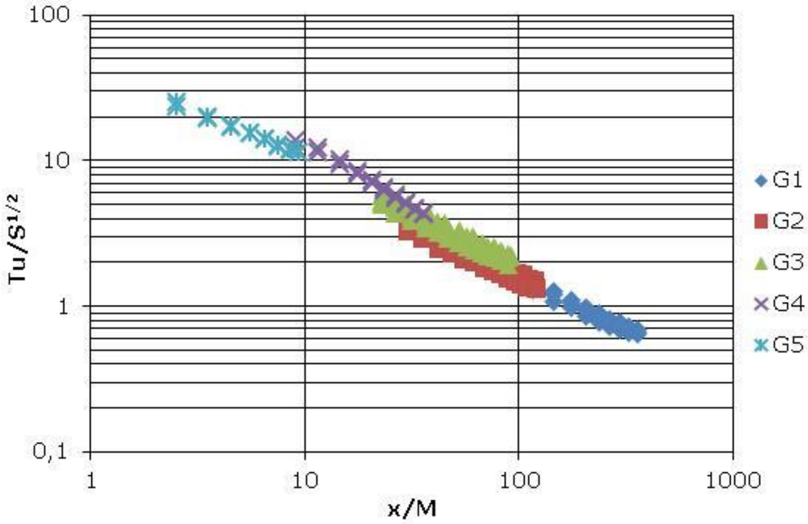

Fig. 7. The decay power law (7b) behind grids.

It is worth mentioning the results for the correlation given by Roach as the formula (6), suggest the decay coefficient c and the exponent n both depend on the Reynolds number based on grid dimension, M:

$$c = \varphi_c \, \mathrm{Re}_M^{-zc}, \qquad n = \varphi_n \, \mathrm{Re}_M^{-zn} \tag{9}$$

Recent experiments seem to confirm this assumption (Fig. 8a and 8b). (The same correlation was made for the Reynolds number based on the rod diameter, *d*.)

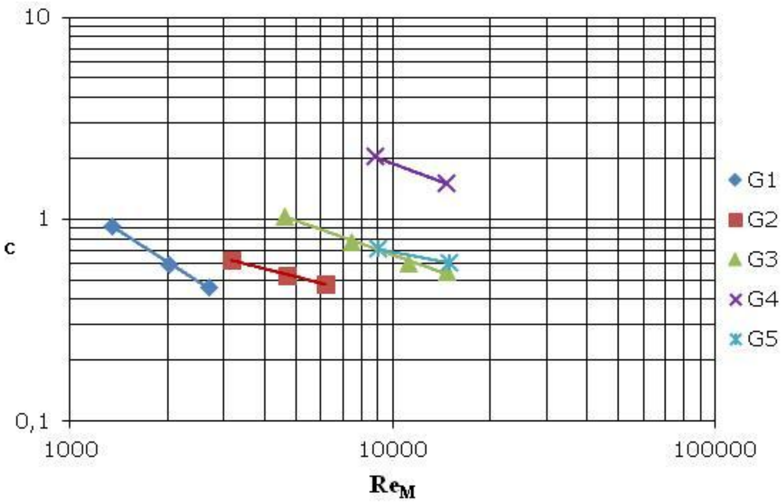

Fig. 8a. Coefficient *c* of the power law (6) behind grids.

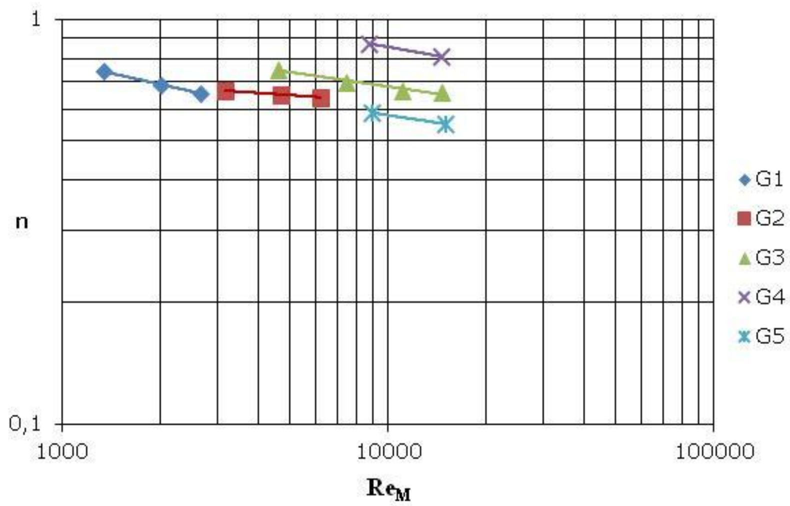

Fig. 8b.Exponent *n* of the power law (6) behind grids.

The factors *zc, zn* (which were avaraged for all grids) and φ_c, φ_n are shown in Table 3. The factors φ_c and φ_n increase consequently with the grid dimensions, except for the Grid5.

Grid number	φ_c	zc	φ_n	zn
1	51.2	0.58	1.78	0.12
2	70.6		1.86	
3	137.1		2.14	
4	388.2		2.68	
5	148.1		1.82	

Tab. 3. Factors from expressions (9) for different grids.

CONCLUSIONS

To investigate the isotropy and homogeneity of turbulence in the flow behind different grids the skewness factor of the flow velocity distribution function and also the transverse variation were determined, respectively. The results showed, there is an initial distance in the bar wakes to which the flow is strongly anisotropic and inhomogeneous. The effect of wake is the stronger, the greater are the parameters of the grid.

Few different correlations of decay power law were tested. The most accurate seems to be the relation (8), which has the greatest correlation coefficient. It requires indeed the determining of the virtual beginning of the turbulence, but in our opinion it is very close to zero and can be neglected.

Referring to the relation (6) there is a prediction the decay coefficient c and the exponent n both depend on the Reynolds number based on the grid dimensions.

Citing Valente & Vassilicos (2011) that there is a region of turbulence production behind a grid at the distance of $x \approx 0.68 M^2/d$, we claim the results for Grid 5 don't implement the decay power law until x=20 *cm*.

These results will be used to assess the influence of the different characteristics of turbulence on the laminar-turbulent transition in the boundary layer of a flat plate.

REFERENCES

Batchelor, G.K. (1953): *The theory of homogeneous turbulence,* Cambridge

Batchelor, G. K., Townsend, A. A. (1948): *Decay of isotropic turbulence in the initial period,* Proc. Roy. Soc. A, 193, 539

Comte-Bellot, G., Corrsin, S. (1965): *The use of a contraction to improve the isotropy of grid-generated turbulence,* J. Fluid Mech., Vol. 25, pp. 657-682

Gad-el-Hak, M., Corrsin, S. (1973): *Measurements of the nearly isotropic turbulence behind a uniform jet grid*, J. Fluid Mech., Vol. 62, part 1, pp. 115-143

Mikhailova, N. P., Repik, E. U., Sosedko, Yu. P. (2005): *Reynolds Number Effect on the Grid Turbulence Degeneration Law*, Fluid Dynamics, Vol. 40, No. 5, pp. 714-725

Mohamed, M. S., La Rue, J. C. (1990): *The decay power law in grid – generated turbulence*, J. Fluid Mech., Vol. 219, pp. 195-214

Mydlarski, L., Warhaft, Z. (1996): *On the onset of high – Reynolds – number grid – generated wind tunnel turbulence*, J. Fluid Mech., Vol. 320, pp. 331-368

Roach, P. E. (1986): *The generation of nearly isotropic turbulence by means of grids*, J. Heat and Fluid Flow, Vol. 8, No. 2, pp. 82-92

Valente, P. C., Vassilicos, J. C. (2011): *The decay of turbulence generated by a class of multiscale grids*, J. Fluid Mech., Vol. 687, pp. 300-340