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EXPERIMENTAL VERIFICATION OF CRITICAL FLOW MODEL OF DENSE GASES

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Abstract

The critical mass flow of dense gases strongly depends on real gas effects. In the present work detailed assessment of the critical flow conditions and the limiting mass velocity in the flow of dense gases have been experimentally proved. The computational study was provided by implementation of theoretical model (Górski, 1997) for one dimensional (1D) and non-linear gas dynamic problems. The values of critical flow function C^* for selected refrigerants were predicted based on *Martin–Hou* equation of state. Appropriate sonic flow conditions have been executed in the pressurized closed loop system by using ISO 9300 critical Venturi nozzle CVN). Measurements of critical mass flow for dense superheated vapor of *R-410A* and *R-507A* carried out on laboratory test stand confirmed the accuracy of proposed model and its physical significance. A main goal of the investigations is a set of charts $C^*(T_0, p_0)$ and tables developed for an assumed range of stagnation temperature T_0 and pressure p_0 in the upstream fluid-flow.

INTRODUCTION

Real gas phenomena occurring in the flow of gases and vapors are still an opened and difficult question (Emmons, 1958). The good backgrounds for an analysis of real dense gas effects in the flow were proposed by Vulis (1950). Later, it was proved by Thompson (1977) that in many cases, apart of discrepancy between experimental and analytical results, can appear some *retrograde phenomena* in the flow. Such phenomena can be observed in the flow of large molecular mass fluids as fluorocarbons and higher order hydrocarbons. Other certain problems are involved during calibration of Venturi tubes and orifices for high-pressure gas flow measurements. Solutions utilizing low-grade or waste energy sources are being introduced more and more commonly in the energy conversion systems. A significant effort of current research is focused on the internal flow modeling and improvement of turbomachinery blading for special applications, such ORC and small CHP units.

Main group of low-boiling temperature agents are inorganic refrigerants as carbon dioxide or blends as *R-410A*, *R507*, and whole range of environmentally harmful replacements for chloro-fluorocarbons. Determination of its physical properties involves an application of multi-parameter EOS's and special computer routines in calculation of necessary parameters. In an engineering practice the most popular and well known is *Martin-Hou* (MH) and modified *Benedict-Webb-Rubin* (MBWR) EOS. Its correctness is confirmed by many authors and can be applied up to high-density region of one component fluid as well as the solutions and saturated region analysis.

Prediction of the mass flow rate and critical flow function is of practical importance since the mass flow rate is essentially associated with the limiting working gas consumption. Critical pressure ratio should be known to establish the operating conditions for safety valves and expansion units. Problem of calculation the critical flow („sonic flow") of dense gas or vapor was treated previously by many authors (Thompson at all., 1977, Leung at all., 1988, Bober at all., 1977). The first part of this work is dedicated to the theoretical prediction of sonic flow conditions and critical flow function at the 1D flow of real gases. The next part consist some experimental tests providing the practical verification and applicability range of the proposed theoretical model.

CRITICAL ISENTROPIC FLOW

At the sonic flow conditions the flow velocity equals to sound speed $w=a_*$, and the static parameters correspond to the critical ones ($T=T^*$, $p=p^*$, $h=h^*$, $a=a^*$). All complete relations in a compressible critical flow of the perfect gases can be found anywhere. At the case of dense gas flow well-known relations presented in the classical books on gas dynamics are in general invalid. It is caused by strong variation of all physical constants such the isentropic index, specific heats and other thermal and caloric properties involving the calculation of the process. In order to improve the existing analytical tools a new approach for calculation of thermodynamic property and process in dense gases has been proposed (Gorski, 1997). An original method of *Virial Compressibility Derivatives* (VCD), gave in this area more simple and directly related results. By using the VCD formalism it is possible to find the basic relations between parameters in the isentropic flow at the form corresponding to an ideal gas model (Gorski, 1997). The obtained general, but only approximate relations between critical (chocked flow) and stagnation parameters are

$$\frac{T^*}{T^0} \cong \left(\frac{2}{\chi+1}\right)^{\frac{\Phi}{\chi-1}}, \quad \frac{p^*}{p^0} \cong \left(\frac{2}{\chi+1}\right)^{\frac{k}{\chi-1}}, \quad \frac{\rho^*}{\rho^0} \cong \left(\frac{2}{\chi+1}\right)^{\frac{1}{\chi-1}} \tag{1}$$

where

$$\Phi = \frac{\rho}{T}\left(\frac{\partial T}{\partial \rho}\right)_s = \frac{z_T}{\bar{c}_v}, \quad k = \frac{\rho}{p}\left(\frac{\partial p}{\partial \rho}\right)_s = \gamma\frac{z_v}{z}, \quad \chi = k + \frac{\rho}{k}\left(\frac{\partial k}{\partial \rho}\right)_s = 2\Gamma - 1 \tag{2}$$

and

$$z=\frac{pv}{\mathrm{R}T}, \quad z_v = z - v\left(\frac{\partial z}{\partial v}\right)_T = z + \rho\left(\frac{\partial z}{\partial \rho}\right)_T, \quad z_T = z + T\left(\frac{\partial z}{\partial T}\right)_v, \quad \gamma = \frac{c_p}{c_v}, \quad \bar{c}_v = \frac{c_v}{\mathrm{R}} \tag{3}$$

In the Equations (1) to (3) the *Grüneisen* parameter Φ and generalized isentropic index χ (or fundamental derivative Γ) are introduced as well as the *Poisson* ratio γ, isentropic exponent k and two Virial Compressibility Derivatives z_T and z_v. These parameters should be found from an equation of state which in the typical case is given as a function of temperature and specific volume $p = p(T, v)$, as for example MH EOS. It is important to note, that in an elementary case of critical perfect gas flow all conditions correspond to the typical relations presented at any textbook on fluid mechanics. An alternative approach to the calculation of real gas isentropic flow in terms of *Mach Number* is presented in the Appendix.

When the critical nozzles are used for measuring the mass flow rate, it is usually assumed that the flow of the gas from plenum to the throat of the nozzle is 1D and isentropic, and the assumption is made that the gas is perfect (Johnson, 1971). The dimensionless parameter which characterizes 1D isentropic flow between inlet and throat section of a Venturi nozzle is the critical flow function *C**. In the real gas flow it can be expressed as (Gorski, 1997)

$$C^{*} = \frac{\dot{m}^{*}\sqrt{RT_{0}}}{A^{*}p_{0}} \cong \sqrt{\frac{k}{z_{0}}}\left(\frac{2}{\chi+1}\right)^{n}, \quad \text{where} \quad n = \frac{\chi+1}{2(\chi-1)} \tag{4}$$

All parameters in the Equation (4) are functions the temperature and pressure and refer both to stagnation (index “o”) and the critical (index “*”) states. This equation has an analogous form to the well-known relation appearing in any textbook of classical gas dynamics. A calculation of a process leading to resolve both connected stagnation and critical characteristic parameters of the flow at critical conditions needs an iterative procedure. It can be simplified when the initially assumed values for a given stagnation state are made. After few steps, calculation results converge to the mean values of process become „constants” in allowed range of error. These approach confirmed simple PC procedures. The complete analysis needs an iterative solution of the energy equation along an isentrope for the sonic flow conditions (*Ma* =1)

$$h_{0} = h + \frac{w^{2}}{2} = h^{*} + \frac{a_{*}^{2}}{2} = \text{idem.} \tag{5}$$

The adequate graphical interpretation of the equation (5) is presented in the Fig. 1.

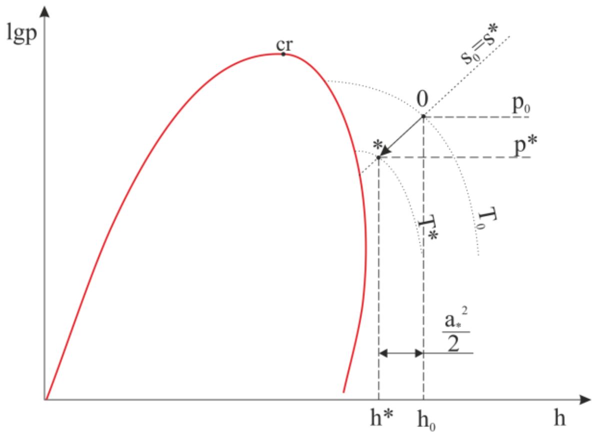

Fig. 1. Calculation an isentropic critical flow of dense vapor

For selected technical gases the value of *C** has been calculated and tabulated (see, ISO-9300). It is evident that in practical calculation of critical parameters at the sonic flow of refrigerants and hydrocarbons appears a great discrepancy between ideal gas model analysis and real gas simulation. Additionally, at high pressure refrigerant flow there are no precarious information about real gas behavior in range of thermal parameters. Main assumption make in that circumstances are linear change of critical flow function *C** in many cases independent from pressure, instead of nonlinear changes as takes place in reality with strong connection to the pressure. Example calculation results for refrigerant mixtures confirm a great discrepancy between ideal gas values and more realistic data derived from thermal EOS’s. According to our recent experimental tests the critical flow function in the refrigerants flow does change over 30% in a comparison to ideal gas flow where the sonic conditions depends only on

Poisson constants for a particular gas. In order to prove the robustness and quality of the proposed method for prediction the sonic flow conditions and calculation the critical flow function in the flow of dense gases or vapors, a special laboratory tests have been carried out.

EXPERIMENTAL TESTS

A main goal of this work was an experimental verification of theoretical results and to compare data from laboratory tests to the computational ones. Rabczak (2007) developed an original test-bench in order to attain the sonic conditions in a closed refrigeration circuit. This facility is operating based on the *Critical Venturi Nozzles* (CVN). Superheated vapors of refrigerants *R-410A* and *R-507A* were used as working media. Based on the computational study the selection of two ISO-9300 Venturi nozzles, with the throat diameter d = 0.8 (only for *R- 507A*) and 1.0 mm (both refrigerants) were made. The critical nozzle of 0.8 mm diameter was used for the upper range of stagnation pressures p_0 > 1.5 MPa. At the smaller pressures, a 1.0 mm throat nozzle was taken in order to preserve the heat capacity of the continuous-flow calorimeter. A steady-state flow calorimeter method was applied for selective measuring of the thermal and flow parameters of refrigerant blends. This calorimetric method ensures precise and stable measurement conditions in the main circuit.

The principal schema of the research meter circuit and instrumentation is shown in the Figure 2.

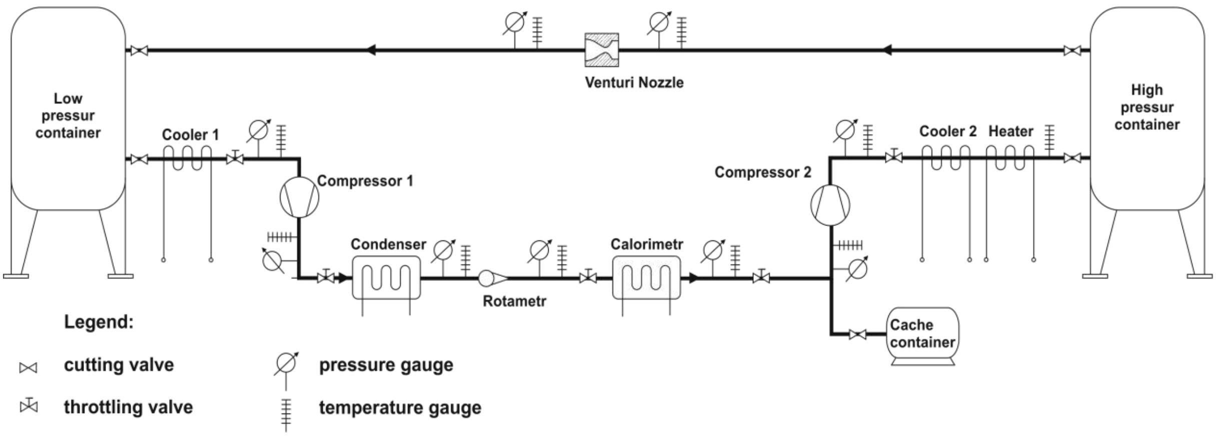

Fig. 2. Schema of the experimental loop at COCh Laboratory (Rabczak, 2007).

Figure 3 presents a comparison of theoretical computation results of critical flow function and the experimental data. The practical range of operating conditions in the upstream flow was limited to p_0 = 35 bar and T_0 =110^{o}C, and superheated vapor area to avoid two-phase phenomena. The critical nozzle of 0,8 mm diameter throat was used for higher range of stagnation pressures. At the stagnation pressure less than 15 bars, the nozzle of 1,0 mm throat was selected to preserve thermal capacity limitations of the flow-calorimeter.

RESULTS AND DISCUSSION

Obtained experimental data are with closely agreement with computational based on the MH equation of state. The average discrepancy between theoretical and experimental results is less then 1,0%. It confirms that the proposed theoretical model is valid and formulated in accordance to the real physics. Experimental data are with closely agreement with selected theoretical model based on MH EOS for refrigerants mixtures, especially in high density region. In region of relatively low pressure experimental results are in better correlation to ideal gas behavior than to general real gas model.

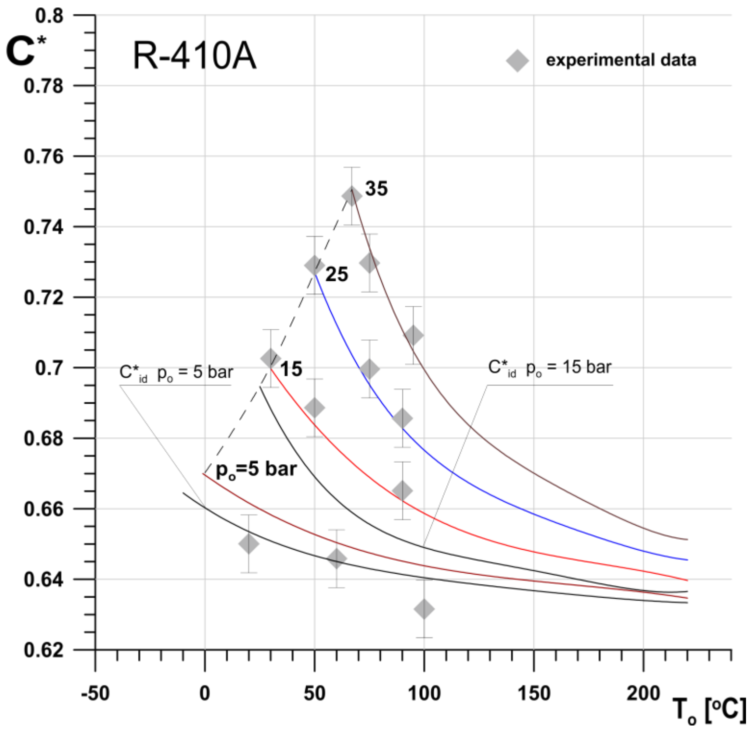

Fig. 3. Critical flow function for *R-410A* - theoretical and experimental data (Rabczak, 2007).

If one looks on the relation between experimental data and obtained from the theoretical model of critical gas flow (Figure 4), it shows that only a few results are outside of the confidence interval. These points determine the critical flow conditions at the low stagnation pressure, below 0.5 bar and are corresponding to an ideal gas behavior.

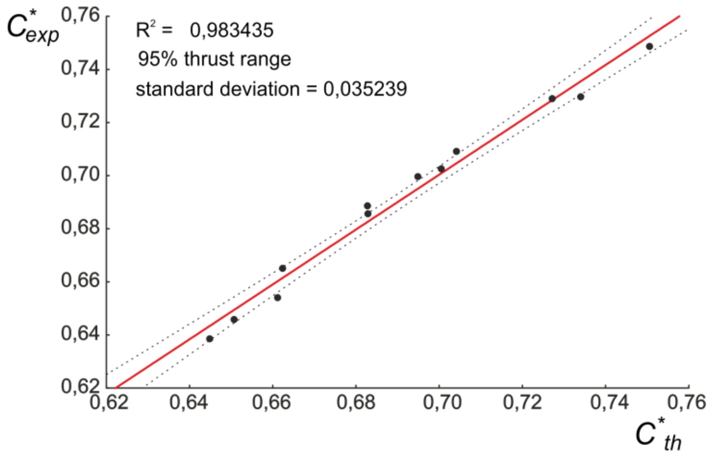

Fig. 4. Theoretical and experimental critical flow factor *C** for *R-410A* (Rabczak, 2007).

Below are two plots of the critical flow factor *C** for *R-744* (CO2) and *R-134a*, calculated from the thermodynamic data based on MH EOS, see Figure 5.

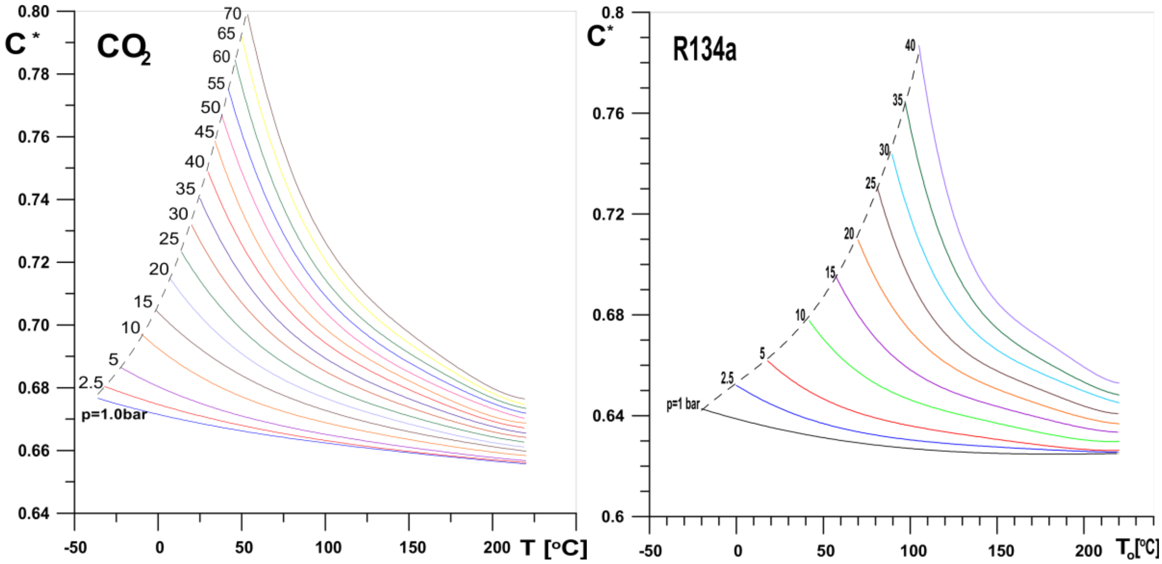

Fig. 5. Critical flow function C^* for *R-744* and *R-134a* (Rabczak, 2007).

The first look on real gas behavior of *R-744* and *R-134a*, according to expectation results from the previous experimental validation of the theoretical model, can be compared based on the molecular shape parameter (*accentric factor* ω). Refrigerants having a similar value of this parameter should also demonstrate similar behavior at the isentropic flow, see Figure 6. Therefore, based on the thermodynamic similarity conditions, the critical flow function can be predicted for other refrigerants (Cornelius & Srinivas, 2004).

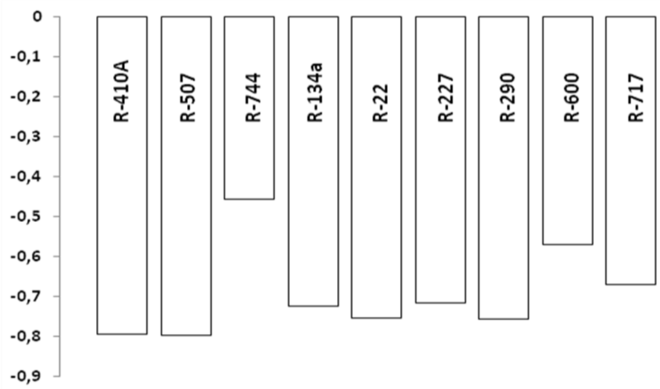

Fig. 6. Range of accentric factor ω for refrigerants.

Refrigerant blends *R-410A* and *R-507* belong to the same group of refrigerants, and if the shape of molecule is considered, the critical flow behavior one of them can be predicted based on the other data. The same relation is among another group of refrigerants represented in the Figure 6. The main group consist for example *R-134a, R-22, R-227, R-290*, but for other, natural components (*R-744*, *R-717* and *R-600*), no such direct relation takes place.

CONCLUSIONS

From experimental data analysis and comparison to critical flow model for dense gas phenomena in the flow of refrigerants can be make some final results summarized below:

1. The critical flow function C^* for dense refrigerant vapor strongly depends on the upstream stagnation conditions. Analytical solutions involve numerical procedures and application of high-quality data based on the advanced methods of applied thermodynamics.

2. The computation results for *R-410A* and *R-507A* are in a good agreement with experimental data. At stagnation pressure exceeding $p_0 > 0.5$ MPa, the perfect gas model is not applicable in the flow of dense and superheated vapor of refrigerants.
3. The future work will aim to develop more universal correlations for determining the critical flow conditions and function C^* for a group of similar fluids. It will be done based on the correlations of generalized corresponding states and extended similarity principles.
4. A natural continuation of this work would be to extend to the examination of the chocking condition into the two-phase domain and two-phase flow.

NOMENCLATURE

A - area [m^2]
a - speed of sound [m/s]
C^* - critical flow function [-]
c_p - specific heat (p=*idem*) [J/kg K]
c_v - specific heat (v=*idem*) [J/kg K]
h - specific enthalpy [kJ/kg K]
k - isentropic exponent [-]
$\dot{m}$ - mass flow [kg/s]
p - pressure [Pa]
R - gas constants [J/kg K]
T - temperature [K]
w - mean velocity of flow [m/s]
z - compressibility factor [-]
z_T, z_v - VCD's, equation (5) [-]
Γ - fundamental derivative [-]
γ - Poisson constant [-]
ρ - density [kg/m^3]
χ - generalized isentropic index [-]
Φ - Grüneisen parameter [-]
ω – accentic factor [-]

Indices
s - isentropic
T - isothermal
v - isochoric
o - stagnation state
$*$ - critical flow ($Ma = 1$)

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A p p e n d i x

Example of calculation the isentropic flow of dense gas (CO2)

a) An iterative solution based on MBWRS equation of state

Initial stagnation state conditions (01):

p_{01} = 7.0 MPa, T_{01} = 360 K, $\rightarrow \rho_{01}$ = 128.61 kg/m^3, h_{01} = 3.670 kJ/kg, s_{01} = - 0.73931 kJ/kgK, c_{v01} = 0.80619 kJ/kgK, $z_{01} = (pv/RT)_{01}$ = 0.80028;
- auxiliary stagnation parameters:

z_{v01} = 0.63031, z_{T01} = 1.3161, $c_{p01} = (c_v + Rz_T^2/z_v)_{01}$ = 1.3252 kJ/kgK, $\gamma_{01} = (c_p/c_v)_{01}$ = 1.6441,

$k_{s01} = (\gamma\, z_v/z)_{01}$ = 1.2949, $a_{s01} = (k_s\, p/\rho)_{01}^{1/2}$ = 265.49 m/s, $\Phi_{01} = (Rz_T/c_v)_{01}$ = 0.30844.

Initial static state conditions (1):

p_1 = 2.0 MPa, T_1 = 265 K, $\rightarrow \rho_1$ = 48.130 kg/m^3, h_1 = -56.031 kJ/kg, s_1 = - 0.73931 kJ/kgK,

c_{v1} = 0.73232 kJ/kgK, R = 188.92 J/kgK, $z_1 = (pv/RT)_1$ = 0.83003;
- auxiliary static parameters:

z_{v1} = 0.66602, z_{T1} = 1.2345, $c_{p1} = (c_v + Rz_T^2/z_v)_1$ = 1.1646 kJ/kgK, $\gamma_1 = (c_p/c_v)_1$ = 1.5903,

$k_{s1} = (\gamma\, z_v/z)_1$ = 1.2761, $a_{s1} = (k_s\, p/\rho)_1^{1/2}$ = 230.27 m/s, $\Phi_1 = (Rz_T/c_v)_1$ = 0.31847.

Characteristic flow parameters:
- Enthalpy change: $\Delta h_s = h_{01} - h_1$ = 59.801 kJ/kgK,
- Flow velocity: $w_1 = \Delta h_s^{\,½}$ = 345.84 m/s,
- Mach Number: $Ma_1 = w_1/a_{s1}$ = 1.5019.

Approximate stagnation parameters – calculation results:

From the approximate relations in an isentropic gas flow (Gorski, 1997):

$$\frac{T_{01}}{T_1} \cong 1 + \Phi_1 Ma_1^2/2 = 1.3584 \quad \rightarrow \quad T_{01} = 360.0\,\mathrm{K},$$

$$m_T = \left[\frac{z_T}{z_v}\left(\frac{\gamma}{\gamma-1}\right)\right]_1 = 4.0069 \quad \rightarrow \quad \frac{p_{01}}{p_1} \cong \left(\frac{T_{01}}{T_1}\right)^{m_T} = 3.4122 \quad \rightarrow \quad p_{01} = 6.824\,\mathrm{MPa},$$

$$\frac{\rho_{01}}{\rho_1} \cong \left(\frac{T_{01}}{T_1}\right)^{1/\Phi_1} = 2.6467 \quad \rightarrow \quad \rho_{01} = 127.39\ \mathrm{kg/m^3}.$$

b) Critical flow of CO2 (sonic conditions):

The approximate relations in the critical dense gas flow (Gorski, 1997):

$$\frac{T_{01}}{T_*} \cong 1 + \frac{\Phi_1}{2} = 1.1592 \quad \rightarrow \quad T_* = 307.2\,\mathrm{K}, \qquad \frac{\rho_{01}}{\rho_*} \cong \left(\frac{T_{01}}{T_*}\right)^{1/\Phi_1} = 1.5902 \quad \rightarrow \quad \rho_{01} = 80.11\,\mathrm{kg/m^3},$$

$$\frac{p_{01}}{p_*} \cong 1.8075 \quad \rightarrow \quad p_* = 3.87\,\mathrm{MPa}, \quad \rho_* w_* = 19090\,\mathrm{kg/m^2s}, \quad p_* + \rho_* w_*^2 = 8.80\,\mathrm{MPa},$$

thus $w_* = a_{s*} = 248.28\,\mathrm{m/s}$ and $Ma = 1$.

Comments: *Indices are corresponding to each particular thermodynamic state along an isentrope during 1D expansion of the carbon dioxide.*