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SKEW SYMMETRIC, CONSERVATIVE FINITE DIFFERENCE SCHEMES

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Skew symmetric finite difference schemes for compressible and incompressible flows are subject of academic research since they where proposed in the early eighties (Feiereisen, et al., 1981; Tadmor, 1984). They allow to strictly avoid numerical damping. They however find little application in simulations of engineering practice. This might be partly due to the different, uncommon concepts compared to finite volume (FV) schemes, due to the sometimes complex implementation, but also due to the lack of easy to use concepts for handling central issues like transformed grids, time integration, boundary conditions and block-block interfaces. We deal with these issues and present our approach for the problems mentioned above. This aims for academic DNS and LES calculations and might become interesting for high quality engineering computations in the near future.

The skew symmetric discretization builds on rewriting the Navier-Stokes or Euler equations

$$\partial_t \rho + \partial_x(\rho u) = 0 \tag{1}$$

$$\frac{1}{2}\left(\partial_t \rho \cdot\ + \rho \partial_t \cdot\ \right) u + \frac{1}{2}\left(\partial_x u\rho \cdot\ + u\rho\partial_x \cdot\right) u + \partial_x p = 0 \tag{2}$$

$$\frac{1}{\gamma - 1}\partial_t p + \frac{\gamma}{\gamma-1}\partial_x\left(up\right) - u\partial_x p = 0. \tag{3}$$

in the skew symmetric form, in one dimension given by

Here ρ, p, γ are density, pressure, the adiabatic index and u_i (i =1,2,3) are the velocity components. The dots in the momentum equation (2) mark, that the derivatives act also on the u right of the parenthesis. The unusual time operator in the momentum equation is a consequence of the skew symmetric rewriting of the equation.

When using skew symmetric derivative matrices it can be shown that the nonlinear transport conserves the kinetic energy in the discrete case, as in the analytical theory. This is in contrast to the majority of numerical schemes with the exception of spectral methodes. Further the equations also conserve mass, total momentum and total energy, even though equations (2) and (3) are clearly not in divergence form, i.e. are not equivalent to FV. We present an easy approach to keep these two central properties on structured, but *arbitrarily distorted* grids in two and three dimensions.

The time has to be discretized carefully to keep the exact conservation. One way to do it is due to Morinishi (Morinishi, 1987) by rewriting it in (2) as

This $$\frac{1}{2}\left(\partial_t \rho \cdot + \rho \partial_t \cdot\right) u = \sqrt{\rho}\partial_t(\rho u). \quad (4)$$

can be discretized in time by an adopted, implicit mid-point rule, which gives a time integration scheme similar to one obtained by (Subbareddy, 2009), being fully conservative for finite time steps. Other time integration schemes developed by us, fully and partly conservative will be compared.

Boundary conditions can be introduced in different ways in finite difference (FD) schemes. It has two, often connected aspects. One is *which* values are to be set at the boundary, which is for the Euler case often done by characteristics, the second is *how* to set the values, e.g. use interpolation schemes, half sided derivative stencils or ghost points. Sometimes, but not always, these different approaches can be shown to be equivalent. We use one sided stencils for the numerical boundary conditions and set the remaining values direct or by analyzing the characteristics. We discuss stability issues and suggest a scheme.

Closely connected to boundary conditions is the block-block interface, which will be covered in a short outlook.

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