STOCHASTIC MODEL UPDATING BASED ON AN EFFICIENT NOVEL METHOD

Qiang Rui[*,**a)], Hongyan Wang[**,*] & Huajiang Ouyang [***]
[*] *Department of Mechanical Engineering, Academy of Armored Force Engineering, Beijing 100072, China*
[**] *School of Mechanical and Vehicle Engineering, Beijing Institute of Technology, Beijing 100081, China*
[***] *Department of Engineering, University of Liverpool, Brownlow Hill, Liverpool L69 3GH, UK*

Summary In this paper, a computationally efficient approach for stochastic model updating is developed based on the combination of Stochastic Response Surface Method (SRSM) and Monte Carlo inverse error propagation. The SRSM model is determined using the Hermite polynomial chaos expansion and regression-based efficient collocation method. A three degree-of-freedom numerical model is employed to illustrate the implementation of the method.

INTRODUCTION

In the past 30 years, model updating methods have been widely used for improving theoretical models for practical structures. To consider the variability that exists not only from measurement noise, but also from the variation of structural parameters, stochastic model updating has become a popular topic. As far as the authors are concerned, Monte Carlo simulation (MCS)-based approach [1, 2] and perturbation approach [3, 4] are the two kinds of most commonly employed uncertainty inverse propagation methods in stochastic model updating. Unfortunately, both of these two methods have their drawbacks, the former depends on a large number of stochastic numerical simulation experiments for the estimation of probability distribution of output parameters, the latter limited to small levels of uncertainties and sensitive to the original estimated parameter values. Therefore, these two method are not applicable for the model updating of complex structure of industrial scale.
In this paper, a computationally efficient approach to stochastic model updating is presented. The framework is based on the combination of the SRSM and the MCS-based inverse error propagation method in stochastic model updating for the first time. A model updating procedure including parameter variability is implemented on the determined stochastic surrogate model with the application of polynomial chaos expansions and efficient regression-based collocation method. A three degree-of-freedom numerical model is used to verify the effectiveness of the presented method.

A NOVEL STOCHASTIC MODEL UPDATING METHOD

Stochastic model updating

In stochastic model updating, the adjustment of parameters includes two parts, the updating of parameter means and the updating of parameter covariances. In order to minimise residual containing the difference between many sets of analytical and experimental data, the adjustment of parameter mean values at the j-th iteration can be expressed as

$$\Delta\bar{\theta}_j = \left[\bar{\mathbf{S}}_j^{\mathrm{T}}\bar{\mathbf{W}}_\varepsilon\bar{\mathbf{S}}_j + \lambda^2\bar{\mathbf{W}}_\theta\right]\bar{\mathbf{S}}_j^{\mathrm{T}}\bar{\mathbf{W}}_\varepsilon\bar{\mathbf{r}}_j \tag{1}$$

where $\bar{\theta}$ is the mean value vector of updating parameters. $\bar{\mathbf{W}}_\varepsilon$ and $\bar{\mathbf{W}}_\theta$ are weighting matrices corresponding to mean modal parameter estimation error and mean value parameter error respectively. $\bar{\mathbf{S}}_j = \bar{\mathbf{W}}_\varepsilon^{1/2}(\partial\mathbf{z}(\bar{\theta})_j/\partial\bar{\theta})_j$ represents the sensitivity matrix of parameter mean values at iteration step j .
Similarly, using the definition of Froenius norm, the adjustment of the parameter covariance matrix can be expressed by

$$\mathrm{Cov}(\theta_{j+1},\,\theta_{j+1}) = \mathrm{Cov}(\theta_j,\,\theta_j) + \mathbf{T}_j\Delta\mathrm{Cov}(\mathbf{z},\,\mathbf{z})_j\mathbf{T}_j^{\mathrm{T}} \tag{2}$$

where $\mathbf{T}_j = \left[\bar{\mathbf{S}}_j^{\mathrm{T}}\mathbf{W}_\Sigma\bar{\mathbf{S}}_j\right]^{-1}\bar{\mathbf{S}}_j\mathbf{W}_\Sigma$ is the transformation matrix at the j-th iteration step, $\mathbf{W}_\Sigma = \mathbf{W}_{\mathrm{Cov}}^{\mathrm{T}}\mathbf{W}_{\mathrm{Cov}}$, $\Delta\mathrm{Cov}(\mathbf{z},\,\mathbf{z})_j = \mathrm{Cov}(\mathbf{z}_m,\,\mathbf{z}_m) - \mathrm{Cov}(\mathbf{z}(\theta)_j,\,\mathbf{z}(\theta)_j)$.

Stochastic Response Surface Method

The SRSM provides a kind of computationally efficient statistical surrogate model for uncertainty forward and inverse propagation. Following Isukapalli [5], the application of the SRSM consists of the following steps: (1) Represent the uncertain inputs parameters; (2) Approximate the physical model outputs; (3) Estimate the unknown polynomial chaos expansions coefficients; (4) Estimate the output statistics and model verification.
Using the form of polynomial chaos expansions (PCE), the SRSM is used to develop an approximate surrogate reduced model for outputs. As normal standard random variables ($SRVs$) are used , outputs can be approximated by a PCE on the set of random input variables $\{\xi_i\}_{i=1}^n$, given by

$$z = a_0 + \sum_{i_1=1}^{n} a_{i_1}\Gamma_1(\xi_{i_1}) + \sum_{i_1=1}^{n}\sum_{i_2=1}^{i_1} a_{i_1}a_{i_2}\Gamma_2(\xi_{i_1},\,\xi_{i_2}) + \sum_{i_1=1}^{n}\sum_{i_2=1}^{i_1}\sum_{i_3=1}^{i_2} a_{i_1}a_{i_2}a_{i_3}\Gamma_3\left(\xi_{i_1},\,\xi_{i_2},\,\xi_{i_3}\right) + \cdots \tag{3}$$

[a)] Corresponding author. E-mail: ruiqiang2006@163.com.

where z is random output of the model, the a_0, a_{i_1}, a_{i_2}, $\cdots$, are deterministic coefficients to be estimated, and the $\Gamma_p(\xi_{i_1}, \xi_{i_2}, \cdots, \xi_{i_p})$ are Hermite polynomials of degree p , given by

$$\Gamma_p\left(\xi_{i_1}, \xi_{i_2}, \cdots, \xi_{i_p}\right) = (-1)^p \mathrm{e}^{\frac{1}{2}\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{\xi}} \frac{\partial^p}{\partial\xi_{i_1}\partial\xi_{i_2}\cdots\partial\xi_{i_p}} \mathrm{e}^{-\frac{1}{2}\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{\xi}} \tag{4}$$

$\boldsymbol{\xi}$ is the vector of p normal random variables $\{\xi_{i_k}\}_{k=1}^{p}$, representing random input variables.

NUMERICAL EXAMPLE

For illustrative purpose, the proposed procedure is implemented in a three degree-of-freedom mass-spring system as show in Figure 1. In this example, the assumed mean values for three random variables are, $\bar{\theta}_1 = \bar{\theta}_2 = \bar{\theta}_5 = 2.0\,\mathrm{N/m}$, and standard deviations, $\sigma_{\theta_1} = \sigma_{\theta_2} = \sigma_{\theta_5} = 0.3\,\mathrm{N/m}$. The corresponding 'true' mean values and standard deviations are $\theta_1 = \theta_2 = \theta_5 = 1.0\,\mathrm{N/m}$ and $\sigma_{\theta_1} = \sigma_{\theta_2} = \sigma_{\theta_5} = 0.2\,\mathrm{N/m}$, respectively. Consequently, the relative error is 100% for mean values and 50% for standard deviations.

Figure 2 shows the convergence of frequency clouds and confidence ellipses of estimated initial values to the updated values of parameters using the 2$^{\mathrm{nd}}$-order SRSM method. The ellipses shown are one, two and three-sigma confidence ellipses, respectively. It is obvious that, after 10 iterations the confidence ellipses in Figure 2 (b) overlay quite closely with each other. The small deviation between the statistical properties of simulated system outputs and assumed experimental data indicates good convergence of model parameters to the 'real' system parameters.

The convergence comparison of mean values and standard deviation of parameters for the 2$^{\mathrm{nd}}$-order SRSM is presented in Figure 3 . It is obvious that the mean values of parameters can converge much closer to the 'real' values than the standard deviations of parameters.

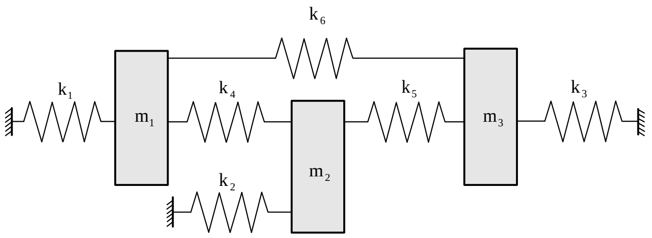
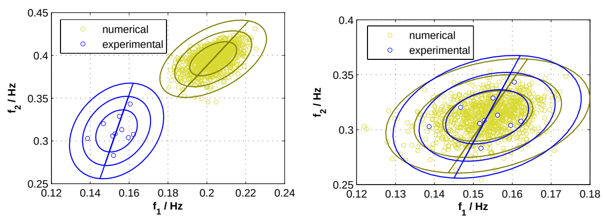


Figure 1. Three degree-of freedom system model



(a) Initial stage

(b) Iteration step 10

Figure 2. Initial and updated frequency clouds

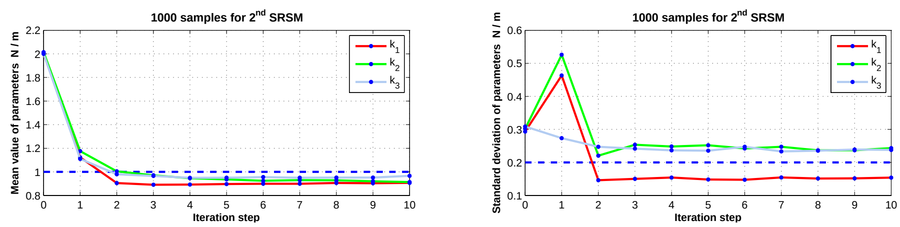


Figure 3. Convergence of means and standard deviations of updating parameters

CONCLUSIONS

An efficient stochastic model updating algorithm has been presented in this paper. Using a three-degree-of-freedom system as numerical example, a computationally efficient solution is obtained.

References

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