A MULTISCALE APPROACH BASED ON THE PGD TO SOLVE NONLINEAR TIME-DEPENDENT PROBLEMS

D. Néron, P. Ladevèze, M. Cremonesi, P.A. Guidault
LMT-Cachan (ENS Cachan/CNRS/UPMC/PRES UniverSud Paris)
61, avenue du Président Wilson, F-94235 Cachan Cedex, France

Summary This work concerns the Proper Generalized Decomposition (PGD) which is used to solve problems defined over the time-space domain and which are possibly nonlinear. The problematics which is addressed herein is the use of the PGD in a time-space multiscale strategy developed in the framework of the LATIN method. This strategy is a mixed, multilevel domain decomposition method which leads to resolutions both on the refined scale (the microscale) and on the coarse scale (the macroscale) with respect to time as well as space. In previous works, the PGD technique has been applied to approximate the fields defined at the microscale only, allowing to reduce significantly the computational and the storage costs of the strategy. The purpose of this work is to extend the PGD approximation to the macroscale in order to decrease the CPU cost of the homogenized operator introduced to accelerate the convergence of the iterative algorithm.

INTRODUCTION

Numerical simulation has been playing an increasingly important role in science and engineering. However, when dealing with high-fidelity models, the number of degrees of freedom can lead to systems so large that direct techniques are inapplicable. Model reduction techniques constitute an efficient way to circumvent this difficulty by seeking the solution of a problem in a reduced-order basis (ROB), whose dimension is much lower than the original vector space. A posteriori methods usually consist in defining this ROB by the decomposition of the solution of a surrogate model relevant to the initial model (see e.g. [1, 2, 3, 4, 5]). A priori methods follow a different path by building progressively an approximate separated representation of the solution, without assuming any basis (see e.g. [6, 7, 8, 9, 10]).

This work focusses on the Proper Generalized Decomposition (PGD) which belongs to the second family and is used herein to solve problems defined over the time-space domain and which are possibly nonlinear. For solving such problems, the PGD, originally introduced as the "radial loading approximation" in [11], consists in seeking a separated time-space representation of the unknowns and the iterative LATIN method is used to generate the approximation by successive enrichments [12]. At a particular iteration, the ROB which has been already formed is first used to compute a reduced-order model (ROM) and find a new approximation of the solution. If the quality of this approximation is not sufficient, the ROB is enriched by determining a new functional product using a greedy algorithm. The PGD has been applied for solving many types of problems in the context of the LATIN method and allowed to decrease the CPU cost drastically.

A MULTISCALE APPROACH

A multiscale computational strategy involving homogenization both in time and in space has been presented in [13] and enhanced in [14] to solve efficiently structural mechanics problems. This strategy is a mixed, multilevel domain decomposition method which leads to resolutions both on the refined scale (the microscale) and on the coarse scale (the macroscale) with respect to time as well as space. The problem on the microscale is solved using the iterative LATIN solver while the purpose of the macroresolutions is to accelerate the convergence of this iterative algorithm. In [13, 14], the PGD technique has been used to approximate the fields defined at the microscale which allowed to reduce significantly the computational and the storage costs of the strategy.

EXTENSION OF THE MODEL REDUCTION TECHNIQUES TO THE MACROPROBLEM

The purpose of this work is to extend the PGD approximation to the macroscale. The macroresolutions are defined through the construction of a homogenized operator which maps the macropart of the displacement fields to the macropart of the forces at the interfaces. This operator depends on the time and the space variables but also on the macroquantities. As it changes at each iteration of the LATIN algorithm, it construction can be CPU expensive, as well as the resolution of the macroproblem. Then, we propose herein to apply the PGD technique to the construction of the homogenized operator by considering as coordinates, not only the space and the time variables, but also the macroquantities.

First numerical examples show that introducing this new approximation does not affect the convergence of the strategy but allows a significant decrease of the computational cost. The extension to more general problems is under investigation.

References

[1] Y. Maday and E. M. Ronquist. The reduced-basis element method: application to a thermal fin problem. *SIAM Journal on Scientific Computing*, 26(1):240–258, 2004.

[a] Corresponding author. E-mail: neron@lmt.ens-cachan.fr.

[2] K Kunisch and L Xie. Pod-based feedback control of the burgers equation by solving the evolutionary hjb equation. *Computers & Mathematics With Applications*, 49(7-8):1113–1126, 2005.

[3] T. Lieu, C. Farhat, and A. Lesoinne. Reduced-order fluid/structure modeling of a complete aircraft configuration. *Computer Methods In Applied Mechanics and Engineering*, 195(41-43):5730–5742, 2006.

[4] Max D. Gunzburger, Janet S. Peterson, and John N. Shadid. Reduced-order modeling of time-dependent pdes with multiple parameters in the boundary data. *Computer Methods In Applied Mechanics and Engineering*, 196(4-6):1030–1047, 2007.

[5] M. Bergmann and L. Cordier. Optimal control of the cylinder wake in the laminar regime by trust-region methods and pod reduced-order models. *Journal of Computational Physics*, 227(16):7813 – 7840, 2008.

[6] J. Ladevèze. Algorithmes adaptés aux calculs vectoriels et parallèles pour des méthodes de décomposition de domaines. In *Actes du troisième colloque Tendances Actuelles en Calcul de Structures*, pages 893–907. Pluralis, 1985.

[7] A. Ammar, B. Mokdad, F. Chinesta, and R. Keunings. A new family of solvers for some classes of multidimensional partial differential equations encountered in kinetic theory modeling of complex fluids: Part II: Transient simulation using space-time separated representations. *Journal of Non-Newtonian Fluid Mechanics*, 144(2-3):98–121, 2007.

[8] Anthony Nouy. Recent developments in spectral stochastic methods for the numerical solution of stochastic partial differential equations. *Archives of Computational Methods In Engineering*, 16(3):251–285, 2009.

[9] P. Ladevèze, D. Néron, and J.-C. Passieux. On multiscale computational mechanics with time-space homogenization. In *Multiscale methods—Bridging the scales in Science and Engineering*, chapter Space Time Scale Bridging methods, pages 247–282. Oxford University Press. Ed. J. Fish, 2009.

[10] F. Chinesta, P. Ladevèze, and E. Cueto. A short review on model order reduction based on proper generalized decomposition. *Archives of Computational Methods in Engineering*, 18(395-404), 2011.

[11] P. Ladevèze. New algorithms: mechanical framework and development (in french). Technical Report 57, LMT-Cachan, 1985.

[12] P. Ladevèze. *Nonlinear Computationnal Structural Mechanics—New Approaches and Non-Incremental Methods of Calculation*. Springer Verlag, 1999.

[13] P. Ladevèze and A. Nouy. On a multiscale computational strategy with time and space homogenization for structural mechanics. *Computer Methods in Applied Mechanics and Engineering*, 192:3061–3087, 2003.

[14] P. Ladevèze, J.-C. Passieux, and D. Néron. The LATIN multiscale computational method and the Proper Generalized Decomposition. *Computer Methods in Applied Mechanics and Engineering*, 199:1287–1296, 2010.