FORCED VIBRATIONS ANALYSIS AND THERMAL FATIGUE LIFE PREDICTION FOR INELASTIC LAYERED STRUCTURES

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Summary A method for predicting the responses of inelastic non-linear systems to the mechanical and electrical excitations, as well as the elevated temperatures arising due to the dissipative heating is suggested. As an example, the forced vibrations of simply supported and roller-supported beams containing piezoactive layers are investigated. The thermal fatigue life is estimated assuming the beam fails if the temperature exceeds the Curie point for piezoceramics, and the safe dissipative heating levels under harmonic loading are determined.

INTRODUCTION

The forced-vibration analysis of structures occupies a significant place in the dynamics of deformable systems. An accurate prediction of the dynamic response is a serious challenge, since the material of a structure may become plastic under intensive loading and/or exhibit viscous properties. Variable viscoelastoplastic behaviour should be studied when designing metal dampers for the vibrations of building structures under wind and seismic loads, devices for suppressing vibrations of pipelines, test specimens in low-cycle fatigue tests, etc. It is well known that the inelastic deformation of a material is accompanied by the release of heat due to the dissipation of mechanical energy. As a result, complex coupled thermo-electromechanical processes must be investigated to correctly predict the structural response and influence it in a desired manner.

There are currently two approaches to solve such problems: the so called “complete” and “approximate” formalisms. One employs the constitutive equations valid for arbitrary or, at least, rather wide classes of loading histories [1,2]. The other approach is based on simplified models for specific deformation history [5,7]. Within the framework of unified flow theories used in the first approach, the time dependent inelastic behaviour of crystalline solids is represented as part of a combined elastic-inelastic formulation that may not rely on a yield criterion or loading and unloading conditions. The same equations are applicable for all circumstances such as straining at prescribed rates, creep under constant stress, and stress relaxation under constant strain. In the present research, the formalism developed on the basis of a typical unified flow theory [8] is used for the benchmarking purpose. The second approach was developed especially for one of the most important cases of periodic processes – for cyclic (particularly harmonic) loading. In the case of harmonic deformation, all basic types of constitutive equations – integral (hereditary) or differential – have an algebraic form written in terms of complex stress and strain amplitudes. The relationship between these quantities is expressed in terms of complex-value moduli [3].

METHODOLOGY

In the case of long-term inelastic deformation, the complexity of the solution obtained within the framework of complete problem statement is due to the necessity of storing a large body of data and performing extensive computations to allow for the deformation history. To overcome these difficulties, in the practically important case of harmonic loading, an approximate monoharmonic model of thermo-electro-mechanical processes was used. This simplified model is based on the concept of complex moduli, which are determined by a modified technique of equivalent linearization [7]. In terms of these moduli, the initial problem is reduced to a scleronomous system of equations for complex amplitudes of mechanical and electrical variables – displacements, stresses, total and inelastic strains, voltage and electric current. The single frequency approximation can be particularly useful for the investigation of the problems where the amplitudes of the main field variables play a major role, when the peculiarities of the hysteresis loop and specific behaviour over the vibration cycle are not relevant.

In this paper, the model based on the monoharmonic approximation and the concept of complex moduli was developed for thin wall structures consisting of elastic or viscoelastic piezoactive and inelastic electrically passive layers undergoing harmonic electro-mechanical loading. The nature of the approximations imposes certain limitations on the model use. The model was proved to be a powerful technique if the loading and boundary conditions allow for neglecting the influence of geometrical nonlinearity and hence ensure a symmetrical vibration cycle. If geometrical nonlinearity is significant, the second harmonic has to be taken account of. It was shown using the results of [4,9,10] that the approximate model correctly estimates the amplitudes of the electric and mechanical parameters for small or moderate inelastic deformation of the passive layers. The nature of the approximate model does not allow considering large inelastic strains. Also, the approximate approach is not designed to describe the transient mechanical response of structures. However, in the framework of coupled electro-mechanical theory, it is perfectly capable to estimate

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correctly the amplitudes of the main field variables for steady state regimes. The developed method is very effective for cyclically stable materials: it is more than a hundred times faster [4], and demands much less computational resources, than approaches involving direct integration of constitutive equations.

RESULTS

As an example, the forced vibrations and dissipative heating were considered for roller-supported (Fig. 1) and simply supported beams with the middle layer made of aluminium alloy and the outer layers made of piezoactive material. The piezoceramics PZT was chosen as a material of the piezoactive layers. Generally, the complex moduli of materials can be determined either experimentally or obtained numerically (within the framework of chosen material model) as a response to the harmonic loading. The complex moduli for aluminium alloy were taken from [7] where the detailed procedure for calculating them is also described. Following [6], the cyclic behaviour of PZT was assumed to be a viscoelastic one with relatively small loss moduli. Complex moduli for this material were assumed to be independent of the frequency, temperature and strain intensity. The developed formalism of thermo-electro-mechanical problem was used for estimating the self-heating caused by the electromechanical losses in piezoactive layers and the mechanical losses in electrically passive metal layer. The heating temperature evolution with time was computed. It was shown that even very small temperature increases due to the dissipation of electromechanical energy over the single vibration cycles may lead to a significant temperature rise for the multi-cycle processes.

To avoid dangerous overheating, safe vibration regimes were found. For this purpose, the thermal fatigue life characteristic, N_θ, was introduced. It corresponds to the classical fatigue life, but with the Curie temperature taken as the failure criterion. A typical curve M_0–N_θ for frequency $f = 32$ Hz (the fastest heating regime) is shown in Fig. 2. The line divides the plane into two regions: a region above the curve where thermal failure can occur at a sufficiently high number of cycles and a region below the curve where thermal failure will not occur regardless of the number of cycles. For each value of the controlled parameter, the safe operating regime can easily be established from the appropriate curve. Taking account of the information presented in Fig. 2, one can easily estimate the useful life expectancy of a structure for a specified loading level. To prevent overheating, active suppression of the vibration can be used. It was shown [4,9,10], that in the cases of electric and mechanical excitation, the beam responds similarly to both types of loading. Hence, it is possible to damp mechanically excited vibrations by choosing the appropriate amplitude of voltage applied to the piezoactive layers, as detailed in [4,9,10].

The developed method was also successfully used for describing the thermal state under active damping regimes for other thin wall structures with piezoelectric layers for the case of small to moderate inelastic strains in the metal layer.

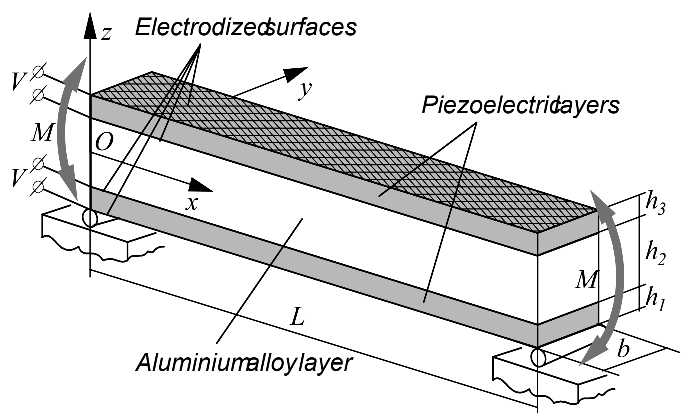

Figure 1: Beam geometries and the loading scheme.

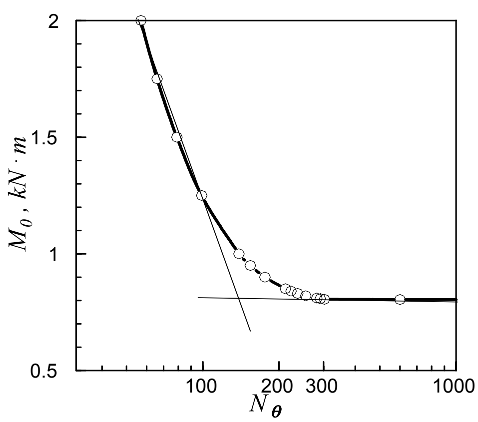

Figure 2: Typical thermal fatigue curve.

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