NONLINEAR VIBRATIONS OF LAMINATED CIRCULAR CYLINDRICAL SHELLS: COMPARISON OF DIFFERENT SHELL THEORIES

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Summary The geometrically nonlinear forced vibrations of laminated circular cylindrical shells are studied by using the Amabili-Reddy higher-order shear deformation theory. An energy approach based on Lagrange equations is used to obtain the equations of motion. The numerical results obtained by using the Amabili-Reddy shell theory are compared to those obtained by using an higher-order shear deformation theory retaining only nonlinear term of von Kármán type and the Novozhilov classical shell theory.

INTRODUCTION

A few studies are available in the literature on geometrically nonlinear vibrations of laminated circular cylindrical shells, and no one on forced nonlinear vibrations. Iu and Chia [1] studied geometrically nonlinear free vibrations of cross-ply laminated circular cylindrical shells by using a modified version of the Donnell nonlinear shell theory. Ganapathi and Varadan [4] studied the nonlinear free vibrations of laminated shells by using a first-order shear deformation theory and the finite element method. Classical shell theories, which neglect shear deformation and rotary inertia, give inaccurate natural frequencies for moderately thick laminated shells and plates. In order to overcome this limitation, shear deformation theories have been introduced. The nonlinear version of the higher-order shear deformation theories usually takes into account only von Kármán type nonlinear terms, i.e. nonlinear terms involving only the normal displacement. For this reason, Amabili and Reddy [3] recently developed an accurate and consistent nonlinear higher-order shear deformation theory that overcomes this limitation.

THEORY AND DISCRETIZATION

Lagrangian description is used. The stress-strain can be transformed from material to shell coordinates (x, θ, z) by

$$\begin{Bmatrix} \sigma_x \\ \sigma_\theta \\ \tau_{\theta z} \\ \tau_{xz} \\ \tau_{x\theta} \end{Bmatrix}^{(k)} = [Q]^{(k)} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \gamma_{\theta z} \\ \gamma_{xz} \\ \gamma_{x\theta} \end{Bmatrix}. \tag{1}$$

Here different strain-displacement relationships can be sued. The elastic strain energy U_S of the shell is given by

$$U_S = \frac{1}{2}\sum_{k=1}^{K}\int_0^{2\pi}\int_0^{L}\int_{h^{(k-1)}}^{h^{(k)}} \left(\sigma_x^{(k)}\,\varepsilon_{xx} + \sigma_\theta^{(k)}\,\varepsilon_{\theta\theta} + \tau_{x\theta}^{(k)}\,\gamma_{x\theta} + \tau_{xz}^{(k)}\,\gamma_{xz} + \tau_{\theta z}^{(k)}\,\gamma_{\theta z}\right)(1 + z/R)\,\mathrm{d}x\,R\,\mathrm{d}\theta\,\mathrm{d}z, \tag{2}$$

where K is the total number of layers in the laminated shell and ($h^{(k-1)}$, $h^{(k)}$) are the z coordinates of the top and bottom surfaces of the k-th layer. After satisfaction of boundary conditions and discretization, the Lagrange equations of motion are obtained as follows

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T_S}{\partial \dot{q}_j}\right) - \frac{\partial T_S}{\partial q_j} + \frac{\partial U_S}{\partial q_j} = Q_j, \qquad j = 1,\ldots,\bar{N}, \tag{3}$$

where T_S is the kinetic energy of the shell, q_j is a generalized coordinate and Q_j the associated generalized force that includes damping and external excitation.

NUMERICAL RESULTS

The dimensions and material properties of a simply supported, imperfection-free, laminated circular cylindrical shell made of graphite/epoxy layers are: R= 0.15 m, L = 0.52 m, h = 0.03 m, E_1= 50×10^9 Pa, E_2= 2×10^9 Pa, G_{12}= G_{13}= 1×10^9 Pa, G_{23}=0.4×10^9 Pa, ν_{12}=ν_{23}=0.25 and ρ = 1500 kg/m^3. This is a thin shell, being R/h =50. The shell is made of four layers 0/90°/90°/0 of the same thickness. The fundamental mode is now obtained for n = 2 and its natural frequency is 535.47 Hz. A nondimensional excitation f=0.006 is used for the fundamental mode and the nonlinear response is presented in Figure 1 for both the Amabili-Reddy and the Novozhilov theories. In the present case, since the shell is thick, an important difference is observed between the two shell theories. In particular, the Novozhilov theory under-predicts the nonlinearity of the shell since it neglects the rotary inertia and shear deformation. The natural frequency estimated by Novozhilov theory is 575.89 Hz, with an error of 7.5 %. Figure 2 compares results obtained for the same mode n=2 but using the Amabili-Reddy and the standard higher-order shear deformation (HOSD) theories. Also in this case an important difference is observed between the two shell theories, but the HOSD theory over-predicts the softening nonlinearity of the shells. Therefore the Novozhilov and HOSD theories give results for mode n =2 that are plotted on opposite sides with the respect to the more accurate results predicted by the Amabili-Reddy theory.

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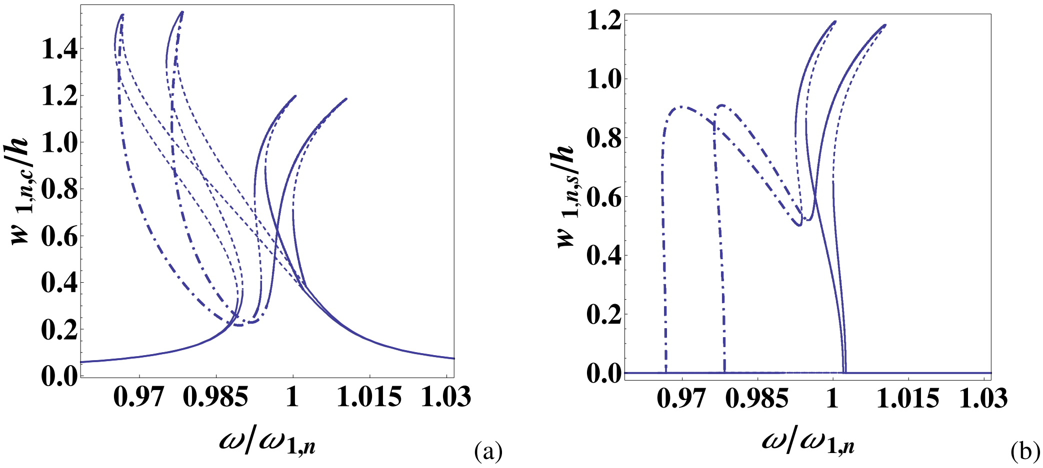

Figure 1. Comparison of the forced vibration responses computed with the Amabili and Reddy and Novozhilov theories; thick shell, n=2, fundamental mode. ▬▬, stable periodic solution; ▬ • ▬, quasi-periodic stable solution; ---, unstable periodic solution. (a) Driven generalized coordinate $w_{1,n,c}$; (b) companion generalized coordinate $w_{1,n,s}$.

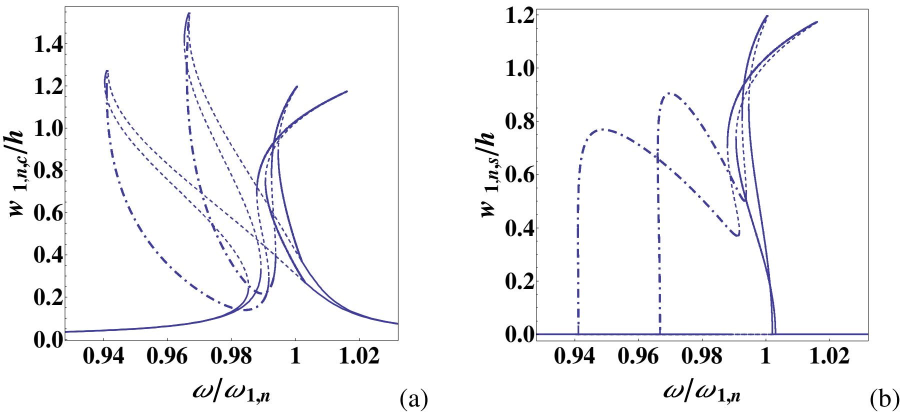

Figure 2. Comparison of the forced vibration responses computed with the Amabili and Reddy and higher-order shear deformation (HOSD) theory; thick shell, n=2, fundamental mode. ▬▬, stable periodic solution; ▬ • ▬, quasi-periodic stable solution; ---, unstable periodic solution. (a) Driven generalized coordinate $w_{1,n,c}$; (b) companion generalized coordinate $w_{1,n,s}$.

CONCLUSIONS

A main contribution of the present study is the comparison of numerical responses computed by using three different geometrically nonlinear shell theories: the classical Novozhilov theory (neglecting shear deformation and rotary inertia), the usual version of higher-order shear deformation theory that neglects in-plane nonlinear terms (HOSD) and the higher-order shear deformation theory recently developed by Amabili and Reddy. Results show that the Amabili-Reddy and Novozhilov theories give good results for thin (R/h = 50) laminated shells, while the HOSD gives accurate results only when the number of circumferential waves n is not small. On the other hand, for thick (R/h = 5) laminated shells, the Amabili-Reddy theory should be used in order to have accurate results.

References

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