DISPERSION RELATION ESTIMATION FOR THE GUIDED WAVE USING TIME-FREUQENCY REPRESENTATION BASED METHOD

Yang Yang[1 a)], Zhike Peng[2] & Guang Meng[3]
[1,2,3]*State Key Laboratory of Mechanical System and Vibration, Shanghai Jiao Tong University, Shanghai, 200240, P.R. China*

Summary Guided waves are widely used to detect the fault in structure during non-destructive detection, or determine the material properties of the composite or layered materials. However, its dispersive nature complicates the analysis and application in practise. The guide wave propagation in the structure is characterized by the dispersive relation between wavenumber versus frequency. In order to reveal the dispersion property of the guide wave, Lamb wave propagating in plate is measured experimentally and analysed by a time-frequency representation based dispersion relation estimation method. The results show the estimated dispersion relation agrees with the theoretical dispersion curve.

Keywords : guided wave, dispersion relation, time frequency representation, non-destructive detection

INTRODUCTION

Guided wave is acknowledged as one of most encouraging tools for material characterization in structural health monitoring or non-destructive detection application. It is widely used to evaluate the integrity of structure component and determine the material property, i.e., composite or layered material for its merits of low attenuation, high sensibility to property of the material and the defect, spread all over the surface and the inter of structure. Pleural of relevant researches have been undergoing in various disciplines since 1980s, see Chimenti for an overview [1]. It is critical to understand or quantify the dispersive characterization of the guided wave propagating in the structure, i.e., the nonlinear Lamb wave potentially relevant to the micro defect evaluation. Usually, the dispersive curve of the each guided Lamb mode is obtained by solving the characteristic equations, i.e., the dispersion relationship of guided Lamb wave propagating in a homogenous, isotropic plate is described by the Rayleigh-Lamb equations [2], which reveal an implicit relation between wavenumber k and frequency ω. The group velocity of the Lamb wave $c_g=d\omega/dk$, which results in the propagation time of the Lamb wave in certain distance is the function of frequency. However, the very dispersive nature complicates its analysis and application in practise. In the case of complicated structure in which the wave propagating model is difficult to establish, the theoretical dispersive curve cannot be computed so that the utilization of the guided wave is limited. In this study, the nonlinear dispersive curve of an experimental measured Lamb wave is estimated based on the time frequency representation (TFR) based method. The results that agree with the theoretical dispersive curve validate the reliability and robustness of the parameter estimation method for the Lamb wave.

LAMB WAVE MEASUREMENT EXPERIMENT

The Lamb wave signals are measured by a point source/receiver, non-contact laser system. The data collected with sampling frequency of 100MHz. In order to improve the SNR, the responses are averaged after consecutive 32 shots with repetition frequency of 2Hz [3]. The overview of the laser system is shown in Fig 1. The TFR of the measured wave signal obtained by short-time Fourier transform (STFT) associated with theoretical dispersion curves (pink line and red dot line) for the specimen of thickness 3.7mm with 145mm propagation distance are plotted in Fig 2. Only two modes, s_0 (symmetric mode) and a_0 (anti-symmetric mode) are investigated.

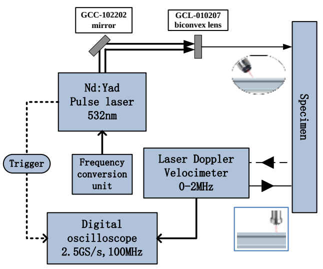


Fig. 1 Systematic overview of the experiment

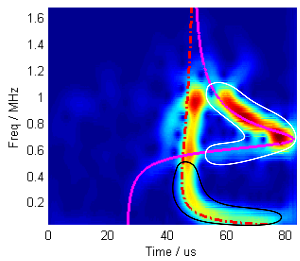


Fig 2, The TFR associated with theoretical dispersion curves

TFR BASED DISPERSION ESTIMATION METHOD

In order to investigate the dispersive character of the Lamb wave, the time-frequency analysis (TFA) is applied. Conventional TFA methods use the kernel function of the time to approximate the instantaneous frequency of the non-stationary signal. However, the dispersive relation of Lamb wave reveals non-monotonic group delay (GD), which cannot find a definite inverse function of the instantaneous frequency. Therefore, we defined the standard polynomial

[a)] Corresponding author. Email: emma002@sjtu.edu.cn.

chirplet transform [4] in frequency domain (PCT-f), which is used to formulate the dispersion parameter estimation (DPE) method. The PCT-f is basically defined to be the dual STFT of the signal spectrum with time-rotating and -shifting operation, i.e., $\Theta^R(\omega,\beta_1,\ldots,\beta_n)$ and $\Theta^S(\omega,\omega_0,\beta_1,\ldots,\beta_n)$.

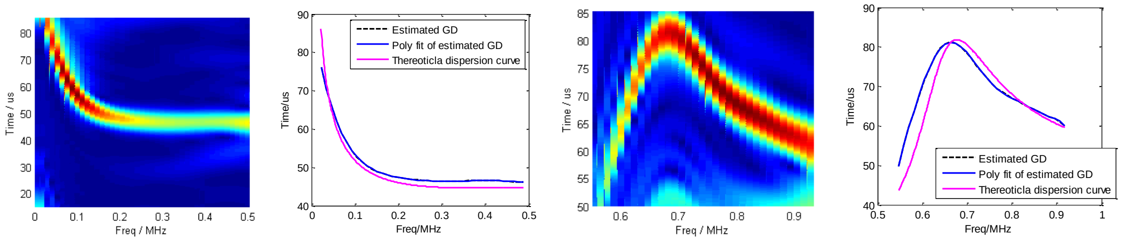
$$C^{PCTF}(\omega_0,t;\beta_1,\ldots,\beta_n,\sigma)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(\omega)\Theta^R(\omega,\beta_1,\ldots,\beta_n)\Theta^S(\omega,\omega_0,\beta_1,\ldots,\beta_n)W_\sigma(\omega-\omega_0)\exp(j\omega t)d\omega$$

with $\Theta^R(\omega,\beta_1,\ldots,\beta_n)=\exp\left(-j\sum_{i=2}^{n+1}\frac{1}{i}\beta_{i-1}\omega^i\right)$ and $\Theta^S(\omega,\omega_0,\beta_1,\ldots,\beta_n)=\exp\left(j\sum_{i=2}^{n+1}\beta_{i-1}\omega_0^{i-1}\omega\right)$

The iterative DPE first generates an initial TFR using PCT-f and extract the energy peak from the TFR to be estimated GD, then the estimated GD is approximated by the polynomial function of the frequency to obtain the parameters for the PCT-f. The procedure is iterated until the poly-approximation of the estimated GD changes less than a threshold. The eventually obtained parameters are the estimated parameters of the dispersion curve of Lamb wave.

RESULTS ANALYSIS

The two proportions of Lamb mode s_0 and a_0 are considered, which are highlighted by the white curve and black curve, respectively. The 6-order and 16-order polynomial approximation are used by the PCT-f, with which totally 2 and 4 iterations of DPE are conducted for the two proportions, respectively. The threshold is set to 1%. The TFRs obtained by the PCT-f and the estimated GD are shown in Fig 3. Comparing to Fig 2, the energy concentration in the local region of Lamb mode s_0 and a_0 are extensively enhanced as shown in Fig 3(a) and Fig 3(c). Clearly, the estimated GD or its poly-approximation is closed to the corresponding proportion of the theoretical dispersive curve as shown in Fig 3(b) and Fig 3(d). The terminating condition and the relative error of the estimated GD to the theoretical GD in the each iteration for both regions are listed in Table 1 and Table 2. It can be seen that the relative errors are less than 2.5% and 5% for the black and while circle region, respectively.



(a) (b) (c) (d)

Fig 3, (a) The TFRs obtained by PCT-f and (b) estimated GD in black circle region; (c) The TFRs obtained by PCT-f and (d) estimated GD in white circle region.

Table 1, The terminating condition and the relative error of estimated GD to theoretical GD at black circle region

i	1	2
ζ(%)		**0.91**
ε_{th}(%)	2.70	2.26

Table 2 The relative error of estimated GD to theoretical GD at white circle region

i	1	2	3	4
ζ(%)		4.11	2.80	**0.94**
ε_{th}(%)	3.63	3.83	4.27	5.09

CONCLUSIONS

In this study, the dispersive property of Lamb wave propagating in plate is investigated. The Lamb wave that is measured experimentally by the non-contact laser system demonstrates that dispersion parameters estimation (DPE) based on the time frequency representation (TFR) method. The parameters of the Lamb mode are estimated by the polynomial approximation iteratively, with which the poly-approximation of the estimated GD is proved to be a good estimation of the local dispersion curve. The results show that the TFR of the Lamb wave signal is significantly improved and the local dispersive curve estimation is relative accurate, which implies the effectiveness of the DPE method in the parameter estimation of the guided wave propagation.

References

[1] D. E. Chimenti, “Guided waves in plates and their use in materials characterization”, *Applied Mechanics Reviews*, **50**(5): 247-284, 1997.
[2] J. D. Achenback : Wave Propogation in Elastic Solids. Elsevier, NY, USA 1999.
[3] J. Acoust. Soc. Am.”Time-frequency representations of lamb waves”, *Journal of the Acoustical Society of America*,106(5): 1841-1847, 2001
[4] Z. Peng, G. Meng, F. Chu et al, “Polynomial chirplet transform with application to instantaneous frequency estimation”, *IEEE Transaction on Instrumentation and Measurement*, 60: 3222-3229, 2011