ON CONTROLLING OF DOUBLE PENDULUM BY VIBRATION OF SUSPENSION POINT

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Summary In this work multi-link simple pendulum motion in gravity field with an arbitrary three dimensional periodic vibration of the suspension point is considered. In the focus of our interest were the controlling parameters of suspension point vibrations necessary for pendulum stabilization at an arbitrary set point. The expression for effective potential energy for a simple pendulum in the high frequency limit is obtained. The double pendulum is considered. The controlling parameters for this case are found as well. Also it is found where the pendulum can't be stabilized. All results were checked by computer modeling of exact equations of motion. The methods used can be applied to multi-link pendulums with more elements.

INTRODUCTION

The multi-link pendulum is considered as several concatenated simple pendulums. The simple pendulum is a material point of mass m oscillating on a weightless inextensible rod of length l. The suspension point with a radius vector $\mathbf{r}_0\ \ x_1, x_2, x_3$ moves according to the prescribed periodic law in time with the frequency ω. The pendulum moves in gravitational field.

The problem under consideration has a rich history. The possibility of stabilizing the mathematical pendulum at the upper point due to vertical high frequency vibrations of the suspension point was first investigated by Stephenson [1]. In more detail, pendulum oscillations with vertical and horizontal vibrations were studied independently by Bogolyubov and Kapitsa [2–4]. The asymptotic solutions of the problem on flat pendulum oscillations for arbitrary vibrations are given in [5, 6]. In [7–9], the problem of determining stable periodic solutions for high frequency suspension point vibrations is reduced to the condition of minimum of the effective potential energy

$$U_{ef} = \frac{m\left\langle \mathbf{v}_\tau^{\,2} \right\rangle}{2} - m\ \ \mathbf{g},\mathbf{R}\ \ ,\ \mathbf{v}_\tau = \mathbf{v} - \mathbf{R}\frac{\mathbf{v},\mathbf{R}}{l^2}, \tag{1}$$

here $\mathbf{v}$ is a suspension point velocity and $\mathbf{R}$ is the radius defining the direction of the rod.

Because the pendulum length is constant $|\mathbf{R}| = l$, the vector $\mathbf{R}$ is described by two spherical coordinates θ and φ.

Thus, U_{ef} is the function of two coordinates θ, φ, and six parameters $\left\langle v_i v_j \right\rangle$; i, j = 1, 2, 3. The purpose of this work is to find the controlling parameters of vibrations $\left\langle v_i v_j \right\rangle$ necessary for one-link and two-link pendulum stabilization at a desired position. Method used is applicable to pendulums with more elements.

SIMPLE PENDULUM

Neglecting the term $\left\langle v^2 \right\rangle$, which is constant on the entire sphere, Eq. (1) for the effective potential energy can be rewritten as

$$U_{ef} = U_{vib} + U_g = -\frac{m\left\langle\ \mathbf{R},\mathbf{v}\ ^2 \right\rangle}{2l^2} - m\ \ \mathbf{R},\mathbf{g}\ \ . \tag{2}$$

The following expression for effective force due to vibrations corresponds to the term U_{vib}:

$$\mathbf{F}_{vib} = -\frac{\partial U_{vib}}{\partial \mathbf{R}} = \frac{m}{l^2} V\mathbf{R},\quad V = \left\langle \mathbf{v}\mathbf{v}^T \right\rangle = \left\langle v_i v_j \right\rangle,\quad i, j = 1,2,3 \tag{3}$$

Substituting Eq. (3) into equilibrium equation $\mathbf{F}_{vib} + m\mathbf{g} = \frac{T}{l}\mathbf{R}$ (where T is the reaction force of rod), we obtain

$$V\mathbf{R} + l^2 = \alpha l^2 \mathbf{R},\quad \alpha = \frac{T}{ml}. \tag{4}$$

Let's consider only the case when the suspension point is vibrating along vector $\mathbf{c}$

$$\mathbf{v} = \mathbf{c} l f\ \ t\ \ / \sqrt{\left\langle f^2\ \ t\ \ \right\rangle}, \tag{5}$$

where $f\ \ t$ is an arbitrary periodic function. Then the matrix V in Eq. (3) can be rewritten as $V = l^2 \mathbf{c}\mathbf{c}^T$. So for the equilibrium equation (4) we obtain.

$$\mathbf{c},\mathbf{R}\ \ \mathbf{c}+\mathbf{g}=\alpha\mathbf{R}\Rightarrow\mathbf{c}=\frac{\alpha\mathbf{R}-\mathbf{g}}{\sqrt{\ \alpha\mathbf{R}-\mathbf{g},\mathbf{R}}} \tag{6}$$

Thus the problem about pendulum equilibrium at set point is solved: in order that the pendulum placed at the point $\mathbf{R}$ remain there in the equilibrium state, the suspension point should be vibrated by law (5), (6), with an arbitrary value of parameter α.

DOUBLE PENDULUM

Let $\mathbf{R}_1$, $\mathbf{R}_2$ be the vectors defining the direction of the rods we want to stabilize our pendulum at (the nearest and the farthest to the suspension point respectively), $\mathbf{v}_0$, $\mathbf{v}_A$, $\mathbf{v}_B$ are the vibration velocities of the suspension point, weight A and B respectively (Fig. 1) .

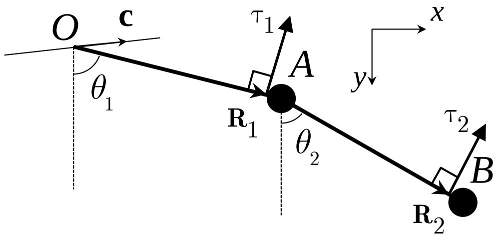

Fig. 1. Double pendulum

It is very useful to consider the double pendulum as two separate simple pendulums with vibrating suspension point and with external forces acting on them. By using the concept of vibrational force we can obtain equilibrium equations for the pendulum similar to eq. (4)

Let's consider only the case when the suspension point is vibrating along vector $\mathbf{c}$

$$\mathbf{v}_O = l\mathbf{c}f\ \ t\ \ /\sqrt{\left\langle f^2(t)\right\rangle}. \tag{7}$$

In this case the solution of the equilibrium equations will be next:

$$\mathbf{c}=A\ \ \alpha\mathbf{R}_1-2\mathbf{g}\ \ \sqrt{-\frac{2\ \ \mathbf{R}_1,\tau_2\ \ \ \mathbf{R}_1,\mathbf{R}_2}{\mathbf{g},\tau_2\ \ l^3}},\quad \alpha=-\frac{\mathbf{g},\tau_2\ \ l^2}{\mathbf{R}_1,\tau_2\ \ \ \mathbf{R}_1,\mathbf{R}_2}+\frac{\mathbf{g},\mathbf{R}_1}{l^2},$$

$$A=E-\frac{\mathbf{R}_1,\mathbf{R}_2}{2l^4}\mathbf{R}_2\mathbf{R}_1{}^T. \tag{8}$$

This is the solution of the problem: for the $\mathbf{R}_1,\mathbf{R}_2$ to be an equilibrium position we should vibrate the suspension point by law (7), where $\mathbf{c}$ is defined by Eq. (8).

Stability

It is easy to find stability condition for simple pendulum. It is $\delta^2 U_{ef}>0$, where U_{ef} is defined by Eq. (2). This inequality can be solved analytically. In the case, when the suspension point is vibrating by the law (5), we get the inequality for parameter α (Eq.(7)):

$$\alpha>\frac{g_{\mathbf{R}}+\sqrt{g_{\mathbf{R}}{}^2+4g_\tau{}^2}}{2l},$$

where $g_{\mathbf{R}}, g_\tau$ are projections of $\mathbf{g}$ onto the vectors $\mathbf{R}$ and τ respectively.

In the case of double pendulum it is much harder to find analytical solution, so we obtained it numerically.

CONCLUSION

Through our work we developed and widely used the method of vibrational forces. By using this method we have found parameters of vibration that stabilize simple and double pendulum at any desired position. Methods used for double pendulum can be easily applied to pendulums with more elements.

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