PERIODIC OSCILLATIONS AND STABILITY OF THIN VARIABLE STIFFNESS COMPOSITE LAMINATED PLATES

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Summary Forced, geometrically non-linear, periodic vibrations of thin Variable Stiffness Composite Laminated (VSCL) plates are analysed. The stiffness varies in the plates' domain, because curvilinear reinforcement fibres are employed, instead of straight fibres. The similarities and differences between the response to harmonic forces of constant stiffness and variable stiffness composite laminates in the same materials are analysed. The stability status and the bifurcations of oscillations of the diverse composites are differentiated.

INTRODUCTION

Tow placement machines allow for the variation of the orientation of fibres along a lamina [1] and in this way one obtains a composite laminated panel where the stiffness varies from point to point (VSCL). It is, for example, possible to apply a fibre as shown Figure 1. Composite laminates are time and again employed in demanding situations, where an understanding of their dynamic behaviour in the non-linear regime is important. We here investigate geometrically non-linear vibrations of thin VSCL plates and, in particular, the steady-state response of these plates to harmonic forces. For constant force amplitudes, the excitation frequency is varied – as a parameter – and the non-linear counterparts of the Frequency Response Functions are obtained. The periodic oscillations of non-linear VSCL plates are investigated in detail and compared with the ones of traditional constant stiffness laminates in the same original materials.

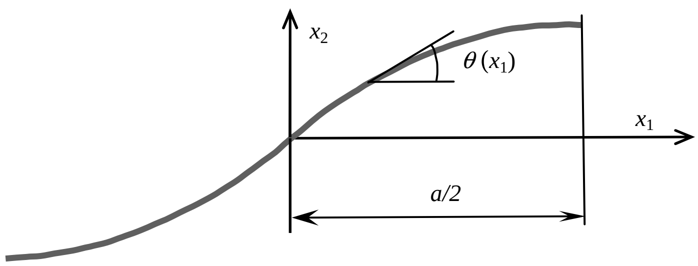

Figure 1: Fibre with variable orientation $\theta(x_1)$. *a* is a reference dimension of the plate.

FORMULATION SYNOPSIS

A fixed Cartesian coordinate system and an equivalent single layer, thin plate, theory are adopted. To derive the model, which is constituted by ordinary differential equations of motion, each displacement component of a point in the middle plane is written as follows

$$u_i^0(\xi,\eta,t) = \mathbf{N}^{u_i}(\xi,\eta)^T \mathbf{q}_{u_i}(t) \tag{1}$$

where ξ and η represent non-dimensional coordinates, $\mathbf{N}^{u_i}(\xi,\eta)$ are vectors of displacement shape functions and $\mathbf{q}_{u_i}(t)$ are vectors of generalized coordinates. It is considered that the fibre angle varies linearly with the longitudinal coordinate x_1 such that, in lamina k, the orientation of fibres is defined by

$$\theta^k(x_1) = \frac{2\left(T_1{}^k - T_0{}^k\right)}{a}|x_1| + T_0{}^k \tag{2}$$

where $T_0{}^k$ is the fibre angle at x_1=0 and $T_1{}^k$ the fibre angle at $x_1=\pm a/2$. Due to this variation, the stress-strain relation changes with x_1, unlike what occurs in traditional laminates.
To account for large displacement amplitude vibrations, non-linear relations between strains and the displacements are adopted. Then, the virtual work principle is employed, and equations of motion of the following form are obtained

$$\mathbf{M}\ddot{\mathbf{q}}(t) + (\alpha\mathbf{M} + \beta\mathbf{K}_\ell)\dot{\mathbf{q}}(t) + \mathbf{K}_\ell\mathbf{q}(t) + \mathbf{K}_{n\ell}(\mathbf{q}(t))\,\mathbf{q}(t) = \mathbf{P}\cos(\omega t) \tag{3}$$

The linear and non-linear stiffness matrices, $\mathbf{K}_\ell$ and $\mathbf{K}_{n\ell}$, now depend on a variable fibre angle; hence, they differ from the ones of traditional laminates. The mass matrix $\mathbf{M}$ and the vector of generalised external forces, of amplitude $\mathbf{P}$ and frequency ω, are equal to the ones of usual laminated composites [2]. A Rayleigh type damping is considered in equation (3).

The shooting method [3] is here applied in the time domain, in order to solve the following boundary value problem: what is the phase space state that is equal to itself at the beginning of a cycle and after one period? For this purpose it is necessary to change the equations of motion to first order ordinary differential equations:

$$\begin{bmatrix} \mathbf{0} & \mathbf{M} \\ \mathbf{M} & (\alpha\mathbf{M}+\beta\mathbf{K}_{\ell}) \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{y}}(t) \\ \dot{\mathbf{q}}(t) \end{Bmatrix} + \begin{bmatrix} -\mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{\ell}+\mathbf{K}_{n\ell}(\mathbf{q}(t)) \end{bmatrix} \begin{Bmatrix} \mathbf{y}(t) \\ \mathbf{q}(t) \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{P}\cos(\omega t) \end{Bmatrix} \tag{4}$$

The periodic solutions of the equations of motion are hence determined "exactly" (depending solely on numeric precision). As a by-product of the solution procedure, the monodromy matrix is also computed and, from the eigenvalues of the latter, the stability of the solutions is determined and the bifurcations characterized.

NUMERICAL TESTS

A very short sample of the data computed is shown in Figure 2. This portrays the non-dimensional velocities against the non-dimensional displacements of central points of diverse composite layered plates. The plates are in the same constitutive materials, but in one of the plates the fibres are rectilinear and in the remaining plates the fibres have curvilinear patterns. The plates are excited by harmonic forces at the respective linear natural frequency; the force is uniformly distributed.

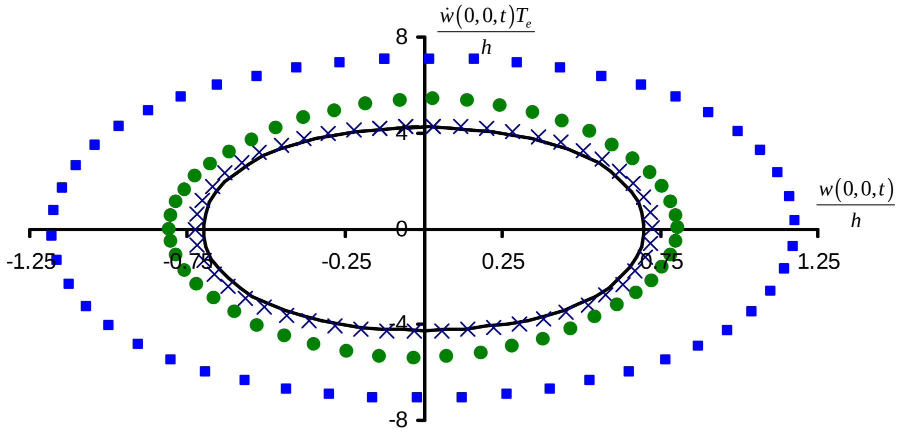

Figure 2: Non-dimensional displacement versus non-dimensional velocity of middle point of plates, excited by harmonic forces at the respective linear fundamental frequency. The fibre orientations are characterized by T_0 and T_1 with: T_0=45° and: —— T_1=0°, ╳ T_1=15°, ● T_1=45°, ■ T_1=90°.

It is evident from Figure 2 that a variation only in T_1 (T_0 and T_1 define the fibre angle variation in each layer) can lead to very diverse response amplitudes. The differences between the dynamic behaviour of constant stiffness and variable stiffness laminates become even more pronounced when interactions with higher vibrations modes arise. These interactions are due to energy transfer, which is endorsed by non-linearity. Because the higher modes of VSCL can have quite different shape from the ones of traditional laminates [4], the oscillations in the non-linear regime also occur with clearly different shapes. Also of interest are the stability status of solutions and the degree of hardening spring, which can be different from one VSCL to another, just by changing T_1.

CONCLUSION

Periodic vibrations of thin Variable Stiffness Composite Laminated plates, actuated by harmonic forces and in the geometrically non-linear regime, are investigated. Differences in the dynamic behaviour of VSCL and constant stiffness laminates are analysed. We focus in particular in the vibration amplitudes, modal interactions, stability status, bifurcations of periodic motions and in the degree of hardening spring.

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