MODES OF VIBRATION OF DAMAGED BEAMS BY A NEW *p*-VERSION FINITE ELEMENT

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Summary Free periodic vibrations of damaged beams, possibly vibrating with large amplitudes, hence in the geometrically non-linear regime, are analysed. The type of damage considered is localized and represented by a notch or indentation. The two main interests of this study are: (1) a new *p*-version finite element that efficiently deals with discontinuities, as abrupt variations in geometry, is presented; (2) the modes of vibration of notched beams in the linear regime and their variation with the vibration amplitude are studied.

INTRODUCTION

We are here interested in understanding how do more or less small notches, which may occur due to damage, affect the modes of vibration of beams, vibrating either with small or with moderately large amplitudes. The problem is important in practice, also but not only, because modal analysis is often used for damage detection [1]. To carry out the investigation, a new *p*-version finite element is employed. This element is in itself noteworthy, since it provides accurate results with a moderate number of degrees of freedom and represents an advance with respect to traditional *p*-elements. The latter are known to be very efficient in smooth problems, but not so efficient when structures present discontinuities [2].

FORMULATION

A first-order shear deformation, or Timoshenko, model is adopted and therefore the displacement field is:

$$u_1(x_1,x_3,t)=u_1^0(x_1,t)+x_3\theta^0(x_1,t) \qquad u_3(x_1,x_3,t)=u_3^0(x_1,t) \tag{1}$$

where $u_i(x_1, x_3, t)$ represents the displacement component along axis x_i and $\theta^0(x_1,t)$ is the rotation of the cross section. Superscript 0 indicates axis x_1, which crosses the centroids of undamaged cross sections (Figure 1).

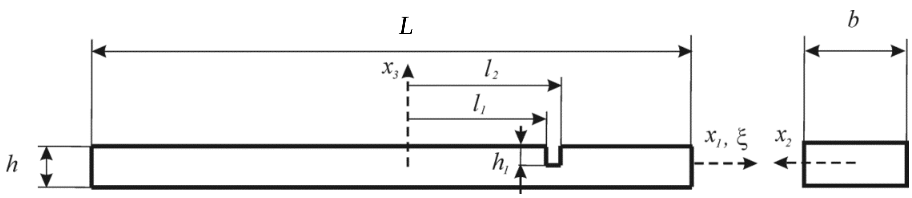
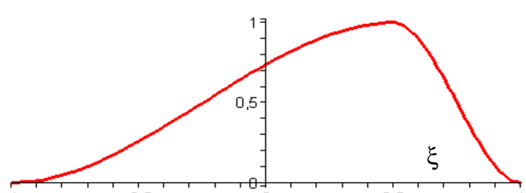


Figure 1: Notched beam, notch location and depth, local and global axis.

The displacements components of points on axis x_1 are expressed in the form:

$$u_1^0(\xi,t)=\mathbf{N}^{u_1}(\xi)^{\mathrm{T}}\mathbf{q}_{u_1}(t) \qquad u_3^0(\xi,t)=\mathbf{N}^{u_3}(\xi)^{\mathrm{T}}\mathbf{q}_{u_3}(t) \qquad \theta^0(\xi,t)=\mathbf{N}^{\theta}(\xi)^{\mathrm{T}}\mathbf{q}_{\theta}(t) \tag{2}$$

where $\mathbf{N}^{\mathbf{i}}$ are vectors of element shape functions and $\mathbf{q}_i$ are generalised coordinates; ξ is a non-dimensional coordinate (-1≤ξ≤1). As traditional in the *p*-version FEM, to improve the accuracy of the discretization, the number of shape functions and generalised coordinates in a finite element are increased. It is known that the *p*-version FEM is very efficient when the structure is smooth, hence without discontinuities [2-4]. We here propose the introduction of two new shape functions, which improve the efficiency of the *p*-version FEM in the presence of discontinuities, as localised damage. These new shape functions allow for additional flexibility in a crack or notch location. As an illustration of the new shape functions, Figure 2 portrays them for a discontinuity located at ξ=0.5.



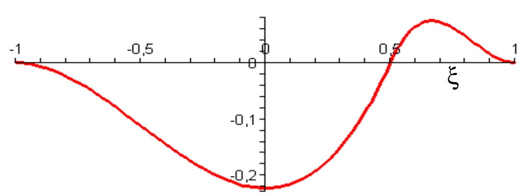


Figure 2: Example of new displacement shape functions

Elastic, isotropic and homogeneous materials are considered and the kinematic relations are of the Von Kármán type. The time domain equations of motion are derived by the principle of virtual work and are of the form

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\mathbf{K}_{\ell}\,\mathbf{q}(t)+\mathbf{K}_{n\ell}\big(\mathbf{q}(t)\big)\mathbf{q}(t)=\mathbf{0} \tag{3}$$

where $\mathbf{q}(t)$ are the generalized displacements, $\mathbf{M}$ is a constant mass matrix, $\mathbf{K}_\ell$ is a constant stiffness matrix, and matrix $\mathbf{K}_{n\ell}(\mathbf{q}(t))$ represents the geometrically non-linear stiffness. The procedure followed to obtain these matrices and their form is very much the one of [3], but there are changes in the matrices due to the notch. The most important of these changes, is the appearance of new couplings between longitudinal deformation and bending.

The influence of the notch location on the vibration modes, in the linear and non-linear regimes, is analyzed. The linear modes of vibration result from the usual generalized eigenvalue problem. The evolution of these modes with the vibration amplitude is investigated by the harmonic balance and continuation methods [4]: vector $\mathbf{q}(t)$ is written as a truncated Fourier series and a set of algebraic equations of motion is obtained and solved. The backbone curve, which relates the frequency with the vibration amplitude, is defined.

SAMPLE RESULTS

As an illustration of the results obtained, Table 1 shows the accuracy of the new *p*-version finite element and the relatively reduced number of degrees of freedom it requires. This table gives linear natural frequencies computed with the new *p*-element, with a classic *p*-element that employs the shape functions of [3] and [4], and with the *h*-version FEM (ANSYS software with a refined mesh). This particular beam is clamped-clamped and with the following geometric and material properties: L=1.33 m, b=h=25.3 mm, l_1=0.2225 m, l_2=0.2475 m; E=203.91 GN/m^2, ρ=7800 kg/m^3, (E and ρ stand for Young modulus and mass density). The first mode shapes of the damaged and undamaged beams are compared in Figure 3, where one can see the localized variation of the shape due to the notch.

Table 1: linear natural frequencies (Hz)

	Undamaged beam			12mm deep notch			
Mode	ANSYS BEAM189	Classic *p*FEM (15 sf)	New *p*FEM (15 sf)	ANSYS BEAM189	Classic *p*FEM (70 sf)	Classic *p*FEM (28 sf)	New *p*FEM (28 sf)
1	74.979	74.986	74.986	73.975	74.064	75.230	73.957
2	206.01	206.05	206.05	194.50	195.15	205.68	194.30
3	402.04	402.21	402.21	399.11	399.47	402.08	399.29

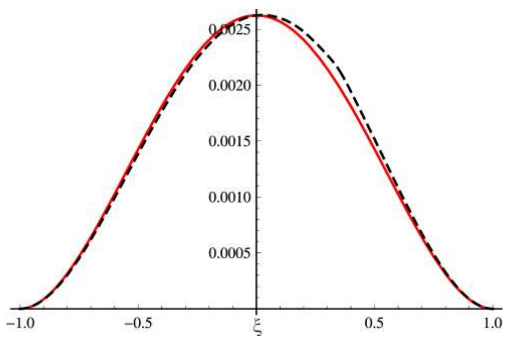


Figure 3: First transverse mode shape of the undamaged (red line) and damaged beam (dashed line)

CLOSURE

A new *p*-element for damaged beams that leads to a numerically efficient model is employed to investigate the modes of vibration of notched beams. The efficiency of this element is shown by comparisons with traditional *h* and *p* version elements. Linear modes of vibration of damaged beams and their evolution with the vibration amplitude, i.e., into the geometrically non-linear regime, are studied. New coupling terms exist due to the damage.

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