ROTATING EIGENMODES ON AN IMPERFECT CIRCULAR PLATE

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Summary The axisymmetric structure of an ideal circular plate leads to mode pairs with identical natural frequencies with congruent mode shapes which are rotated against each other for $\pi/2n$ with n as the number of nodal diameters. With an excitation of a centrically clamped circular plate at two locations at the outer radius with a certain phase difference and the natural frequency of a mode, the mode shape will rotate.

INTRODUCTION

If a plane, homogeneous, axisymmetric circular plate which is fixed at the centre and free at the outer radius is excited at one point at the outer radius with a harmonic force at the exact natural frequency of one of the fundamental modes with nodal diameters, the mode shape will have maximum amplitude at the excitation point, no matter where the excitation is applied at the circumference [1-3].
For an imperfect circular plate the mode pair is split into two single modes with slightly different natural frequencies. The mode shapes are still congruent but with fixed orientation independently of the location of the excitation point. Depending on the frequency, phase and location of the excitation the mode shape of modes with nodal diameters will rotate, which is presented in this paper.

THEORY

A circular plate which is clamped at the centre and free at the outside and excited with a natural frequency at one point at the outer radius the standing wave can be explained by the superposition of a left propagating wave and a right propagating wave (Eq. 1)

$$w_1 = w_{1\_left\_wave} + w_{1\_right\_wave} = e^{i(kx+\omega t)} + e^{i(kx-\omega t)} \tag{1}$$

where w is the normal displacement, k is the wave number, x is the location along the circumference, ω is the circular frequency and t is the time. The standing wave of a second excitation at the different position $x - x_0$ with a phase difference of ϕ can also be written in Eq. 2

$$w_2 = w_{2\_left\_wave} + w_{2\_right\_wave} = e^{i(k(x-x_0)+\omega t+\phi)} + e^{i(k(x-x_0)-\omega t-\phi)} \tag{2}$$

The superposition of these two standing waves can be expressed by the sum of w_1 and w_2 shown in Eq. 3

$$w = w_1 + w_2 = e^{i(kx+\omega t)} + e^{i(kx-\omega t)} + e^{i(k(x-x_0)+\omega t+\phi)} + e^{i(k(x-x_0)-\omega t-\phi)} \tag{3}$$

With $\phi = n \cdot x_0$ (x_0 in angular dimension) the overlap of the two standing waves will result in a traveling wave.

EXPERIMENTAL SETUP

An aluminium circular plate with a radius of 220 mm and a thickness of 2 mm has been taken for the experimental validation. The setup is shown in figure 1. The plate is clamped with a screw in the centre and free at the outside. The excitations were realized with magnets fixed to the circular plate and coils which were connected to amplifiers and signal generators. A laser scanning vibrometer from Polytec was used for the measurement of the vibrations and visualization of the mode shapes.

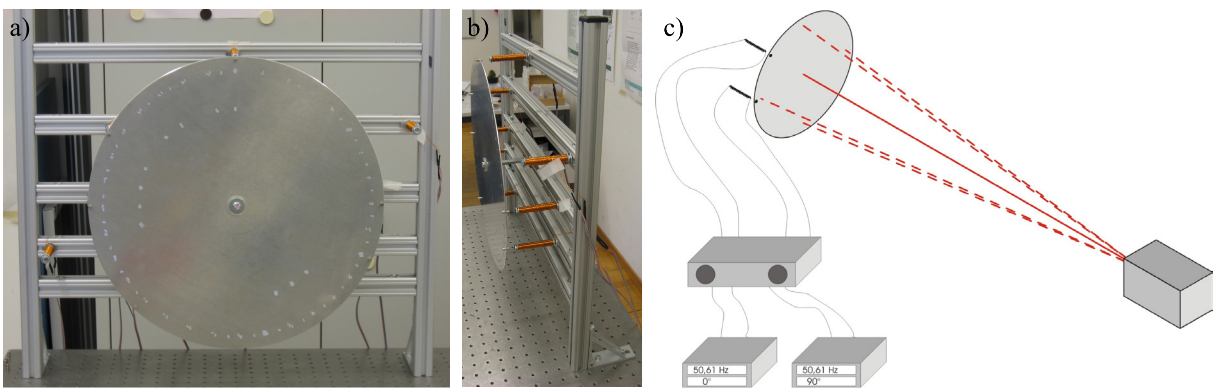

Figure 1: Experimental setup with eight coils mounted on the frame to realize the different locations for the excitation for the different modes. Front view of the plate a) and side view b). A schematic diagram of the experimental setup with the two signal generators, the amplifier, the coils and the magnets which are fixed on the plate and the laser scanning vibrometer c).

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RESULTS

According to the theory each mode pair has one natural frequency. Because of the mistuned and imperfect circular plate the mode pairs don't have the identical natural frequency for both mode shapes anymore, but two slightly different frequencies. In order to obtain rotational modes, a frequency between f_1 and f_2 is chosen for the excitation i.e. $f_{exc.}$=50.61 Hz. In the measured amplitude spectrum of the 2-0 mode shown in figure 2, the two peaks can be seen which correspond to the eigenfrequencies.
For the 2-0 mode experiments a phase difference for the two excitation signals of $\phi = 90°$ has been taken and the locations adapted to this by taking an angle of $x_0 = 45°$ in between. The used frequency was 50.61 Hz. In figure 3 a sequence in time over nearly half a period of the excitation signal of the mode shape from the 2-0 mode is shown for a constant phase difference for the two excitation signals. The phase φ between excitation and velocity varies with time. The counter clockwise rotation of the mode shape is shown. The direction of the rotation is dependent on the sign of the phase at the two excitation locations. With this method also experiments for the 3-0 mode, 4-0 mode and 5-0 mode were performed. The visualizations of these mode shapes is shown in the figure 4. For the 3-0 mode the excitation parameters $x_0 = 30°$, $\phi = 90°$ and for the 4-0 mode $x_0 = 22.5°$, $\phi = 90°$ were used. For the 5-0 mode parameter $\phi = 45°$ and $x_0 = 45°$ were used for the rotating modes.

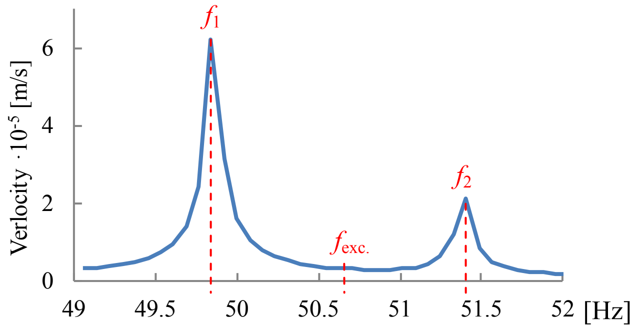

Figure 2: Measured amplitude spectrum of the 2-0 Mode showing the two peaks coming from the symmetrical imperfection.

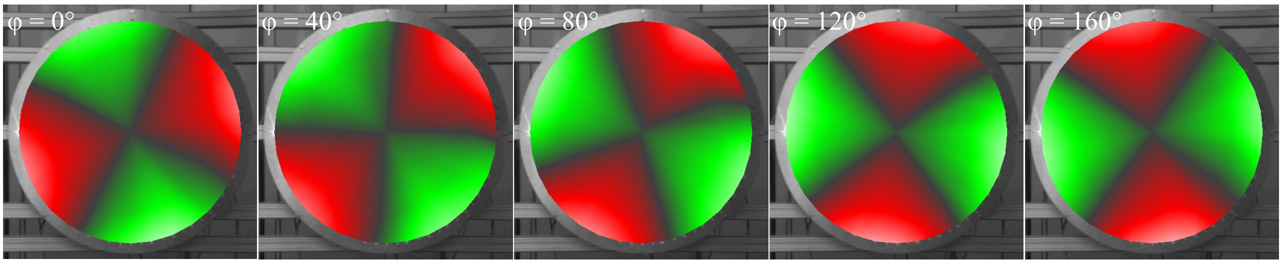

Figure 3: Visualization of the measured mode shape of the 2-0 Mode for different phases φ. The red colours are for positive amplitudes and the green colours for negative amplitudes.

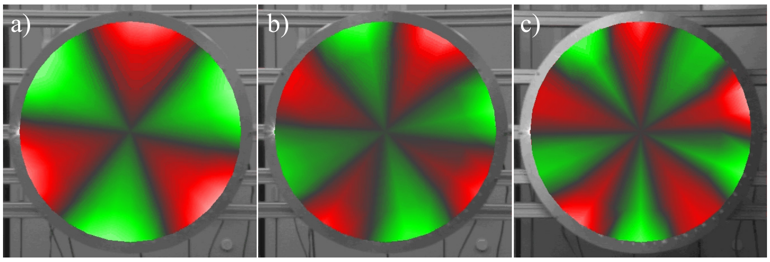

Figure 4: Visualization of the measured mode shapes of the 3-0 Mode a), 4-0 Mode b) and 5-0 Mode c).

CONCLUSIONS

With an excitation at two points at the outer radius of a centre clamped circular plate, the mode shapes of modes with nodal diameters can rotate. The rotation of the mode shape has been visualized with a scanning laser vibrometer.

References

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