SINGULARITY ANALYSIS ON STOCHASTIC P-BIFURCATIONS

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Summary Stochastic P-bifurcation is one of the focus problems in the field of the randomly excited nonlinear systems. Except the bifurcation with the variation of the peak number, another type of bifurcation is proposed here to study the case that the relative height of different peaks changes. The parametric conditions for both types of stochastic P-bifurcation are derived by applying the singularity method. Then the general expressions of these conditions are verified by an example, bistable Duffing-van der Pol equations under white noise excitation. These results are useful not only in finding the critical conditions for P-bifurcation but also in designing the controller for probability density function.

INTRODUCTION

Stochastic P-bifurcation has been an important topics in the field of randomly excited nonlinear systems. Bistable nonlinear system, like the Duffing–Van der Pol oscillator, has by many researchers[1,2]. Commonly the bifurcation condition was determined by finding the variation of the extrema number of the probability density function. The problem is that this method may fail when the extrema number is larger than 3. On the other hand, two peaks of the probability density curve may exchange their relative height with the parameters changing, which results in qualitatively change in power spectral density as shown in [2]. The problem is that how can we get the critical conditions for this case. To answer these two questions, we propose a classification for P-bifurcation and derive their condition by applying the singularity theory[3] in the following sections.

CLASSIFICATION AND PARAMETER CONDITIONS FOR P-BIFURCATION

Without loss of generality, one can assumes that the stationary probability density function (PDF) of a stochastically excited nonlinear system is in the following form

$$p(a) = P(a,\alpha)\exp(Q(a,\alpha)) \tag{1}$$

where a is the response amplitude and α is the parameter vector. When α varies, the probability density curve may change qualitatively, i.e. a P-bifurcation occurs. If the peak number changes, the bifurcation is denoted by P_H. If the relative height of different peaks exchanges, the bifurcation is named by P_{DL}. The former has been studied by many researchers, the latter is proposed here firstly. Figure 1 gives the typical probability density curves when a P-bifurcation happens due to the parameter's change. In both cases the middle diagrams represent the PDFs at the bifurcation points. About the critical conditions for these two types of P-bifurcation, we have the following theorem.

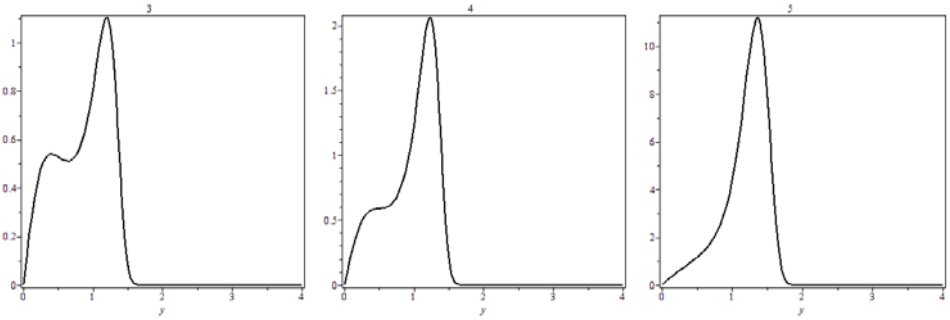
Therorem: P_H or P_{DL} bifurcation appears when the parameter α lies in the transition set

$$\alpha_H = \{\alpha \in R^m \,|\, P' + PQ' = 0, P'' + 2P'Q' + PQ'' + PQ'^2 = 0\} \tag{2}$$

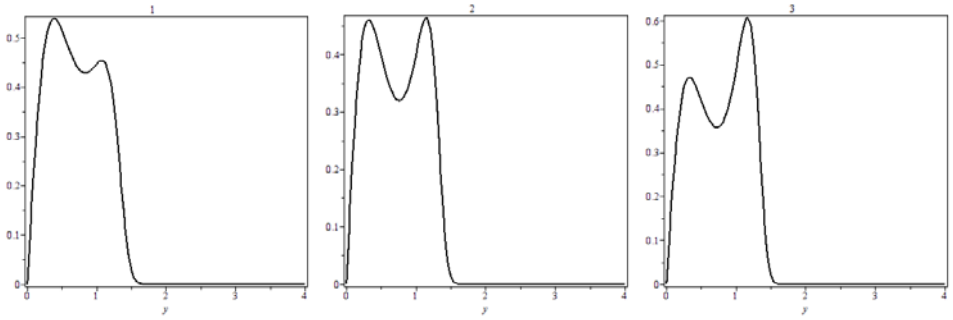
or

$$\alpha_{DL} = \{\alpha \in R^m \,|\, p = P(a,\alpha)\exp(Q(a,\alpha)), P' + PQ' = 0, a = \xi,\eta,\xi \neq \eta\} \tag{3}$$

respectively. Here $(\,)' = \dfrac{d(\,)}{da}$。



(a) Bifurcation with variation of peak number



(a) Bifurcation with variation of peak ratio

Figure1 Two types of P-bifurcation and their PDFs (a) P_H (b) P_{DL}

3. EXAMPLE FOR STOCHASTIC P-BIFURCATION

Consider the bistable Duffing-van der Pol equation under excitation of Gauss noise $n(t)$.

$$\ddot{x}-(\varepsilon+\alpha_1 x^2-\alpha_2 x^4)\dot{x}+x+x^3=n(t) \tag{4}$$

The transition sets for its P-bifurcations are shown in Figure 2(a). The black curve is same as given in [2]. The red curve is correct because it really detect the condition that two peaks exchange their relative height. That indicates conditions given in the Theorem are true.

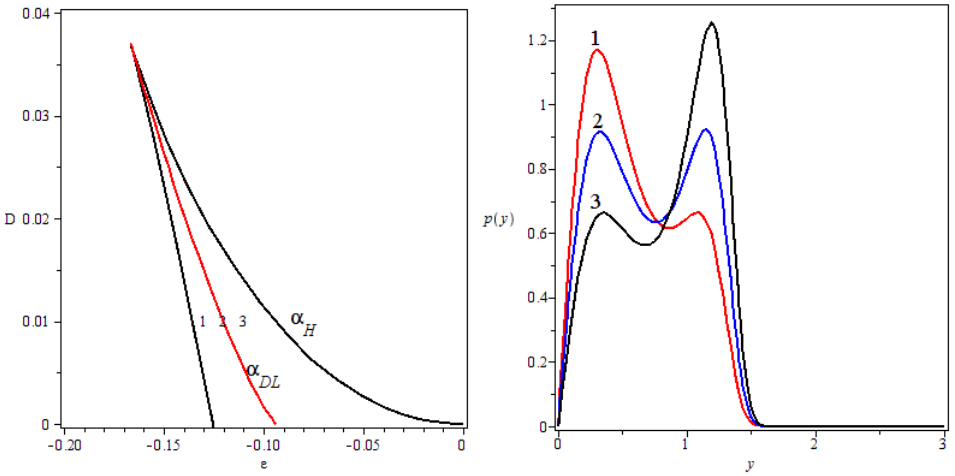


Figure 2 (a) Transition sets and (b) three-peak PDFs for D=0.01

CONCLUSIONS

Stochastic P-bifurcation can be divided into two groups. P_H bifurcation denotes the case that peak number of PDF curves varies due to the parameters' variation. P_{DL} bifurcation denotes the case that relative height of different peaks changes with the variation of the parameters. The general formulas of the critical parametric conditions for these two types of P-bifurcation are given and verified example.

References

[1] Golubitsky M. and Schaeffer D.G.: Singularities and groups in bifurcation theory (I). Springer, 1985

[2] Xu Y., et al.: Stochastic bifurcations in a bistable Duffing--Van der Pol oscillator with colored noise. Physical Review E 83(5):056215, 2011.

[3] Zakharova A., et al.: Stochastic bifurcations and coherencelike resonance in a self-sustained bistable noisy oscillator. Physical Review E 81(1):011106, 2010.