CHAOTIC THRESHOLDS FOR THE RECENTLY PROPOSED ROTATING PENDULUM SYSTEM

Ning Han*, Qingjie Cao*,**a) & Yushu Chen*
*School of Astronautics, Harbin Institute of Technology, Harbin 150001 China
**Department of Mathematics and Physics, Shijiazhuang Tiedao University, Shijiazhuang 050043 China

Summary We investigate the the nonlinear dynamics of the recently proposed rotating pendulum which exhibits both smooth and discontinuous dynamics depending on the value of a smoothness parameter. This is a strong irrational system coupled with triangular nonlinearity due to geometry configuration, which is characterized by both conventional pendulum and SD oscillator. We introduce a cylindrical approximate system which is topologically equivalent to the original system. This approximation provides a methodology to avoid the conventional barrier associated with the irrational nonlinearity. Melnikov method is employed to obtain the chaotic thresholds for the coexisted first-type and second-type homoclinic orbits under the perturbation of viscous damping and external harmonic forcing within smooth regime.

INTRODUCTION

This is a cylindrical system comprising an inverted pendulum linked by an oblique spring (capable of resisting both tension and compression). Even the stiffness of the spring is linear, the resistance force supplied to the system is strong irrational nonlinearity due to geometry configuration. If the system is perturbed by both viscous damping and external harmonic forcing, the dimensionless form of the system [1] can be written as the following.

$$\ddot{x} + \xi\dot{x} - g\sin x + \omega_0^2 h\sin x(1 - \frac{1}{\sqrt{1+h^2-2h\cos x}}) = f_0\cos\omega t, \tag{1}$$

which is very difficult to get the chaotic threshold analysis directly. In the following discussions we introduce the following system which bears significant similarities ($h > 1$) to the original system of double pitchfork bifurcation at $x = 0$ and $x = \pm\pi$.

$$\ddot{x} + \xi\dot{x} - \sin x(g - \omega_0^2 h\frac{h^2-h-1}{h^2-1} + \frac{\omega_0^2 h}{h^2-1}\cos x) = f_0\cos\omega t. \tag{2}$$

Letting $\alpha = g - \omega_0^2 h\frac{h^2-h-1}{h^2-1} < 0; \beta = \frac{\omega_0^2 h}{h^2-1} > 0; \alpha+\beta > 0$, the solution of homoclinic orbits can be obtained as the following. The first type homoclinic orbits connected with $(0,0)$ can be written as

$$(x_\pm^{hom1}(t), y_\pm^{hom1}(t)) = (\pm 2a\cot(\sqrt{\frac{-\alpha}{\alpha+\beta}}\cosh(\sqrt{\alpha+\beta}t)), \mp\frac{2\sqrt{-\alpha}\sinh(\sqrt{\alpha+\beta}t)}{1+\frac{-\alpha}{\alpha+\beta}\cosh^2(\sqrt{\alpha+\beta}t)});$$

The second-type homoclinic orbits crossing $(-\pi,0)$ and $(\pi,0)$ can be written as

$$(x_\pm^{hom2}(t), y_\pm^{hom2}(t)) = (\pm 2a\tan(\sqrt{\frac{-\alpha}{\beta-\alpha}}\sinh(\sqrt{\beta-\alpha}t)), \pm\frac{2\sqrt{-\alpha}\cosh(\sqrt{\beta-\alpha}t)}{1+\frac{-\alpha}{\beta-\alpha}\sinh^2(\sqrt{\beta-\alpha}t)}).$$

CHAOTIC THRESHOLD

In this section chaotic thresholds for the first and the second type Homoclinic orbits of system (1) can be obtained by using Melnikov method[2]. Numerical simulations are carried out to show the efficiency of proposed method, which clearly demonstrates the predicated chaotic attractors of pendulum-type, SD-type[3] and their mixture depending on the coupling of the nonlinearities. The Melnikov function of system (2) for the first-type ($hom1$) homoclinic orbits can be obtained as

$$M_1(\tau, f_0, \xi, \omega) = -\xi\int_{-\infty}^{+\infty}(y_\pm^{hom1}(t))^2dt + f_0\sin\omega\tau\int_{-\infty}^{+\infty}y_\pm^{hom1}(t)\sin\omega t dt = -\xi I_1^{hom1} + f_0\sin\omega\tau I_2^{hom1}. \tag{3}$$

It can be seen that $M_1(\tau, f_0, \xi, \omega) = 0$ has simple zero for τ if and only if the following inequality holds:

$$\frac{f_0}{\xi} > \left|\frac{\int_{-\infty}^{+\infty}(y_\pm^{hom1}(t))^2dt}{\int_{-\infty}^{+\infty}y_\pm^{hom1}(t)\sin\omega t dt}\right| = R_{hom1}(\omega), \tag{4}$$

where

$$I_1^{hom1} = 4\sqrt{\beta+\alpha} - \frac{-4\alpha}{\sqrt{\beta}}\ln(\sqrt{\frac{\beta}{-\alpha}} + \sqrt{\frac{\beta+\alpha}{-\alpha}}); I_2^{hom1} = 2\pi\sin(\frac{\omega}{\sqrt{\beta+\alpha}}\sinh^{-1}\sqrt{\frac{\beta+\alpha}{-\alpha}})sech\frac{\pi\omega}{2\sqrt{\beta+\alpha}},$$

a) Corresponding author. E-mail: qingjiecao@hotmail.com.

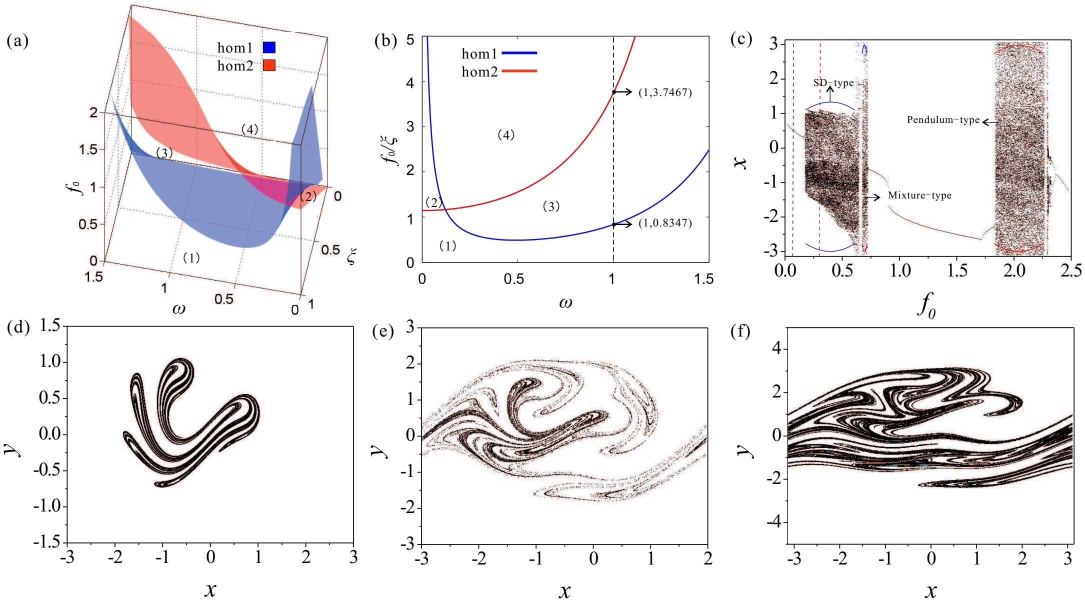

Figure 1. (a) and (b). Melnikov chaotic boundary for coexisted first and second type homoclinic orbits; (c). bifurcation diagram for f_0 versus x, $f_0 = 0.0588$ (blue line), $f_0 = 0.2623$ (red line); (d) SD-type chaotic attractor for $f_0 = 0.38$; (e) mixture-type chaotic attractor for $f_0 = 0.67$; (f) pendulum-type chaotic attractor for $f_0 = 2$.

The Melnikov function of system (2) for the second-type ($hom2$) homoclinic orbits can be obtained by the same method.

$$I_1^{hom2} = 4\sqrt{\beta-\alpha} + \frac{-4\alpha}{\sqrt{\beta}} \ln\left(\sqrt{\frac{\beta}{-\alpha}} + \sqrt{\frac{\beta-\alpha}{-\alpha}}\right); I_2^{hom2} = \pm 2\pi \cos\left(\frac{\omega}{\sqrt{\beta-\alpha}} \sinh^{-1}\sqrt{\frac{\beta}{-\alpha}}\right) sech \frac{\pi\omega}{2\sqrt{\beta-\alpha}}.$$

In order to further check the chaotic boundary, we take $\omega_0^2 = \frac{21+\sqrt{45}}{1+\sqrt{45}}$ and $h = \frac{1+\sqrt{45}}{2}$ as an example. The Melnikovian detected chaotic boundary for the coexisted the first and second type homoclinic orbits is plotted in Fig.1(a) and (b). The chaotic boundaries divide the plane of parameters into four regions marked (1), (2), (3) and (4). The chaos do not exist in region (1); If the chaos exists in this system, the other regions (2), (3) and (4) exhibit different type chaotic attractors. Numerical simulations are carried out to verify the efficiency of the chaotic criteria for $\xi = 0.07, \omega = 1$ taken fixed in this section. Bifurcation diagram of system (1) for f_0 versus x is plotted in Fig.1(c). When f_0 increases beyond above threshold value (blue) reaching $f_0 = 0.24$, system (1) jump to the area of chaotic motion which bears the SD-type attractor as shown in Fig.1(d). It can also be seen clearly from the bifurcation diagram, Fig.1(e), that there exists a mixture-type attractor when f_0 takes values between 0.66 and 0.68. Fig.1(f) plot a pendulum-type attractor for $f_0 = 2$, respectively.

CONCLUSIONS

The chaotic boundary of this cylindrical system have been presented by means of a novel analytical method which successfully avoids the barrier of the associated irrational nonlinearity. The efficiency of the proposed method have been checked by using numerical simulations, which clearly demonstrates the predicated chaotic attractors of pendulum-type, SD-type and their mixture depending on the coupling of the nonlinearities. The future study on the complicated nonlinear dynamics of this cylindrical pendulum is being carried out by the current authors in two aspects: the first is the chaotic behaviours of discontinuous regime and the second is the Hopf bifurcations.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the financial support from the Natural Science Foundation of China (Grant No. 11072065 and 10872136).

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