THE PROBABILISTIC SOLUTION OF NONLINEAR EULER-BERNOULLI BEAM EXCITED BY POISSON WHITE NOISE

Guo-Kang Er[a)]

Faculty of Science and Technology, University of Macau, Macau SAR, P.R.China

Summary With Gallerkin's method, the stretched Euler-Bernoulli beam with its two ends pin-supported is described as multi-degree-of-freedom system. The probabilistic solution of the MDOF nonlinear stochastic systems excited by Poisson white noise is investigated with the state-space-split method and exponential-polynomial closure method proposed by the author. Numerical results show that the results obtained with the state-space-split method and exponential-polynomial closure method agree well with the simulation for the stretched Euler-Bernoulli beam excited by uniformly distributed Poisson white noise.

INTRODUCTION

The investigation on the probabilistic solutions of large or multi-degree-of-freedom (MDOF) nonlinear stochastic dynamic (NSD) systems was attractive in the last decades because other statistic or reliability analysis is based on the probabilistic solutions of the system responses. Excited by Poisson white noise, the probability density function (PDF) of the responses of NSD system is governed by generalized Fokker-Planck-Kolmogorov (GFPK) equation which exact solution is not obtainable for MDOF NSD systems. Therefore, approximate methods were utilized for solving the GFPK equation or the first and second moments of the system responses. Equivalent linearization method (EQL) was frequently employed to obtain the approximate mean and second moment of the responses of weakly NSD systems [1, 2]. The advantage of the EQL method is that it is applicable for MDOF systems. Monte Carlo simulation is another method that is applicable for the numerical solution of MDOF NSD systems, but the computational efficiency, numerical stability, convergence, round-off error, and requirement for large sample size can be challenges with Monte Carlo simulation in analyzing large systems [3, 4]. Recently, a new methodology named state-space-split (3S) method was proposed for the approximate solutions of some reduced Fokker-Planck-Kolmogorov equations [5, 6]. With the 3S method, the large NSD system can be reduced to some small systems which probabilistic solutions can be further analyzed with the exponential polynomial closure (EPC) method [7-9]. The whole solution procedure is named 3S-EPC method. In this paper, the 3S-EPC method is extended to analyze the large NSD systems excited by Poisson white noise. The probabilistic solutions of the stretched nonlinear Euler-Bernoulli beam with hinge supports at its two ends are analyzed when uniformly distributed force is Poisson white noise and applied on the top of the beam to show the effectiveness of the 3S-EPC method in this case.

PROBABILISTIC SOLUTIONS OF STRETCHED EULER-BERNOULLI BEAM

Consider the Euler-Bernoulli beam with hinge supports at its two ends and excited by uniformly distributed load being Poisson white noise as shown in Figure 1. The governing equation of this beam is

$$\rho\ddot{Y}(x,t) + \dot{Y}(x,t) + EIY^{(4)}(x,t) - \frac{EA}{2L}Y''(x,t)\int_0^L Y'^2(x,t)dx = W(t) \tag{1}$$

where $Y(x,t)$ is the deflection of the beam at time t at the location with distance x from the left-hand side of the beam; ρ is the mass density of the material; c is the damping constant; E is the Young's modulus of the beam material; I is the moment inertia of the cross section of the beam; A is the area of the cross section of the beam; L is the length of the beam; $W(t)$ is a zero-mean Poisson white noise with mean pulse arrival rate λ and the mean square of pulse amplitude $E[Z^2]$. In order to analyze the beam with Gallerkin's method, $Y(x,t)$ is expressed as

$$Y(x,t) = \sum_{i=1}^{s} a_i(t)\sin\frac{(2i-1)\pi x}{L} \tag{2}$$

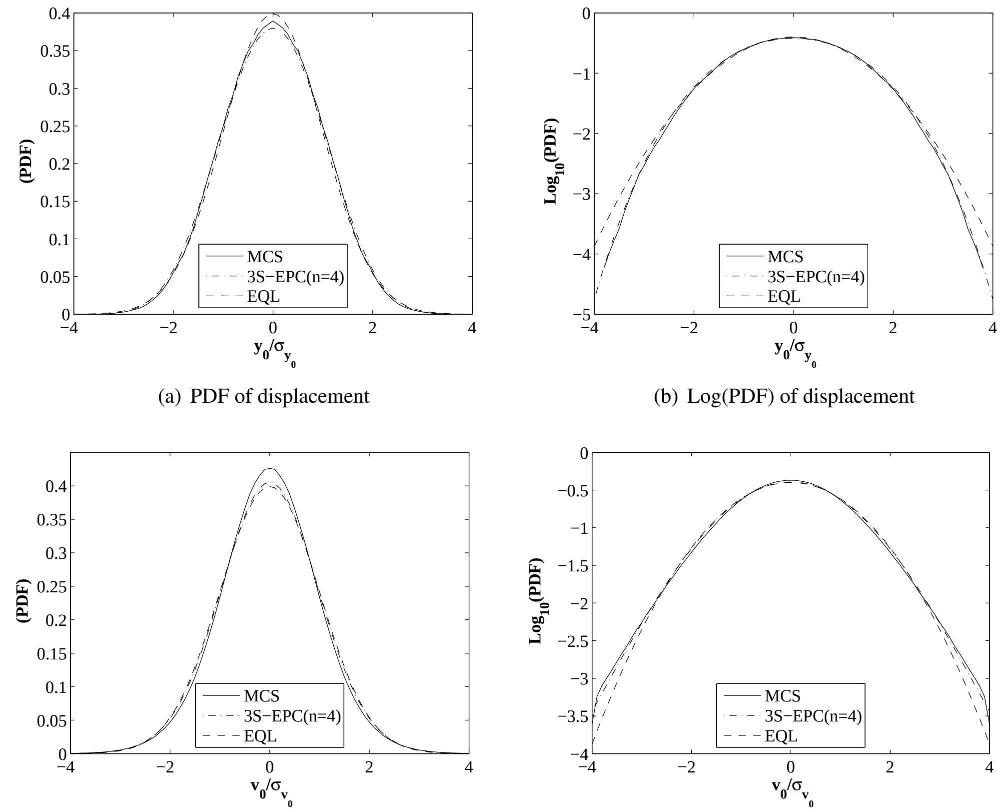
with which the following MDOF NSD system can be obtained.

$$\ddot{a}_i(t) + \alpha\dot{a}_i(t) + \beta a_i(t) + \gamma(2i-1)^2 a_i(t)a_j^2(t) = f_i W(t) \qquad i = 1, 2, \cdots, s \tag{3}$$

where $\alpha = c/\rho$; $\beta = EI\pi^4/(\rho L^4)$; $\gamma = EA\pi^4/(4\rho L^4)$; and $f_i = 4/[(2i-1)\pi\rho$.

Based on Eqs. (1) and (2), the PDFs of the deflection $Y_0 = Y(0.5L, t)$ and velocity $\dot{Y}_0 = \dot{Y}(0.5L, t)$ can be analyzed with 3S-EPC method when polynomial degree n equals 4 in the EPC procedure. Given $L = 5m$, $E = 2.1 \times 10^{11}pa$, $I = 2.17 \times 10^{-4}m^4$, $A = 8.6 \times 10^{-3}m^2$, $\rho = 7.85 \times 10^3 kg/m^3$, $c = 10^3$, $\lambda = 0.1$, $E[Z^2] = 2 \times 10^4$, and $s = 4$, the PDFs and logarithmic PDFs of Y_0 and $\dot{Y}_0$ obtained with 3S-EPC, Monte Carlo simulation, and equivalent linearization are shown and compared in Figs. (a)-(d). The number of samples used in simulation is 10^8. It is seen from the figures that the results obtained with 3S-EPC are close to Monte Carlo simulation while those obtained with equivalent linearization deviate a lot from Monte Carlo simulation.

[a)] Corresponding author. E-mail: gker@umac.mo



(a) PDF of displacement

(b) Log(PDF) of displacement

(c) PDF of velocity

(d) Log(PDF) of velocity

CONCLUSIONS

The 3S-EPC method is extended to analyze the probability density functions of the responses of large nonlinear stochastic dynamic systems with polynomial type of nonlinearity when the excitations are Poisson white noise. Numerical analysis is conducted for the probabilistic solutions of the stretched Euler-Bernoulli with hinge supports at its two ends and excited by Poisson white noise uniformly applied on the top of the beam. The random nonlinear partial differential equation of the stretched Euler-Bernoulli is reduced to multi-dimensional NSD systems with Gallerkin's method. From numerical analysis it is seen that the probability density functions of the deflection and velocity of the beam are close to Monte Carlo simulation even in the tails of the PDF solution. It can be concluded that the 3S-EPC method is also applicable for analyzing the probabilistic solutions of the responses of large NSD systems excited by Poisson white noise in the presented case.

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