EFFECT AND COMPENSATION OF TIME DELAY ON SEMIACTIVELY CONTROLLED CABLE

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Summary The parametrically excited instability of the cable vibration is noticeable affected by increasing the time delay in control over a certain value, which effects need to be taken into account for the cable instability control. And the time delay compensation is treated by expresses the delayed control action using a truncated Taylor's series in terms of the current control action and its derivatives. The results of the stability and the time delay compensation are discussed using a numerical example.

INTRODUCTION

In order to increase the critical wind speed to avoid galloping of flexible cable, it is necessary to introduce additional damping. One of the feasible methods of introducing additional damping to cable is suing active TMD control mechanisms [1-3]. The time delay in generating the active control action due to the actuator's dynamics is, however, a major practical problem which may destabilize the active controlled structure [4,5]. The present paper focuses mainly on the semi-active optimal control of the parametrically excited instability of a stay cable installed transversely with an MR damper. The transformations of the displacements are used for deriving the partial differential equation of the parametrically excited transverse vibration of the controlled inclined stay cable with sag under support motions.

EQUATION OF MOTION

The equation of transverse motion of the controlled cable is derived as follows[3]

$$\ddot{w} + \frac{c}{m}\dot{w} - \frac{k_1 + k_2(t)}{m}\frac{\partial^2 w}{\partial z^2} + \frac{k_3}{m}\int_0^1 w dz = f(z,t) + f_c \tag{1}$$

with boundary conditions: $w = 0, z = 0; w = 0, z = 1$, where w is the nondimensional transverse displacements of

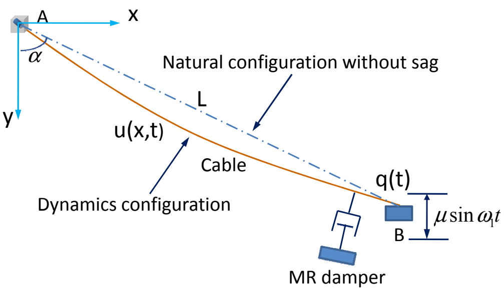

Figure 1. Cable- structure system.

the cable, z is the nondimensional vertical coordinate of cable, m is the mass per unit of cable length, c is the damping coefficients, $f(z,t)$ is the excitation under support motion and f_c is the control force in the transverse cable direction produced by the MR damper, $k_1 = T_x/(L^2 sin\alpha), k_2(t) = EAw_B(t)\cos\alpha/L^2, k_3 = EAm^2g^2/T_x^2$, T_x is the horizontal cable tension in static equilibrium, L is the cable length, α is the angle between the vertical axis and the axis from A to B, E is the elastic modulus, A is the cross-sectional area of the cable.

Discrete model

In here, according to the boundary conditions, an evolution solution of nondimensional displacement w to Eq.(1) can be expressed as $w(z,t) = \sum_{i=1}^{\infty}\phi_i(z)q_i(t)$ where $q_i(t)$ is the i-th modal displacement, and $\phi_i(z) = \sin(i\pi z)$. Application of the Galerkin method yield the following nonlinear ordinary differential equation describing the cable mode vibration

$$\ddot{q}_i + \frac{c}{m}\dot{q}_i + \frac{k_1 + k_2(t)}{m}(i\pi)^2 q_i + \sum_{k=1}^{\infty}\frac{2k_3}{m}\frac{[1-(-1)^i][1-(-1)^k]}{ik\pi^2}q_k = f_i(z,t) + f_{ci}, \quad i = 1, 2, \cdots, \infty \tag{2}$$

where $f_i(z,t) = \int_0^1 2f(t)\sin(i\pi z)dz$, $f_{ci} = \int_0^1 2f_c\sin(i\pi z)dz$.

Model of semiactive control with time delay

In practice, only one MR damper can be installed transversely on the cable near the lower support B. The control force f_c in Eq.(1) of the MR damper with time delay is expressed as $f_c = (-k_a w(z_c, t-\tau) - c_a\dot{w}(z_c, t-\tau))\delta(z - z_c)$. Then

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the system can rewritten as the following form (consider the first order)

$$\ddot{q} + \frac{c}{m}\dot{q} + kq = 4f(t) - k_a \sin(\pi z) q(t-\tau) - c_a \sin(\pi z)\dot{q}(t-\tau) \tag{3}$$

where $k = [\frac{T_x}{mL^2 \sin\alpha} + \frac{EA}{mL^2}\mu\cos\alpha\sin(\omega_1 t)]\pi^2 + \frac{8EAmg^2}{\pi^2 T_x^2}$ and $f(t) = \{cL\mu\omega\cos(\omega_1 t)\cos\alpha - L[c\mu\omega\cos(\omega_1 t) - m\mu\omega^2\sin(\omega_1 t)]\sin\alpha - mg\mu\sin(\omega_1 t)\cot\alpha + \frac{mgEA}{T_x}\mu\sin(\omega_1 t)\cos\alpha\}/mL.$

STABILITY DUE TO THE TIME DELAY AND COMPENSATION

By characteristic equation analysis for this Eq.(3) we can obtain the stability due to the time delay effect.

$$\sin(\omega\tau)(\omega^2 - k) + \frac{c\omega}{m}\cos(\omega\tau) - c_a\sin(\pi z)\omega = 0 \tag{4}$$

An inclined stay cable in a cable-stayed bridge is considered as an example. The periodic motion of deck or support is represented by $w_B(t) = \mu\sin(\omega_1 t)$(μ and ω_1 are respectively the amplitude and frequency of the nondimensional vertical support displacement) so that the stay cable is subjected to the parameter excitation of period $T = 2\pi/\omega_1$. A parameter space, for instance, the plane (τ, ω) of support-motion frequency and amplitude can be divided into stable regions and unstable regions. The unstable regions consisting of unstable points describe the overall instability of the cable. The parameter values are $L = 129.2$m, $A = 71.97 \times 10^{-4}$m^2, $\alpha = 0.984$rad, $m = 58.9$kg/m, $c = 0.002$, $E = 2.0 \times 10^5$Mpa, $T_x = 3300\sin\alpha$kN, $\mu = 0.3 \times 10^{-4}$ to $\mu = 8 \times 10^{-4}$ and $\omega_1 = 1.83$Hz for the cable structure, and $z_c = 0.9$, $c_a = 0.001$, $k_a = 0.002$ for the MR damper. The numerical results on the parametrically excited instability of the semi-actively controlled cable are given in Fig.2.

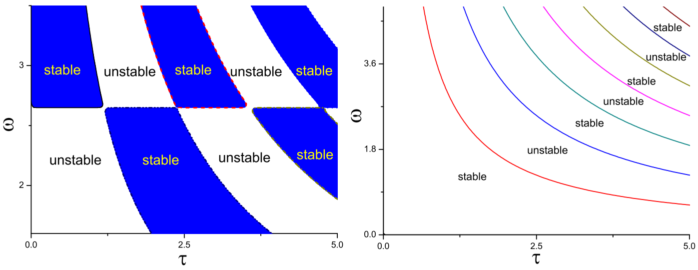

Figure 2. The stable region with increasing amplitude of the periodic support motion.

In order to determine the displacement feedback gain k_a and the velocity feedback gain c_a to the compensate for the time delay existing in the applied control action $w(t-\tau)$ and $\dot{w}(t-\tau)$ of equation (3), the delayed control action can be expressed as a series of the current control action and its derivatives according to Taylor's series as follows: $w(t-\tau) = w(t) - \tau\dot{w}(t) + \frac{\tau^2}{2}\ddot{w}(t) - \frac{\tau^3}{6}\dddot{w}(t) + \cdots$. In this paper, for simplicity, the pole assignment method is used. The design objective is to determine that control law which takes the time delay into account, and gives acceptable controlled response for the cable.

CONCLUSION AND ACKNOWLEDGES

The stability of controlled inclined stay cables under support motions is studied through the cable structure and the time delay effect. And the time delay compensation is treated by expresses the delayed control action using a truncated Taylor's series in terms of the current control action and its derivatives. The controlled response considering the delay is obtained by applying the designed active control on the actual nonlinear model. The results of the stability and the time delay compensation are discussed using a numerical example. The study was supported in the part by the National Natural Science Foundation of China (Grant No. 11032004 and 10972073) and NCET-09-0335.

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