ANALYSIS OF NON-LINEAR VIBRATIONS OF A VISCOELASTIC CYLINDRICAL SHELL UNDER THE CONDITIONS OF INTERNAL RESONANCE

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Summary Non-linear damped vibrations of a cylindrical shell subjected to the different conditions of the internal resonance are investigated. Its viscous properties are described by Riemann-Liouville fractional derivative. The problem is solved via the method of multiple scales. The influence of viscosity on the energy exchange mechanism is analyzed. It is shown that each mode is characterized by its damping coefficient connected with the natural frequency by the exponential relationship with a negative fractional exponent.

INTRODUCTION

In recent years much attention is given to damping features of mechanical systems subjected to the conditions of different internal resonances. Damping properties of non-linear systems are described mainly by the first-order time-derivative of a generalized displacement [1]. However, as it has been shown by Rossikhin and Shitikova [2], who analyzed free damped vibrations of suspension combined system under the conditions of the one-to-one internal resonance, for good fit of the theoretical investigations with the experimental results it is better to describe the damping features of non-linear mechanical systems in terms of fractional time-derivatives of the generalized displacements [3]. The analysis of non-linear vibrations of a two-degree-of-freedom mechanical system, the damping features of which are described by a fractional derivative, has shown [4] that in the case when the system is under the conditions of the two-to-one or one-to-one internal resonance, viscosity may have a twofold effect on the system: a destabilizing influence producing unsteady energy exchange, and a stabilizing influence resulting in damping of the energy exchange mechanism.

In the present paper, non-linear free damped vibrations of a thin cylindrical viscoelastic shell, the damping properties of which are described by the Riemann-Liouville fractional derivatives, are investigated. The dynamic behaviour of the shell is described by a set of three non-linear differential equations. The interaction of modes being in the two-to-one, one-to-one internal resonance, and the internal additive or difference combinational resonances are analyzed.

PROBLEM FORMULATION AND GOVERNING EQUATIONS

Let us consider the dynamic behaviour of a free supported non-linear viscoelastic circular cylindrical shell of radius R and length l, vibrations of which in the cylindrical system of coordinates is described by the following three differential equations written in the dimensionless form:

$$u_{xx} + \frac{1-\nu}{2}\,\beta_1^2 u_{\varphi\varphi} + \frac{1+\nu}{2}\,\beta_1 v_{x\varphi} - \nu\beta_1 w_x + w_x\left(w_{xx} + \frac{1-\nu}{2}\,\beta_1^2 w_{\varphi\varphi}\right) + \frac{1+\nu}{2}\,\beta_1^2 w_\varphi w_{x\varphi} = \ddot{u} + \mu D^\gamma u, \quad (1)$$

$$\beta_1^2 v_{\varphi\varphi} + \frac{1-\nu}{2}\,v_{xx} + \frac{1+\nu}{2}\,\beta_1 u_{x\varphi} - \beta_1^2 w_\varphi + \beta_1 w_\varphi\left(\beta_1^2 w_{\varphi\varphi} + \frac{1-\nu}{2}\,w_{xx}\right) + \frac{1+\nu}{2}\,\beta_1 w_x w_{x\varphi} = \ddot{v} + \mu D^\gamma v, \quad (2)$$

$$\begin{aligned} &\frac{\beta_2^2}{12}\left(w_{xxxx} + 2\beta_1^2 w_{xx\varphi\varphi} + \beta_1^4 w_{\varphi\varphi\varphi\varphi}\right) - \nu\beta_1 u_x - \beta_1^2 v_\varphi + \beta_1^2 w + \frac{1}{2}\,\nu\beta_1 (w_x)^2 + \frac{1}{2}\,\beta_1^3 (w_\varphi)^2 \\ &- w_x\left(u_{xx} + \frac{1-\nu}{2}\,\beta_1^2 u_{\varphi\varphi} + \frac{1+\nu}{2}\,\beta_1 v_{x\varphi}\right) - \beta_1 w_\varphi\left(\beta_1^2 v_{\varphi\varphi} + \frac{1-\nu}{2}\,v_{xx} + \frac{1+\nu}{2}\,\beta_1 u_{x\varphi}\right) \\ &- w_{xx}(u_x + \nu\beta_1 v_\varphi - \nu\beta_1 w) - \beta_1^2 w_{\varphi\varphi}(\nu u_x + \beta_1 v_\varphi - \beta_1 w) - (1-\nu)\beta_1 w_{x\varphi}(\beta_1 u_\varphi + v_x) = -\ddot{w} - \mu D^\gamma w, \end{aligned} \quad (3)$$

subjected to the initial

$$u|_{t=0} = \dot{u}\Big|_{t=0} = 0, \quad v|_{t=0} = \dot{v}\Big|_{t=0} = 0, \quad w|_{t=0} = \dot{w}\Big|_{t=0} = 0, \quad (4)$$

and boundary conditions

$$w|_{x=0} = w\Big|_{x=1} = 0, \quad v|_{x=0} = v\Big|_{x=1} = 0, \quad u_x\Big|_{x=0} = u_x\Big|_{x=1} = 0, \quad w_{xx}\Big|_{x=0} = w_{xx}\Big|_{x=1} = 0, \quad (5)$$

where the $x-$axis is directed along the axis of the cylinder, φ is the polar angle in the plane perpendicular to the $x-$axis, $u = u(x,\varphi,t)$, $v = v(x,\varphi,t)$, and $w = w(x,\varphi,t)$ are the displacements of points located in the shell's middle surface in three mutually orthogonal directions x, φ, r, r is the polar radius, $\beta_1 = l/R$ and $\beta_2 = h/l$ are the parameters defining the dimensions of the shell, h is the thickness, ν is the Poisson ratio, overdots denote time-derivatives, μ is the damping coefficient, D^γ is Riemann-Liouville fractional derivative of the γ-order [3], and lower indices label the derivatives with respect to the corresponding coordinates.

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METHOD OF SOLUTION

Since the set of Eqs. (1)–(3) admits the solution of the Navier type, then the displacements could be represented as

$$u=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}x_{1\,mn}(t)\eta_{1\,mn}(x,\varphi),\ v=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}x_{2\,mn}(t)\eta_{2\,mn}(x,\varphi),\ w=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}x_{3\,mn}(t)\eta_{3\,mn}(x,\varphi), \tag{6}$$

where m and n are integers, $x_{i\,mn}(t)$ are the generalized displacements, and $\eta_{i\,mn}(x,\varphi)$ $(i=1,2,3)$ are the eigenfunctions satisfying the boundary conditions (5).

Substituting (6) into Eqs. (1)–(3), multiplying (1), (2), and (3) by η_{1lk}, η_{2lk}, and η_{3lk}, respectively, integrating over x and φ, and using the orthogonality conditions for linear modes within the domains of $0\le x\le 1$ and $0\le\varphi\le 2\pi$, we are led to a coupled set of nonlinear ordinary differential equations of the second order in $x_{i\,mn}$ $(i=1,2,3)$:

$$\ddot{x}_{i\,mn}+\mu D^{\gamma}x_{i\,mn}+S_{ij}^{mn}x_{j\,mn}=-F_{i\,mn}\quad (i,j=1,2,3), \tag{7}$$

where the summation is carried out over two repeated indices, $F_{i\,mn}$ are nonlinear parts, and S_{ij}^{mn} is a symmetric matrix possessing three real eigenvalues $\Omega_{i\,mn}$ $(i=1,2,3)$.

Assuming that the vibration process occurs in such a way that only three natural modes are excited and dominate over other natural modes, omitting the indices mn, expanding the generalized displacements x_i and the matrix S_{ij} in terms of its eigenvectors, we could utilize Eqs. (7) for investigating free damped vibrations on nonlinear cylindrical shells subjected to the following types of the internal resonance:

(1) the two-to-one internal resonance, when one natural frequency is twice the other natural frequency,

- $\Omega_1=2\Omega_2$, or $\Omega_2=2\Omega_1$, • $\Omega_1=2\Omega_3$, or $\Omega_3=2\Omega_1$, • $\Omega_2=2\Omega_3$, or $\Omega_3=2\Omega_2$;

(2) the combinational resonance of the additive-difference type, i.e.,

- $\Omega_1=\Omega_2+\Omega_3$, or $\Omega_2=\Omega_1-\Omega_3$, or $\Omega_3=\Omega_1-\Omega_2$,
- $\Omega_1=\Omega_2-\Omega_3$, or $\Omega_2=\Omega_1+\Omega_3$, or $\Omega_3=\Omega_2-\Omega_1$,
- $\Omega_1=\Omega_3-\Omega_2$, or $\Omega_2=\Omega_3-\Omega_1$, or $\Omega_3=\Omega_1+\Omega_2$.

Applying further the standard procedure of the method of multiple scales [5] for eliminating secular terms and representing the amplitude functions in their polar form $A_k=a_k\exp(i\varphi_k)$ $(k=1,2,3)$, it is possible to obtain a set of six nonlinear equations in amplitudes a_k and phases φ_k. It is interesting to note that in the case of the two-to-one internal resonance, this set of equations for the cylindrical shell is practically the same as was previously investigated for this type of the internal resonance in a two-degree-of-freedom mechanical system [4].

As for the combinational internal resonance which could occur via utilizing the second-order uniform expansion, then equations for the cylindrical shell differ significantly from those found in [6] when analyzing the combinational resonances of the additive or difference type occurring in the nonlinear plate after applying the third-order uniform expansion.

The analysis of vibrations of such cylindrical shell being under the different conditions of the internal resonance shows that depending on the order of the fractional derivative γ $(0<\gamma<1)$ there may occur three vibrational regimes: (1) steady-state, (2) quasi-stationary, and (3) transient.

CONCLUSIONS

It is shown that the phenomenon of internal resonance could be very critical for circular cylindrical shells, the motion of which is described by three coupled nonlinear differential equations, since the internal resonance of the type two-to-one, as well as additive and difference combinational resonances are always present.

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