Responses of axially moving viscoelastic beam under narrow-band random excitation

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Summary: We investigate the responses of axially moving viscoelastic beam subject to narrow-band random excitation. The method of multiple scales is used to derive the analytical expression of first-order uniform expansion of the solution. We obtain the first-order and the second-order steady-state moments for the non-trivial steady-state solutions. Specially, we discuss the first mode theoretically and numerically. Results show that under certain conditions, as the intensity of the random excitation increases, non-trivial steady-state solution fluctuation will become strenuous, which will result in the non-trivial steady-state solution lose stability and the trivial steady state solution can be a possible.

THE FIRST APPROXIMATE SOLUTION

The equation of axially moving viscoelastic beam to narrow-band random excitation is given as

$$\frac{\partial^2 u}{\partial t^2}+2\gamma_0\frac{\partial^2 u}{\partial t\partial x}+\left(\gamma_0^2+1\right)\frac{\partial^2 u}{\partial x^2}+k_f^2\frac{\partial^4 u}{\partial x^4}=\frac{3}{2}\varepsilon k_1^2\frac{\partial^2 u}{\partial x^2}\left(\frac{\partial u}{\partial x}\right)^2+\varepsilon C\frac{\partial u}{\partial t}-\varepsilon\alpha^2\frac{\partial^5 u}{\partial t\partial x^4}+\varepsilon huf\left(t\right),\tag{1}$$

where $u\left(x,t\right)$ describes the transverse displacement and $f\left(t\right)$ is a narrow-band random excitation which can be described as $f\left(t\right)=\cos\left(\varphi(t)\right)$, $\varphi(t)=\Omega t+\beta W\left(t\right)+\Theta$, $\varphi(t)=\Omega+\beta\xi\left(t\right)$.

We introduce the transformation $M=I$, $G=2\gamma_0\,\partial/\partial x$, $K=\left(\gamma_0^2+1\right)\partial^2/\partial x^2+k_f^2\,\partial^4/\partial x^4$. So Eq. (1) can be written as

$$M\frac{\partial^2 u}{\partial t^2}+G\frac{\partial u}{\partial t}+Ku=\varepsilon C\frac{\partial u}{\partial t}+\frac{3}{2}\varepsilon k_1^2\frac{\partial^2 u}{\partial x^2}\left(\frac{\partial u}{\partial x}\right)^2-\varepsilon\alpha^2\frac{\partial^5 u}{\partial t\partial x^4}+\varepsilon hu\cos\left(\Omega t+\beta W(t)+\Theta\right).\tag{2}$$

In the method of multiple scales, the uniformly approximate solution of Eq. (2) is sought in the form

$$u\left(x,t,\varepsilon\right)=u_0\left(x,T_0,T_1\right)+\varepsilon u_1\left(x,T_0,T_1\right)+...,\tag{3}$$

where $T_0=t$ is a fast time scale and $T_1=\varepsilon t$ is a slow time scale. By denoting the differential operators $D_0=\partial/\partial T_0$ and $D_1=\partial/\partial T_1$, we get

$$\partial/\partial t=D_0+\varepsilon D_1+...,\quad \partial^2/\partial t^2=D_0^2+2\varepsilon D_0D_1+....\tag{4}$$

Substituting Eqs. (3) and (4) into Eq. (2) and comparing coefficients of ε with equal powers, we obtain equations:

$$MD_0^2u_0+GD_0u_0+Ku_0=0,\tag{5}$$

$$MD_0^2u_1+GD_0u_1+Ku_1=C\frac{\partial u_0}{\partial T_0}+\frac{3}{2}k_1^2\frac{\partial^2 u_0}{\partial x^2}\left(\frac{\partial u_0}{\partial x}\right)^2-\alpha^2\frac{\partial^5 u_0}{\partial T_0\partial x^4}+hu\cos\left(\Omega T_0+\beta W(T_1)+\Theta\right).\tag{6}$$

In this paper, the simple support conditions are given as $u\left(0,t\right)=u\left(1,t\right)=0$, $\partial^2u/\partial x^2\big|_{x=0}=\partial^2u/\partial x^2\big|_{x=1}=0.$

When we merely consider the case that the nth mode is directly excited with the assumption that there are no interactions among the modes. The general solution of Eq. (5) consists of one mode corresponding to the frequency:

$$u_0=\phi_n\left(x\right)A_n\left(T_1\right)e^{i\omega_nT_0}+cc\tag{7}$$

In the case of the principal parametric resonance (Namely: $\Omega=2\omega_n+\varepsilon\sigma$), according to [1], in order to eliminate the secular producing terms, it is required that a_n and θ_n vary in the slow time scale according to

$$\begin{aligned}\dot{a}&=-\left(C\chi_{nr}+\alpha^2\delta_{nr}\right)a_n-k_1^2\varsigma_{nr}a_n^3-\frac{h}{2}a_n\sqrt{\mu_{nr}^2+\mu_{ni}^2}\cos\left(\theta_n\right),\\ \dot{\theta}&=\left(\sigma+2C\chi_{ni}+2\alpha^2\delta_{ni}\right)a_n+2k_1^2\varsigma_{ni}a_n^3-ha_n\sqrt{\mu_{nr}^2+\mu_{ni}^2}\sin\left(\theta_n\right)+\beta a_nW'\left(T_1\right),\end{aligned}\tag{8}$$

where χ_{nr}, δ_{nr}, ς_{nr}, μ_{nr} represent the real parts, and χ_{ni}, δ_{ni}, ς_{ni}, μ_{ni} the imaginary parts.

Solving the Eqs. (8) for the amplitude a_n and the phase θ_n, the first-order uniform expansion of the solution of Eq. (2) is defined as

$$u_0=\phi_n\left(x\right)a_n\left(T_1\right)\exp\left[i\frac{\left(\Omega t-\theta_n\left(\varepsilon t\right)\right)}{2}\right]+\bar{\phi}_n\left(x\right)a_n\left(T_1\right)\exp\left[-i\frac{\left(\Omega t-\theta_n\left(\varepsilon t\right)\right)}{2}\right]+O\left(\varepsilon\right).\tag{9}$$

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NON-TRIVIAL STEADY-STATE SOLUTION AND THEIR STABILITY

This part focus on the case of non-trivial steady-state solution of Eqs. (8). When $\beta = 0$, the following frequency–amplitude equation for Eqs. (8) can be obtained

$$\begin{aligned} &4k_1^4\left(\varsigma_{nr}^2+\varsigma_{ni}^2\right)a_{n0}^4+\left[4k_1^2\sigma\varsigma_{ni}+8k_1^2\left(C\varsigma_{nr}\chi_{nr}+\alpha^2\varsigma_{nr}\delta_{nr}+C\varsigma_{ni}\chi_{ni}+\alpha^2\varsigma_{ni}\delta_{ni}\right)\right]a_{n0}^2 \\ &\qquad +4\left(C\chi_{nr}+\alpha^2\delta_{nr}\right)^2+\left(\sigma+2C\chi_{nr}+2\alpha^2\delta_{nr}\right)^2-h^2\left(\mu_{nr}^2+\mu_{ni}^2\right)=0. \end{aligned} \tag{10}$$

Next, we determine the effects of the noise ($\beta \neq 0$). Thus, the solution of Eqs. (8) can be defined as $a_n = a_{n0} + \tilde{a}_n$, $\theta_n = \theta_{n0} + \tilde{\theta}_n$. Substituting a_n and θ_n into Eqs. (8) and neglecting the nonlinear terms, we obtain

$$\begin{aligned} &d\tilde{a}_n=-\tilde{a}_n\left(C\chi_{nr}+\alpha^2\delta_{nr}+3k_1^2a_{n0}^2\varsigma_{nr}-\frac{h}{2}\sqrt{\mu_{nr}^2+\mu_{ni}^2}\cos\theta_{n0}\right)dT_1-\frac{h}{2}a_{n0}\sqrt{\mu_{nr}^2+\mu_{ni}^2}\sin\theta_{n0}\tilde{\theta}_n dT_1, \\ &d\tilde{\theta}_n=4k_1^2\varsigma_{ni}a_{no}\tilde{a}_n dT_1-h\sqrt{\mu_{nr}^2+\mu_{ni}^2}\cos\theta_{n0}\tilde{\theta}_n dT_1+\beta dW\left(T_1\right). \end{aligned} \tag{11}$$

Then, the values of Ea_n^2, $E\left(a_n\theta_n\right)$ and $E\theta_n^2$ can be obtained. So,

$$\begin{aligned} &Ea_n=E\left(a_{n0}+\tilde{a}_n\right)=a_{n0}, \\ &Ea_n^2=a_{n0}^2+E\left(\tilde{a}_n^2\right). \end{aligned} \tag{12}$$

Fig. 1 shows the theoretical frequency-amplitude response on Eqs. (12) and the numerical simulation result of Eqs. (8), which verify the validity of the analyses method mentioned above. From Eqs. (12), we find that under the narrow-band noise excitation, the non-trivial steady-state solution fluctuations will become strenuous with the noise intensity β becomes bigger. That is, the random noise will affect the non-trivial steady-state solution at fixed points.

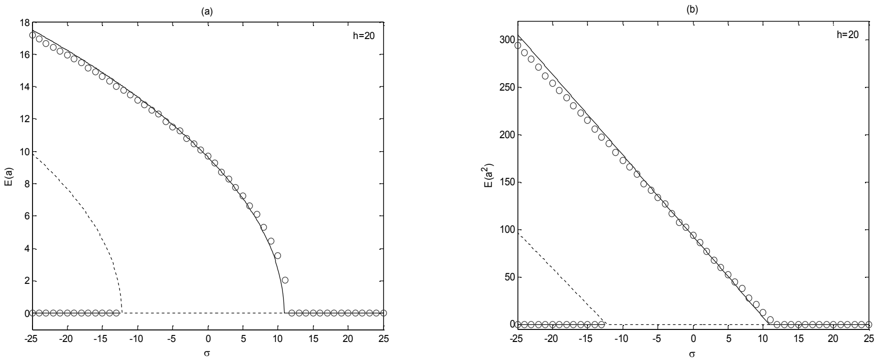

Fig. 1 Frequency-amplitude response of the first mode: C = 0.5, k_1 = 0.01, α = 0.02, h = 20.0 and β = 0.1. "o o o" numerical result, "—" theoretical result, "- - -" unstable solution.

CONCLUSIONS

The method of multiple scales here is applied to investigate the principal parametric resonance of axially moving viscoelastic beam under narrow-band random excitation. Using the solvability condition, we give the largest Lyapunov exponent of the trivial steady-state solution to analyze the almost sure stability of the system, at the same time we derived the first-order and the second-order steady-state moments for the non-trivial steady-state solutions. Theoretical analyses and numerical simulations for the first mode show that: under some conditions, during the intensity of the random excitation increases, non-trivial steady-state solution fluctuation will become strenuous, resulting in the non-trivial steady state solution may lose its stability and the system may have a trivial steady state solution.

References

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