SYNCHRONIZATION MOTION OF NETWORKED HARMONIC OSCILLATORS UNDER LOCAL INSTANTANEOUS INTERACTION

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Summary The main purpose of this paper is to investigate the synchronization motion of networked harmonic oscillators with communications delays under local instantaneous interaction. A distributed algorithm for the synchronization of coupled harmonic oscillators in undirected network topologies is proposed. It is assumed that each oscillator can simultaneously communicate with its neighbors and meanwhile updates its velocity state in the form of jumps at a series of discrete time. A simple convergence criterion for such algorithm over undirected fixed network topologies is derived analytically. It is shown that the networked harmonic oscillators can always be synchronized under instantaneous network connectivity. Subsequently, numerical examples illustrate and also visualize the effectiveness and feasibility of the theoretical results.

INTRODUCTION

In recent years, coordinated behavior and cooperative control of networked mechanical systems derived from some practical engineering problems have attracted a great deal of attention from various fields of science and engineering. In particular, synchronization of coupled harmonic oscillators has been a significant topic in both theoretical research and practical applications. The main reason is that: network is regarded as a good tool to deal with a set of interactional systems, while coordination problems encountered in the engineering world can often be rephrased as synchronization or consensus problems. More importantly, it plays an important role in applications involving multi-agent networks with repetitive movements such as cooperative patrol, mapping, sampling or surveillance. In a recent paper [1], Ren investigated n coupled harmonic oscillators with local interaction of the form

$$\ddot{x}_i(t) + \alpha x_i(t) + \sum_{j=1}^{n} a_{ij}(t)\big(\dot{x}_i(t) - \dot{x}_j(t)\big) = 0, \quad i = 1, 2, \cdots, n. \tag{1}$$

where $x_i(t)$ is the position of the ith oscillator, α is a position gain at time t, and $a_{ij}(t)$ characterizes interaction between oscillator i and j at time t (i.e., $a_{ij}(t) > 0$ if oscillator i and j can exchange the velocity information with each other at time t and $a_{ij}(t) = 0$ otherwise). Particularly, Eq. (1) can represent two objects of mass $m = 1$ connected by a damper systems as shown in Figure 1 for $i = 2$.

Motivated by (1), we in this brief consider n coupled harmonic oscillators with local instantaneous interaction of the form

$$\ddot{x}_i(t) + \alpha x_i(t) + \mu \sum_{j=1}^{n} a_{ij}(t)\big(\dot{x}_i(t-\tau) - \dot{x}_j(t-\tau)\big)\delta(t - t_k) = 0, \quad i = 1, \cdots, n. \tag{2}$$

where $\mu > 0$ is a control parameter that partially assigns coupling strength between oscillators, $\tau > 0$ denotes the time delay for information communicated between oscillator j and oscillator i, the time sequence $\{t_k\}$ forms a strictly increasing sequence i.e., $t_{k-1} < t_k$ ($k \in \mathbb{N}$) with $\lim_{k\to\infty} t_k = +\infty$, and $\delta(t)$ is the Dirac function, which depicts the impulse effects of communication links with respect to velocity information among oscillators at certain coupling moments. The motivation of the present investigation originates from three elementary reasons. Firstly, in some practical applications of multi-agent systems such as sensor networks, mobile ad-hoc networks, computer networking, and telecommunications, each agent has to exchange the information from its neighbors instantaneously at certain moments due to some factors of external disturbances such as switching phenomenon, varying environment, frequency change or other sudden channel noise, etc. [2]-[6]. Secondly, in view of the technological appeal of digital implementations, sampled-data control systems in the form of impulsive model have attracted considerable interests on both theoretical and practical fronts in the background of networked control systems (NCSs) with packet losses. Finally, communication delays between agents should be taken into account, since they naturally arise as a consequence of data transmission and/or packet drop due to the limitation of the network resource, which will inevitably degrade control performance of the NCSs, or even cause the control systems instable. Therefore, it is important, and indeed necessary, to study synchronization in networked harmonic oscillators with communication delays under instantaneous or impulsive interaction regarding analysis and designs of the NCSs in practical engineering applications.

MAIN RESULTS

Let $\mathcal{G}$ denotes a weighted undirected connected graph with an adjacency matrix $\mathcal{A} = [a_{ij}]$, where $a_{ij} = a_{ji} > 0$ if node i and j can communicate with each other, and $a_{ij} = a_{ji} = 0$ otherwise. The eigenvalues of the corresponding Laplacian

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L can be ordered as $0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_n$. Let $r_i(t) = x_i(t)$ and $v_i(t) = \dot{x}_i(t)$ denote the position and velocity of oscillator i respectively. Based on the theory of impulsive dynamical systems, we have the following synchronization results:

Synchronization criterion Assume that $0 < \mu < 1/\lambda_n$ and $0 < \tau \leq T_k = t_k - t_{k-1} \in I$ for all $k \in \mathbb{N}$, where I is a closed subinterval of $(0,\ \pi/2\sqrt{\alpha})$. Then all the states $[\,r_i(t),\ v_i(t)\,]^\top$ $(i = 1,\ 2,\ \cdots,\ n)$ of system (2) will asymptotically converge to the $\frac{2\pi}{\sqrt{\alpha}}$-period synchronization state $[\,\gamma(t),\ \nu(t)\,]^\top = [p^\top r_0 \cos\sqrt{\alpha}t + \frac{1}{\sqrt{\alpha}}p^\top v_0 \sin\sqrt{\alpha}t, -\sqrt{\alpha}p^\top r_0 \sin\sqrt{\alpha}t + p^\top v_0 \cos\sqrt{\alpha}t]^\top$, where $p = \frac{1}{n}1_n$, r_0 and v_0 are respectively the initial position and velocity of system (2).

It is important to emphasize that, different from the existing works concerned with continuous synchronization algorithms, a distinctive feature of the proposed algorithm (2) is that it is applicable to deal with synchronization problems of coupled harmonic oscillators with local instantaneous interaction, in which each oscillator can receive the information on relative velocities of its neighbors in the form of abrupt jumps occurring only at discrete moments. Accordingly, we here call the oscillators in Eq. (2) the instantaneous coupled harmonic oscillators. Furthermore, the fixed topology can easily extended to switching topologies cases.

APPLICATIONS

In order to verify the effectiveness of the theoretical results, we consider a network having 5 agents moving in one dimensional Euclid space with topology $\mathcal{G}$ as shown in Figure 2. Some corresponding parameters are selected as $\mu = 0.2, \tau = 0.2, T_k = 0.4$. Figure 3 shows the numerical simulation results. We further simulate the motion coordination of five multi-agent systems using algorithm (2). Their dynamics are given by $\dot{p}_i = q_i$ and $\dot{q}_i = \omega_i$, $i = 1, 2, 3, 4, 5$, where $p_i = [x_i, y_i]^T$ stands for the position, $q_i = [v_{xi}, v_{yi}]^T$ stands for the velocity, and $\omega_i = [\omega_{xi}, \omega_{yi}]^\top$ is the acceleration input. By applying algorithm (2), ω_{xi} and ω_{yi} can be designed respectively as [1] $\omega_{xi} = -\alpha(x_i - \delta_{xi}) + \sum_{j=1}^{n} \mu a_{ij}(v_{xj}(t-\tau) - v_{xi}(t-\tau))\delta(t - t_k), \omega_{yi} = -\alpha(y_i - \delta_{yi}) + \sum_{j=1}^{n} \mu a_{ij}(v_{yj}(t-\tau) - v_{yi}(t-\tau))\delta(t - t_k)$, where δ_{xi} and δ_{yi} are constant. We choose $\alpha = 2, \mu = 0.15, T_k = 0.5, \tau = 0.4, 0 \leq t \leq 40$. The complete trajectories are shown in Figure 3, circles denote the snapshots at $t = 0$ while the diamonds denote the snapshots at $t = 10, 20, 30, 40$ It is clearly that these five agents synchronize their motions and move on elliptic orbits.

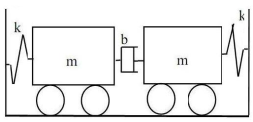

Figure 1. Two objects of mass connected by a damper

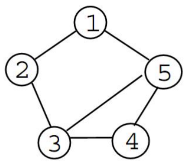

Figure 2. Interaction graph for five agents .

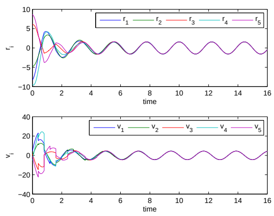

Figure 3. Evolution of oscillator states

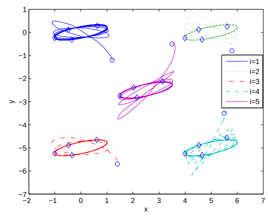

Figure 4. Trajectories of five agents.

CONCLUSIONS

In this paper, we have investigated synchronization problem of networked harmonic oscillators with communication delays under local instantaneous interaction. By using impulsive control theory on hybrid dynamical systems, matrix analysis theory, and algebraic graph theory, a simple yet generic convergence criteria is derived under which all the oscillators can synchronize their motion over fixed interaction topology. Finally, simulations are offered to verify the effectiveness and correctness of the proposed algorithms and show its applications in the motion coordination of multi-agent systems.

References

[1] Ren, W.: Synchronization of coupled harmonic oscillators with local interaction. *Automatica*, 44(12) 3195-3200, 2008.

[2] Nair, S., Leonard, N.: Stable synchronization of mechanical system networks. *SIAM Journal Control and Optimization*, 47(2) 661-683, 2008.

[3] Zhang, W. A., Yu, L.: Stabilization of sampled-data control systems with control inputs missing. *IEEE Transactions on Automatic Control*, 55(2) 447-452, 2010.

[4] Zhou, J., Wu, Q. J., Zhang, H., Xiang, L.: Distributed δ-consensus in directed delayed networks of multi-agents. *Proceeding of the 18th World Congress of the International Federation of Automatic Control*, Milano, Italy, 3304-3309, 2011.

[5] Zhou, J., Zhang, H., Xiang, L., Wu Q.: Synchronization of coupled harmonic oscillators with local instantaneous interaction. *Automatica*, 48, 2012 , in press.

[6] Zhou, J., Wu, Q. J., Xiang, L.: Pinning complex delayed dynamical networks by a single impulsive controller. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 58(12) 2882 - 2893, 2011.