STOCHASTIC AVERAGING OF BOUC-WEN HYSTERETIC SYSTEMS UNDER POISSON WHITE NOISE EXCITATION

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Summary Stationary response of a single-degree-of-freedom (SDOF) oscillator with Bouc-Wen hysteresis driven by Poisson white noise is investigated by using stochastic averaging method. The averaged generalized Fokker-Planck-Kolmogorov (GFPK) equation is derived and approximate solution of it is obtained by using classical perturbation method and exponential polynomial closure (EPC) method separately. Theoretical results of an example are confirmed by using Monte Carlo simulation.

INTRODUCTION

Stochastic averaging method has been widely used in studying stochastic dynamics of different types of hysteretic systems driven by Gaussian random excitations, such as bilinear model [1,2], Bouc-Wen model [2,3], integrable Duhem model [4] and Preisach model [5,6]. Recently, stochastic averaging method of energy envelope and stochastic averaging method of amplitude envelope have been extended into nonlinear systems subject to Poisson white noise [7,8]. However, few works on random dynamics of hysteretic system driven by Poisson white noise excitation have been reported.

In the present paper, stochastic averaging method of energy envelope is extended for predicting stationary response of a Poisson white noise excited Bouc-Wen hysteretic system, a classical system with smooth-type hysteresis. Perturbation method [9] and EPC method [10] proposed to solve GFPK equation directly are used for solving averaged GFPK equation approximately. The complexity of these two methods in solving GFPK equation is reduced significantly via stochastic averaging procedure. More simple and clearer forms of approximate solution are obtained and confirmed by comparing with results of Monte Carlo simulations.

PROBLEM FORMULATION

Consider the following nondimensional SDOF Bouc-Wen hysteretic system excited by Poisson white noise:

$$\ddot{X}+\varepsilon\dot{X}+f(X,\dot{X})=\sqrt{\varepsilon}\xi(t),\ f(X,\dot{X})=\alpha X+(1-\alpha)Z(X,\dot{X}),\quad 0<\alpha<1 \tag{1}$$

in which, the hysteretic component can be modelled by a first-order differential equation,

$$\dot{Z}=b\dot{X}-\left(\gamma\left|\dot{X}\right|Z+\beta\dot{X}\left|Z\right|\right)\left|Z\right|^{m-1} \tag{2}$$

and Poisson white noise $\xi(t)$ can be treated as the formal derivative of the compound Poisson process $C(t)=\sum_{i=1}^{N(t)}Y_iU(t-t_i)$ where $N(t)$ is a Poisson counting process with average arrival rate λ, $U(t\text{-}t_i)$ is unit step function at t_i, Y_i is a identical symmetric-distributed random variable representing the intensity of the i th impulse.

Eq. (1) describes a quasi-Hamiltonian system with Hamiltonian (total energy) $H=\dot{X}^2/2+G(X)$ where $G(X)$ is the potential energy defined in [2, 3] and $g(X)=dG(X)/dX$. Eq. (1) can be replaced by the following Stratonovich stochastic differential equations (SDEs),

$$dX=\left(\partial H/\partial\dot{X}\right)dt,\ dH=-\varepsilon\left(\partial H/\partial\dot{X}\right)^2dt+\left(\partial H/\partial\dot{X}\right)\left[g(X)-f(X,\partial H/\partial\dot{X})\right]dt+\sqrt{\varepsilon}\left(\partial H/\partial\dot{X}\right)\circ dC(t) \tag{3}$$

Following the converting rule from Stratonovich SDEs into Itô SDEs for systems with Poisson white noise excitation derived by Di Paola and Falsone [11] and the evolution equation of probability density associated with the Itô SDEs, the following GFPK equation of probability density $\rho=\rho(x,h,t)$ can be obtained from Eq. (3):

$$\begin{aligned}\frac{\partial\rho}{\partial t}=&-\frac{\partial}{\partial x}\left(\frac{\partial h}{\partial\dot{x}}\rho\right)-\frac{\partial}{\partial h}\left\{\left(-\varepsilon\left(\frac{\partial h}{\partial\dot{x}}\right)^2+\frac{\partial h}{\partial\dot{x}}\left[g(x)-f\left(x,\frac{\partial h}{\partial\dot{x}}\right)\right]+\frac{\varepsilon}{2}\lambda E[Y^2]\right)\rho\right\}+\frac{1}{2!}\frac{\partial^2}{\partial h^2}\left\{\left(\varepsilon\left(\frac{\partial h}{\partial\dot{x}}\right)^2\lambda E[Y^2]+\frac{\varepsilon^2}{4}\lambda E[Y^4]\right)\rho\right\}\\&-\frac{1}{3!}\frac{\partial^3}{\partial h^3}\left\{\frac{3}{2}\varepsilon^2\left(\frac{\partial h}{\partial\dot{x}}\right)^2\lambda E[Y^4]\rho\right\}+\frac{1}{4!}\frac{\partial^4}{\partial h^4}\left\{\varepsilon^2\left(\frac{\partial h}{\partial\dot{x}}\right)^4\lambda E[Y^4]\rho\right\}+o(\varepsilon^3)\end{aligned} \tag{4}$$

With the assumption that dissipated energy per cycle of hysteresis loop is small, the conditional probability density proposed by Stratonovich [12] is introduced:

$$\rho(x\,|\,h,t)=\begin{cases}1\Big/T(h)\sqrt{2h-2G(x)} & \text{for}\quad G(x)<h\\ 0 & \text{for}\quad G(x)>h\end{cases}\qquad T(h)=\oint_{G(x)<h}\frac{dx}{\sqrt{2h-2G(x)}} \tag{5}$$

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Substituting $\rho(x,h,t)=\rho(h,t)\rho(x \mid h,t)$ with $\rho(x \mid h,t)$ in Eq.(5) into GFPK Eq. (4) and integrating Eq. (4) with respect to x yield the following averaged GFPK equation for probability density $\rho=\rho(h,t)$:

$$\frac{\partial \rho}{\partial t}=-\frac{\partial}{\partial h}\left(A_1(h)\rho\right)+\frac{1}{2!}\frac{\partial^2}{\partial h^2}\left(A_2(h)\rho\right)-\frac{1}{3!}\frac{\partial^3}{\partial h^3}\left(A_3(h)\rho\right)+\frac{1}{4!}\frac{\partial^4}{\partial h^4}\left(A_4(h)\rho\right)+o(\varepsilon^3) \tag{6}$$

in which, $A_1(h)$, $A_2(h)$, $A_3(h)$ and $A_4(h)$ have similar integration expressions such as

$$A_1(h)=\frac{1}{T(h)}\oint_{G(x)<h}\left(-\varepsilon\left(\frac{\partial h}{\partial \dot{x}}\right)^2+\frac{\partial h}{\partial \dot{x}}\left[g(x)-f\left(x,\frac{\partial h}{\partial \dot{x}}\right)\right]+\frac{\varepsilon}{2}\lambda E[Y^2]\right)\frac{dx}{\sqrt{2h-2G(x)}} \tag{7}$$

When left-hand side of averaged GFPK Eq. (6) vanishes, the reduced averaged GFPK equation about stationary probability density $\rho=\rho(h)$ can be solved approximately by using two methods:

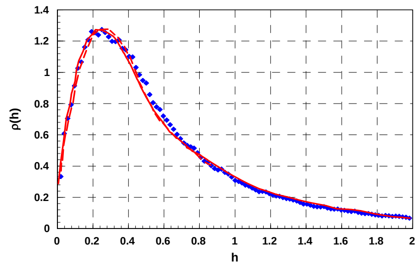
Classical perturbation method

As Cai & Lin proposed [9], substituting the perturbation expansion $\rho(h)=\rho_0(h)+\varepsilon\rho_1(h)+\ldots$ into stationary Eq. (6) and equating term of the same order of ε, the perturbation solution $\rho_0(h),\rho_1(h)$, … can be obtained step by step.

EPC method

As Zhu & Er proposed [10], substituting the approximate solution as an exponential polynomial function $\rho(h) \approx \bar{\rho}(h)=D\exp(\sum_{r=1}^{n}a_r h^r)$ into stationary Eq. (6) and ignoring terms of order ε^3 and higher, the following residual error is obtained: $\Delta_n=R_n(h;a_1,a_2,\cdots,a_n)\bar{\rho}(h)$ in which R_n is a polynomial function of $a_1,a_2,\cdots,a_n$. A set of mutually independent functions $F_s(h)=h^s\rho_0(h),\ s=1,2,...,n$ that span space R^+ can be introduced to make the projection of R_n vanish on R^+, which leads to nonlinear algebraic equations in terms of $a_1,a_2,\cdots,a_n$: $\int_0^\infty F_s R_n dh=0,\ s=1,2,\cdots,n$ .The algebraic equations can be solved with any available method to determine the parameters in $\bar{\rho}(h)$. Once $\rho(h)$ is obtained from stationary Eq. (6), the stationary probability density for system response can be calculated as follows: $\rho(x,\dot{x})=\rho(h)\rho(x \mid h)\left|\partial h/\partial \dot{x}\right|=\rho(h)/T(h)\big|_{h=\dot{x}^2/2+G(x)}$.

ILLUSTRATIVE EXAMPLE



●, results of Monte Carlo simulation
Dashed line, Perturbation solution
Solid line, EPC solution
Parameters:
ε=0.09,b=1.0,$\beta=\gamma$=0.5,m=1,α=0.05
λ=2.0,$Y \sim N(0,4)$

CONCLUSION REMARKS

Stochastic averaging method of energy envelope for Bouc-Wen hysteretic system driven by Poisson white noise has been proposed. Classical perturbation method and EPC method are used separately for solving reduced averaged GFPK equation. More simple and clearer theoretical solutions are obtained and confirmed by comparing with results from Monte Carlo simulation.

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