EXAMINATIONS OF NONLINEAR ISOLATORS USING POWER FLOW APPROACH

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Summary We investigate a nonlinear passive vibration isolation system from a viewpoint of vibrational power flows. The system consists of a vibrating mass, subject to a harmonic excitation force, mounted on a linear base structure of 1 degree-of-freedom through a nonlinear isolator. The isolator spring force behaves a cubic nonlinearity, physically representing a softening or hardening stiffness depending on the sign of the nonlinear parameter. The averaging method is used to derive the response power flow variables of the system. The effects of the nonlinear parameter on the power flows are examined by a comparison with the corresponding linear system. The results and analysis provide an understanding of power flow characteristics of this nonlinear dynamical system. The paper reveals dynamic behaviour of nonlinear isolators and their possible benefits to vibration controls for considerations in engineering designs.

INTRODUCTION AND MATHEMATICAL MODEL

A power flow analysis (PFA) is a widely accepted technique for investigating the dynamics of complex structures and coupled systems, based on the universal law of energy balance and conservation. Vibrational power flow provides a performance indicator incorporating both force and velocity characteristics and thus can be used as a cost function to evaluate the performance of various vibration control systems. In recent years, the PFA approach has been developed to model structural, structural-acoustical and structural-control systems. Most previous studies were conducted on linear systems based on the linear vibration theory, although some reports involving nonlinearities were reported [1-7]. However, practical systems are inherently nonlinear, or designed deliberately to be nonlinear for a better dynamic performance. To reveal the power flow behaviour of nonlinear dynamical systems, fundamental theories with effective techniques as well as various case studies for PFA of nonlinear dynamical systems are required to be developed.

This paper studies a nonlinear vibration isolation system using PFA for a better understanding of the power flow behaviour in nonlinear isolation systems to provide a reference in nonlinear isolator designs. Fig.1 shows the investigated system consisting of a one DOF system of mass m_1, spring k_1 and damper c_1 to model a linear elastic base on which a machine, mass m_2, is mounted through a nonlinear spring and damper c_2. The restoring force F of the nonlinear spring is assumed to be a linear/nonlinear function of its dynamic deflection X, $F(X) = k_2(X + \eta X^3)$ of which $\eta = 0, \eta > 0$ and $\eta < 0$ represent a linear, hardening and softening stiffness system, respectively. A harmonic force of amplitude f and frequency ω is applied on m_2. The dynamic governing equations of the system are obtained as

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 - c_2(\dot{x}_2 - \dot{x}_1) - k_2(x_2 - x_1) - \eta k_2(x_2 - x_1)^3 = 0, \quad (1)$$

$$m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) + \eta k_2(x_2 - x_1)^3 = f\cos\omega t. \quad (2)$$

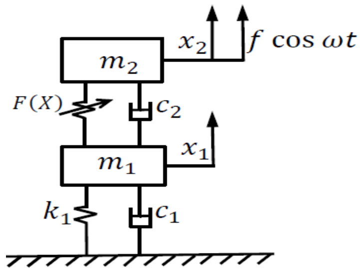

Fig.1. The system model.

After introducing non-dimensional parameters

$$x_0 = \frac{m_1 g}{k_1}, \mu = \frac{m_2}{m_1}, \omega_1 = \sqrt{\frac{k_1}{m_1}}, \omega_2 = \sqrt{\frac{k_2}{m_2}}, \gamma = \frac{\omega_2}{\omega_1}, \xi_1 = \frac{c_1}{2m_1\omega_1},$$

$$\xi_2 = \frac{c_2}{2m_2\omega_2}, \varepsilon = \eta x_0^2, f_0 = \frac{f}{k_1x_0}, y = \frac{x_2 - x_1}{x_0}, x = \frac{x_1}{x_0}, \Omega = \frac{\omega}{\omega_1}, \tau = \omega_1 t,$$

Eqs.(1) and (2) can be transformed to a non-dimensional form in the phase space

$$x' = z, z' = -2\,\xi_1 z - x + \mu\gamma(2\xi_2 w + \gamma y + \varepsilon\gamma y^3), y' = w, w' = \frac{f_0}{\mu}\cos\Omega\tau - x'' - \gamma(2\xi_2 w + \gamma y + \varepsilon\gamma y^3). \quad (3\text{-}6)$$

POWER FLOW ANALYSIS

Adopting an averaging method[8], we assume $x = a\cos(\Omega\tau + \phi), x' = -a\Omega\sin(\Omega\tau + \phi)$, $y = b\cos(\Omega\tau + \theta), y' = -b\Omega\sin(\Omega\tau + \theta)$, where a, b, ϕ and θ are time functions. By replacing corresponding variables in Eqs.(3-6) and averaging them over a period of the excitation load, we derive the following frequency response relationship

$$\left(\gamma(\Omega^2 - 1)\left(1 + \frac{3}{4}\varepsilon b^2\right) - 4\xi_1\xi_2\Omega^2\right)^2 + 4\Omega^2\left(\xi_2(\Omega^2 - 1) + \xi_1\gamma\left(1 + \frac{3}{4}\varepsilon b^2\right)\right)^2 = \frac{\mu^2\gamma^2b^2}{a^2}\left[\gamma^2\left(1 + \frac{3}{4}\varepsilon b^2\right)^2 + (2\xi_2\Omega)^2\right]^2,$$

$$\frac{{f_0}^2}{4\mu^2} = \gamma^2\xi_2^2b^2\Omega^2 + \frac{a^2\Omega^2[(\mu+2)\Omega^2 - 2]}{4\mu} + \frac{b^2}{4}\left(\Omega^2 - \gamma^2 - \frac{3}{4}\varepsilon\gamma^2b^2\right)^2 - a^2\Omega^4\frac{\left(\frac{1}{2}\gamma(\Omega^2-1)\left(1+\frac{3}{4}\varepsilon b^2\right) - 2\xi_1\xi_2\Omega^2\right)}{\mu\gamma\left[\gamma^2\left(1+\frac{3}{4}\varepsilon b^2\right)^2 + (2\xi_2\Omega)^2\right]}. \quad (7,8)$$

Power flow variables can be derived using the resutls of averaging method as

$$\overline{p_{in}} = \xi_1 a^2\Omega^2 + \mu\gamma\xi_2 b^2\Omega^2,\ \overline{p_t}(\Omega) = \xi_1 a^2\Omega^2, \overline{p_d} = \xi_1 a^2\Omega^2 + \mu\gamma\xi_2 b^2\Omega^2, \quad (9\text{-}11)$$

where $\overline{p_{in}}, \overline{p_t}$ and $\overline{p_d}$ represent the time averaged values of input power, transmitted power from m_2 to m_1 and dissipated power of the system, respectively, over a cycle of excitaiton. In these expressions, the input power $\overline{p_{in}}$ exactly equals the disipated power $\overline{p_d}$, which follows the energy conservation law.

The maximum kinetic energy of mass m_1 and m_2 in the steady state motion are expressed respectively as

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$$K_1 = \frac{1}{2}a^2\Omega^2, K_2 = \frac{1}{2}\mu(a^2 + b^2)\Omega^2 + a^2\Omega^2\frac{-\gamma(\Omega^2-1)\left(1+\frac{3}{4}\varepsilon b^2\right)+4\xi_1\xi_2\Omega^2}{\gamma\left(\gamma^2\left(1+\frac{3}{4}\varepsilon b^2\right)^2+(2\xi_2\Omega)^2\right)}. \quad (12,13)$$

Eqs.(10) and (12) suggest that the amount of time averaged power transmitted to the base is proportional to its maximum kinetic energy for a prescribed damping $\xi_1$of the base structure, which indicates that the dynamical character of the base can be described by the time averaged transmitted power. By using the derived formulations and selected system parameters, the power flow variables are plotted against non-dimensional excitation frequency in Fig.2 (dB reference: 10^{-12}). Figs.2(a) and (b) show the cases of which the linearized natural frequency of the isolator is smaller than that of the base structure when $\gamma < 1$ while Figs.2(c) and (d) are for the cases of $\gamma > 1$. Figs.2(a) and (b) demonstrate that a softening stiffness bends the first peaks in both time averaged input and transmitted powers to the low frequency band with a significant reduction of the peak values. Meanwhile, the non-unique solution branches of power flows are encountered in this frequency band due to the introduced nonlinearity. However, the power flow values on these branches are still lower than the peak value of the corresponding linear system. An anti-peak on the solution curve branch ($\varepsilon = -0.5$) of transmitted power is also observed in the low frequency range. Comparing with the softening stiffness, for the hardening stiffness system, the first resonant peaks of power flow curves move towards the higher frequency range with a substantial increase in the peak value of the transmitted power. When $\gamma > 1$, Figs.2 (c) and (d) indicate that adding stiffness nonlinearity may influence power flow characteristics around both resonant peaks, whereas in Figs.2(a) and (b) only power flow close to the first peak is affected. Similar to the previous case, nonlinearity may result in non-unique values of power flow in some frequency ranges with associated jump phenomena. For the softening stiffness isolator, the power flow curves would bend both peaks towards lower frequency range. Hardening stiffness, however, affects power flow when the excitation frequency is close to the second resonant frequency. Except the multi-solutions in the lower frequency range for softening stiffness cases, the power flow variables for both nonlinear cases remain largely unchanged when the excitation frequency is far away from the resonant frequencies. The reason is that in non-resonant frequency ranges the dynamic deflections of both linear and nonlinear systems are relatively small and almost the same so that the power flow values are unaffected by nonlinearity.

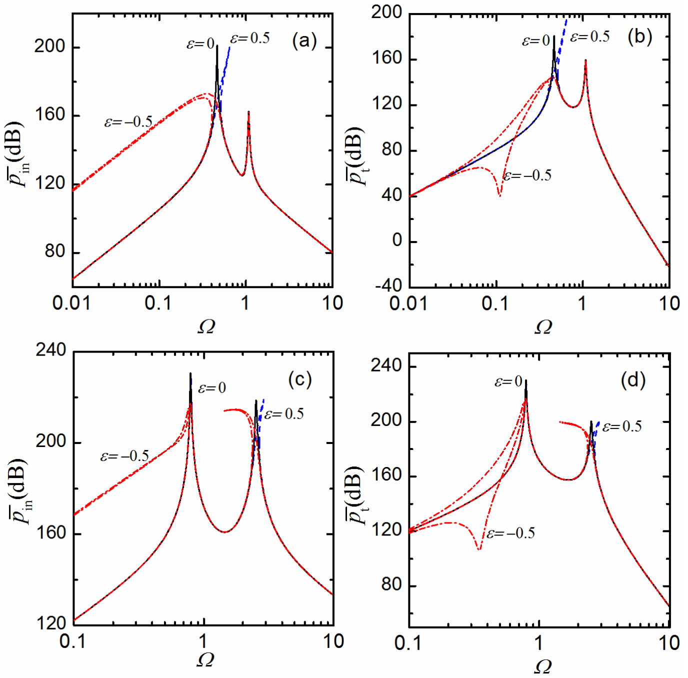

Fig.2. Effects of nonlinear stiffness on power flow for parameters ($\xi_1 = \xi_2 = 0.01, \mu = 0.5$): (a) and (b) $\gamma = 0.5 < 1, f_0 = 0.01$; (c) and (d) $\gamma = 2 > 1, f_0 = 0.1$.

CONCLUSIONS AND DISSCUSSIONS

The dynamics of a nonlinear vibration isolation system is examined from a viewpoint of power flows. It is found that the stiffness nonlinearity in isolator influences power flow locally when the excitation frequency is close to resonant frequencies. Introducing a softening stiffness can reduce the peak values of both input and transmitted powers and bend power flow curves towards a lower frequency range, which may benefit vibration isolations. On the contrary, hardening stiffness would twist the peaks to the higher frequency range and generate a larger power transmission, which is undesirable. This study reveals the important power flow characteristics of nonlinear isolators, which may provide a preliminary basis for further research or practical designs of nonlinear isolation systems. The obtained results are based on an averaged approximation. Therefore, to avoid losing nonlinear information, more detailed numerical simulations are undertaken. To deal with the multiple solutions for nonlinear isolators could be a challenge of further explorations.

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