STABILITY AND STABILIZATION OF THE MOTION OF THE RIGID BODY WITH ELASTIC COMPARTMENTS CONTAINING AN IDEAL FLUID

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Summary Based on the modal analysis, equations of a motion of a rigid body with elastic cylindrical compartments filled with an ideal fluid are deduced. The equation for eigen frequencies and analytical expressions for eigen forms of joint oscillations of fluid and elastic plates are obtained. Conditions of stability and stabilization are studied for the problem of translational motions of a rigid body under influence of an elastic force (Sretensky problem) and for compound pendulum problem. Analytical expressions for added mass and added moment of inertia of a fluid are obtained.

THE STATEMENT OF PROBLEM

Let's consider a mechanical system consisting of a rigid body with a double-connected cylindrical concavity with an arbitrary cross-section S and $m+1$ elastic plates, which divide the concavity into m compartments. *i*-th compartment has height h_i and is completely filled with heavy fluid with density ρ_i ($i=\overline{1,m}$). Plates are isotropic, they have flexural stiffness D_i and are influenced with preliminary membrane tensions T_i in median surface ($i=\overline{1,m+1}$). Fluid is ideal, homogeneous and incompressible, motion of fluids is potential, there are no discontinuities in joint oscillations of fluid and elastic plates. Motion of such mechanical systems is described by complex boundary problems of integro-differential equations [1–3]. For example, Sretensky problem equations look like

$$k_{0i}\frac{\partial^2 W_i}{\partial t^2}+D_i\Delta_2^2 W_i - T_i\Delta_2 W_i + g\Delta\rho_i W_i =$$

$$=\sum_n\left(b_{in}\dot{\tilde a}_{i+1,n}+b_{i-1,n}\dot{\tilde a}_{i-1,n}-a_{in}\dot{\tilde a}_{in}\right)\psi_n-\Delta\rho_i\ddot X x-\rho_i Q_i+\rho_{i-1}Q_{i-1},$$

$$\mathfrak{L}_{\gamma_j}(W_i)=0,\quad \mathfrak{M}_{\gamma_j}(W_i)=0,\quad \nabla W_i<\infty,\quad \int\limits_S W_i dS=\int\limits_S W_{i+1}dS.$$

Here W_i is lateral deflection of *i*-th plate; X is a translation displacements of rigid body from equilibrium position along axis Ox; $\mathfrak{L}_{\gamma_j}$ and $\mathfrak{M}_{\gamma_j}$ are operators of boundary conditions of plates attachment; $\Delta\rho_i=\rho_i-\rho_{i-1}$ ($\rho_0=\rho_{m+1}=0$), $b_{in}=\rho_i/\sinh\kappa_{in}$, $\kappa_{in}=k_n h_i$, $\tilde a_{in}=\dfrac{1}{k_n N_n^2}\displaystyle\int\limits_S\frac{\partial W_i}{\partial t}dS$, $N_n^2=\displaystyle\int\limits_S\psi_n^2 dS$, $a_{in}=\rho_i\coth\kappa_{in}+\rho_{i-1}\coth\kappa_{i-1,n}$; ψ_n and k_n are the eigen functions and according eigen numbers of oscillations of ideal fluid in cylindrical vessel; Q_i – an arbitrary functions of time; γ_j are boundaries of double-connected area S ($j=\overline{1,2}$).

METHODS OF SOLUTION

In linear statement, equations of disturbed motion of a fluid and elastic plates can be reduced to the following generalized wave equation [3, 4]

$$A\frac{\partial^2 W}{\partial t^2}+C\,W=f, \tag{1}$$

here scalar functions W_i are combined into vector $W=(W_1,...,W_{m+1})$; A and C are accordingly inertial and elastic integro-differential operators in a functional space with scalar product $(U,V)=\displaystyle\sum_{i=1}^{m+1}\int\limits_S U_iV_i dS$; f is a disturbance of a fluid and plates motions [1, 2, 5].
The solution of equation (1) can be sought in the form of expansion in series with respect to eigen forms w_k

$$W=\sum_{k=1}^{\infty}p_k(t)w_k, \tag{2}$$

which converge anyway on energy norm or on the mean.
The denumerable system of ordinary differential equations follows from orthogonality conditions of kinetic and potential energies

$$\mu_k(\ddot p_k+\omega_k^2 p_k)=f_k(t),\quad (k=1,2,...) \tag{3}$$

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here $\mu_k = (Aw_k, w_k)$, $f_k(t) = (f, w_k)$, ω_k – eigenfrequency corresponding to eigen form w_k.
The equations of motion for Sretensky problem follow from the system (3) and momentum theorem

$$\mu_k \left(\ddot{p}_k + \omega_k^2 p_k\right) = \sum_{i=1}^{m+1} \Delta\rho_i \tilde{\alpha}_{ik} \ddot{X}, \quad (M_0 + M_{2x}) \ddot{X} = -cX - \sum_{i=1}^{m+1} \Delta\rho_i \sum_{k=1}^{\infty} \tilde{\alpha}_{ik} \omega_k^2 p_k. \tag{4}$$

The equations of motion for compound pendulum problem follow from the system (3) and angular momentum theorem

$$\mu_k \left(\ddot{p}_k + \omega_k^2 p_k\right) = \sum_{i=1}^{m+1} \left(\tilde{\Omega}_{ik} \ddot{\theta} + g\tilde{\alpha}_{ik}\theta\right), \quad \left(J_0 + \tilde{J}_2\right) \ddot{\theta} + \tilde{k}\theta = \sum_{i=1}^{m+1} \Delta\rho_i \sum_{k=1}^{\infty} \left(\tilde{\Omega}_{ik} \omega_k^2 + g\tilde{\alpha}_{ik}\right) p_k. \tag{5}$$

Here X and θ are a translational and rotational displacements of rigid body from equilibrium position,

$$\tilde{\alpha}_{ik} = \int_S x w_{ik} dS = \sum_n C_n \zeta_{ink}, \quad \zeta_{ink} = \frac{1}{N_n^2} \int_S w_{ik} \psi_n dS, \quad C_n = \frac{1}{N_n^2} \int_S x \psi_n dS. \tag{6}$$

Definitions of other variables may be found in papers [1–4]. We obtained follow expressions for added mass and added moment of inertia

$$M_{2x} = M_2 - \sum_{k=1}^{\infty} \frac{1}{\mu_k} \left(\sum_{i=1}^{m+1} \Delta\rho_i \tilde{\alpha}_{ik} \right)^2, \quad \tilde{J}_2 = J_2 - \sum_{k=1}^{\infty} \frac{1}{\mu_k} \sum_{i=1}^{m+1} \left(\tilde{\Omega}_{ik} \Delta\rho_i\right)^2.$$

Apparently from equations (4), (5) and expressions (6), when the concavity has axial symmetry (the direct circular or coaxial cylinder), single-node antisymmetrical oscillations are excited only. From the graphical analysis of the frequency equations for systems (4) and (5) it is follow, that high frequencies of mechanical system oscillations σ_k are close to frequencies of oscillations of plates and a fluid in a motionless vessel ω_k (eigen frequencies). A problem of existing of imaginary roots of frequency equation is especially interesting, because their presence would define to instability of position of equilibrium. When we have one elastic compartment ($m = 1$), it is easy to show the lack of imaginary roots for Sretensky problem [3]. If we have two or more elastic compartments, imaginary roots can appear both in Sretensky problem, and in a compound pendulum problem.

It is shown, that unstable oscillations of a rigid body with the cylindrical elastic compartments containing a fluid always may be stabilized by increasing of a preliminary tension and flexural stiffness of plates [3, 4].

CONCLUSIONS

Based on analysis of obtained frequency equations and conditions of positive definiteness of the potential energy, we found conditions of stability and stabilization of considered mechanical systems motion. For arbitrary number of compartments and various cases of plates attachments, frequency equation and analytical expressions for eigen forms of joint oscillations of plates and fluids are obtained. Necessity of analytical expressions of eigen forms is the main difficulty of proposed approach. Their deriving is not a simple problem. Particularly, it is necessary to have a fundamental system of solutions for supplementary boundary problem for a biharmonic equation [2, 3]. However, such system may be found only for canonical shapes of concavity. In this paper, we have built expressions of eigen forms for cylindrical concavity with circle and ring cross sections. Numerical analysis of obtained analytical expressions are performed for concavities with one and two compartments [4–6]. In common case, eigen frequency equation has the form of determinant with dimension $5m - 1$. When double-connected area degenerates to simple-connected area (ring degenerate to circle) the determinant has dimension $3m - 1$. Simplification of considering common problem takes place when some plates degenerate to membranes or are absent. The main simplification has been obtained in the case when one of the plates becomes rigid. In this case, the static deflections of the plates are equal to zero and vertical oscillations of fluid column are absent.

Thus, obtained resulta can be applied to the problems of transporting and storage of fluid cargoes, as well as to design of amortization elements of skyscropers (Tuned or Active fluid Dampers).

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