DYNAMICS OF OSCILLATORS WITH CONTINUOUS AND DISCONTINUOUS NONLINEARITIES BY HARMONIC BALANCE AND PATH FOLLOWING

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Summary Dynamical systems involving clearances, friction and impact etc. can have both continuous and discontinuous nonlinearities. The dynamics of such systems are investigated by a Multi Harmonic Balance Method (MHBM) combined with an arc length continuation procedure. The example of a harmonically excited smooth and discontinuous oscillator which exhibits rich dynamic behavior such as co-existence of chaotic and periodic motions of different orders, saddle node and symmetry breaking bifurcations which are captured accurately by the MHBM and verified by numerical simulation. The various branches of the harmonic and subharmonic solutions in the frequency domain are also traced. The similarities in the bifurcation behavior of the above oscillator and the Duffing's oscillator is also studied.

INTRODUCTION

The nonlinearities associated with dynamical systems may be smooth or discontinuous. These systems when subjected to external harmonic excitation is found to exhbit periodic, quasiperiodic or chaotic motions. Cao.et.al [1] proposed a nonlinear oscillator having smooth and discontinuous behavior depending on the value of a smoothness parameter(SD oscillator).This oscillator is studied numerically by various researchers and the co-existence of periodic solutions,quasi periodic solutions and chaotic solutions were reported [1, 2, 3].These studies focussed on the time response for different values of the smoothening parameter. The response analysis in the frequency domain has not been considered in the literature.In view of the different types of response exhibited by the system, this paper considers the frequency response of the SD oscillator. The MHBM combines with path following algorithm based on the arc length continuation procedure is used to trace the response plots.

DYNAMIC MODEL AND EQUATIONS OF MOTION

The equation of motion of SD oscillator subjected to harmonic excitation is given by

$$m\ddot{X} + c\dot{X} + 2kX\left(1 - \frac{L}{\sqrt{X^2 + l^2}}\right) = F_0 \cos \Omega t \tag{1}$$

In a nondimensional form it reduces to as

$$\omega^2 x'' + 2\zeta\omega x' + \left(1 - \frac{1}{\sqrt{x^2 + \alpha^2}}\right) x = f_0 \cos \omega\tau_1 \tag{2}$$

where $x = \frac{X}{L}, \alpha = \frac{l}{L} \geq 0, \tau = \omega_0 t, \omega = \frac{\Omega}{\omega_0}, f_0 = \frac{F_0}{2kL}, \tau_1 = \omega t, \zeta = \frac{c}{2m\omega_n}, \omega_0^2 = \frac{2k}{m}$ and the number of primes represents the differentiation with respect to the non dimensional time τ_1. The parameter α is called the smoothening parameter. When $\alpha > 0$ equation(2) represents a smooth system, while if $\alpha = 0$ it represents a discontinuous system. The variation of the nonlinear restoring force for different values of α is shown in Fig.2.

MULTI HARMONIC BALANCE METHOD

The equation of motion of a multi degree of freedom nonlinear system is given by

$$\omega^2\mathbf{m}\ddot{\mathbf{x}} + \omega\mathbf{c}\dot{\mathbf{x}} + \mathbf{kx} + \mathbf{f_{nl}} - \mathbf{f}(\mathbf{p}, \omega, \tau) = \mathbf{0} \tag{3}$$

Periodic solutions of the system are assumed in the form of a truncated Fourier series as

$$\mathbf{x} = \mathbf{X_0} + \sum_{\mathbf{j=1}}^{\mathbf{M}} [\mathbf{X_{2j-1}} \cos(\mathbf{j}\tau) + \mathbf{X_{2j}} \sin(\mathbf{j}\tau)] \tag{4}$$

Substituting equation(4) in equation (3) and applying the Galerkin procedure the incremental equation (5)is generated which can be solved by the Newton Raphson method [4]. A path following algorithm based on arc length parameter is used to trace the response curves with external excitation frequency as the parameter.

$$[J^L]\mathbf{X} + \{\mathbf{G}\} - \{\mathbf{F}\} + \left[\mathbf{J}^L + \mathbf{J}^N\right] \Delta\mathbf{X} = 0 \tag{5}$$

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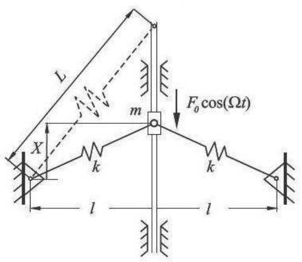

Figure 1. Dynamic model

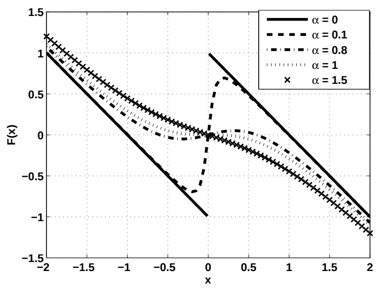

Figure 2. Nonlinear restoring force

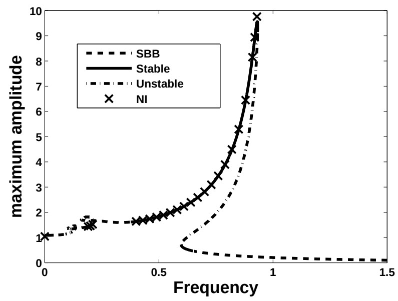

Figure 3. Frequency response for α = 0.8

RESULTS AND DISCUSSIONS

The frequency response plot for $\alpha = 0.8, f_0 = 0.25$ and $\zeta = 0.0141$ is shown in Fig.3. The bifurcation points along the frequency response plots are identified. The system is found to undergo saddle node bifurcation at $\omega = 0.933$ and at $\omega = 0.598$. In between the superharmonics a series of symmetry breaking bifurcations are observed. The frequency ranges between which the symmetry breaking bifurcation observed is ($0.117 \leq \omega \leq 0.185$ and $0.161 \leq \omega \leq 0.393$). The symmetry breaking bifurcation is also observed in the higher frequency values beyond the primary resonance. Fig.4-5 shows the period2 (P2) and period3 (P3) isolated subharmonic solutions obtained by MHBM combined with the arc length continuation. The frequency response plot when $\alpha = 1.1$ is shown in Fig.6. The symmetry breaking bifurcation observed

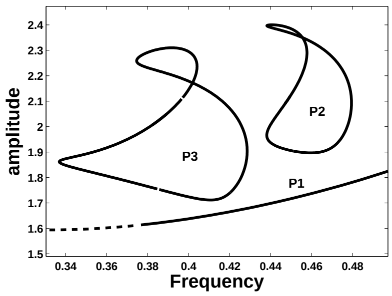

Figure 4. P2 and P3 subharmonics

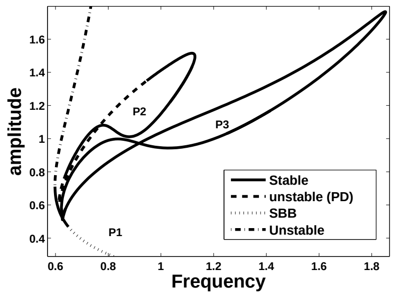

Figure 5. P2 and P3 subharmonics

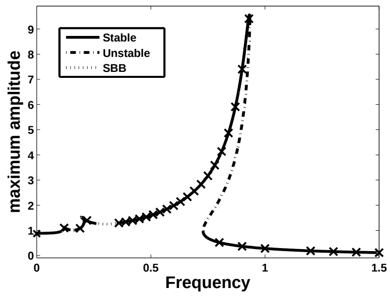

Figure 6. Frequency response at $\alpha = 1.1$

beyond the primary resonance disappears and the original P1 solution get stabilized. The symmetry breaking bifurcation observed at lower frequency values still exists but the frequency range for which it occurs is changed($0.15 \leq \omega \leq 0.162$ and $0.26 \leq \omega \leq 0.35$). Another observation is that with the increase in the value of α the jump up frequency shifts more towards the right while the jump down frequency and the maximum amplitude remains the same. The MHBM solutions are validated by numerical integration and found to have good agreement. The symmetry breaking bifurcation loop around the unstable P1 solution can be obtained by MHBM combined with the path following. It is observed that the motion is chaotic for certain parameter values. For example at $\alpha = 0.8$,$\omega = 0.24$ and 0.9, the motion is chaotic with the largest Lyapunov exponent being 0.051 and 0.147.

CONCLUSIONS

The MHBM is found to be computationally very efficient to obtain the periodic response and bifurcations of the SD oscillator. Apart from the P1 motions, the higher order periodic motions were also obtained by this method. The oscillator is found to have rich dynamics with co-existence of chaotic and periodic solutions of various orders.The smoothening parameter α is found to have significant effect over the bifurcation behavior of the oscillator.

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