Theory and experiment on nonlinear trends of composite laminated plate

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Summary Based on the frequency-response functions derived by using a strong coupled method. The trends of nonlinearity of composite laminated plate are studied, and the results indicate that a simply supported perfect composite laminated plate may have softening and hardening nonlinearity at the same time. The sweep experiment is conducted to verify the theoretical results. The spectrums of the test show that the change of the nonlinear trend may be caused by the variation of the vibration state of the plate.

INTRODUCTION

Composite laminated plates contribute a lot in numerous fields such as aeronautics and astronautics. The large deflection and large amplitude vibration may be occurred in operation of the aerospace vehicles, and this vibration will lead to the serious loss of lives and properties. It is well known that the plate structure have hardening nonlinearity, and some researches show that the plate may have softening nonlinearity because of the imperfection and several boundary conditions. The multiple scales method [1] and the third-order shear deformation plate theory [2] make a great contribution to the dynamics of structures. Yang and Kitipornchai [3] presented that the CSCF and CFCF thermo-electro-mechanically stressed FGM laminated plates have soft-spring behavior at large vibration amplitudes. Amabili [4] theoretically and experimentally studied the effects of geometric imperfections on the trend of nonlinearity and on the natural frequencies. The theoretical and experimental study on the nonlinear trend of a simply supported perfect composite laminated plate is conducted to find some other reasons of the different trends.

FREQUENCY-RESPONSE CURVES

The four dimensional averaged equation in polar form of a simply supported at the four-edge composite laminated thin plate subjected to the transversal and in-plane excitations can be derived with the von Karman strain-displacement relations, the Reddy's third order theory, the Hamilton's principle, the Galerkin's approach and the multiple scales method. The first two modes of the plate are considered and the one-to-two internal resonance, 1/2-subharmonic resonance and principal parametric resonance are involved in this paper. Then, the frequency-response functions are easy to be obtained through the averaged equation.

$$\left(\frac{c_1a_1}{2}\right)^2+\left(-\frac{\sigma_1a_1}{2}+\frac{3\beta_{14}a_1^3}{4}+\frac{a_1a_2^2\beta_{12}}{2}\right)^2=\frac{\beta_{16}^2p_1^2a_1^2}{4}\,, \tag{1a}$$

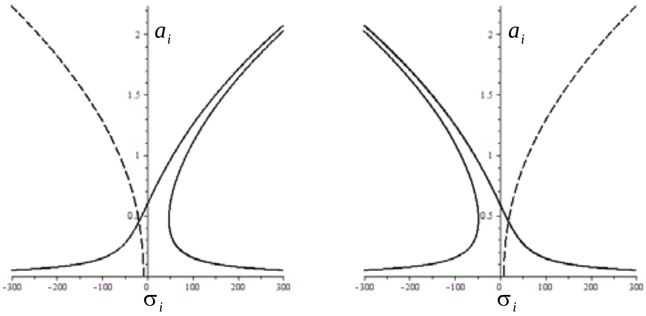
$$\left(\frac{c_2a_2}{2}\right)^2+\left(-\sigma_2a_2+\frac{3\beta_{24}a_2^3}{8}+\frac{\beta_{22}a_1^2a_2}{4}\right)^2=\frac{\beta_{27}^2F_2^2}{4}\,. \tag{1b}$$

Where c_1, c_2, β_{12}, β_{14}, β_{22}, β_{24}, β_{16}, β_{27} are determined by the natural characteristics of the plate, a_1 and a_2 represent the amplitudes of the first two modes, σ_1 and σ_1 are detuning parameters which determined by the resonance relationship $2\omega_1=\Omega_1-\varepsilon\sigma_1$, $\omega_2=\Omega_1-\varepsilon\sigma_2$, $\Omega_1=\Omega_2$, and $0<\varepsilon<<1$, Ω_1 and Ω_2 indicate the in-plane and transversal excitation frequencies. Substituting equation (1a) into (1b) and retaining different variables, we get

$$\left(\frac{c_2}{2}\right)^2+\left(-\sigma_2+\frac{3\beta_{24}}{8}\left(-\frac{3\beta_{14}a_1^2}{2\beta_{12}}+\frac{\sigma_1}{\beta_{12}}\pm\frac{\sqrt{-c_1^2+\beta_{16}^2p_1^2}}{\beta_{12}}\right)+\frac{\beta_{22}a_1^2}{4}\right)^2=\frac{\beta_{12}\beta_{27}^2F_2^2}{-6\beta_{14}a_1^2+4\sigma_1\pm4\sqrt{-c_1^2+\beta_{16}^2p_1^2}}\,, \tag{2a}$$

$$\left(\frac{c_2}{2}\right)^2+\left(-\sigma_2+\frac{3\beta_{24}a_2^2}{8}+\frac{\beta_{22}}{4}\left(-\frac{2\beta_{12}a_2^2}{3\beta_{14}}+\frac{2\sigma_1}{3\beta_{14}}\pm\frac{2\sqrt{-c_1^2+\beta_{16}^2p_1^2}}{3\beta_{14}}\right)\right)^2=\frac{\beta_{27}^2F_2^2}{4a_2^2}\,. \tag{2b}$$

To study the nonlinear characteristics of plates with different materials and dimensions, β_{12}, β_{22}, β_{14}, β_{24} are selected as the different values to draw the frequency-response curves by the software Maple. There, c_1, c_2, β_{16}, β_{27}, p_1, F_2 take fixed values for simplicity of calculation and comparability of those curves, where $c_1=0.1$, $c_2=0.1$, $\beta_{16}=1$, $\beta_{27}=1$, $p_1=0.1$, $F_2=20$. Furthermore, $\sigma_1=\sigma_2$ is assumed, which accord with the physical properties and represent the exact internal resonance. An interesting result obtained after a great number of numerical simulations, the nonlinear trends of the two modes of the plate are different under some parameters. Figure 1(a) shows that the first mode has softening nonlinearity and the second mode has hardening nonlinearity, and the nonlinear trend of the two modes is reversed in Figure 1(b). The behaviors in the two figures all demonstrate the nonlinear trend will be changed because of the present of internal resonance and harmonic resonance. That is, the nonlinear trend is changed because the vibration energy is shifted between two modes. In figure 1, $i = 1, 2$, the dash and solid lines represent the first and second modes.

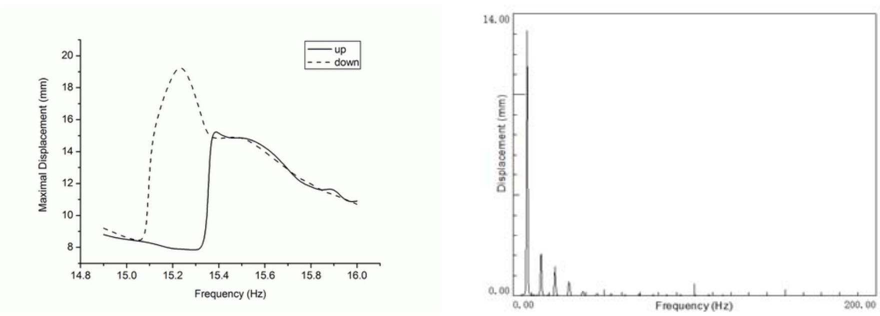


(*a*) (*b*)

Figure 1 Two typical combinations of the frequency-response curve of the two modes. (a) $\beta_{12} = -30$, $\beta_{22} = -80$, $\beta_{14} = -40$, $\beta_{24} = 100$; (b) $\beta_{12} = 30$, $\beta_{22} = 80$, $\beta_{14} = 40$, $\beta_{24} = -100$.

EXPERIMENT

The experiment apparatus of the four-edge simply supported composite laminated plate subjected to the transversal excitation including a Dongling Es-60-445 Vibration Test System, a LMS Test. Lab and a B&K sensor. The testing object is a carbon fiber reinforced composite laminated plate, The plate is swept between 14.5Hz and 16Hz which around the second mode and the sweep rate is 0.01Hz/s, and the experiment including two processes of up sweep and down sweep when the excitation acceleration amplitude is 1.2g. The experimental frequency-response curves which shown in Figure 2(a) is obtained based on the test data, and then the nonlinear tend is naturally present from observing the curves. Figure 2(b) is the amplitude spectrum of a data segment just after the jump at the up sweep and the vibration energy around the first mode is dominant in the spectrum. Combined with spectrums of some other segments, we can conclude that the internal and harmonic resonance changes the nonlinear trend.



(*a*) (*b*)

Figure 2 Experiment results. (a) Experimental frequency-response curves; (b) frequency components after jump.

CONCLUSIONS

The nonlinear trends of composite laminated plates are investigated theoretically and experimentally in this paper. A simply supported perfect plate can present different trends of nonlinearity at different modes according to the numerical simulation. The experiment indicates that the plate shows softening nonlinearity because of the sub-harmonic resonance, which can verify the theoretical results to some degree. It is cannot be denied that the composite laminated plate in the experiment may have some imperfections and the boundary condition of simply supported is not perfect. However, the dominant reasons of the change of the nonlinear trend are the energy shift between two modes from the theoretical analysis and test observation.

References

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