GLOBAL DYNAMICS AND COMPLEXITY: A MODERN PERSPECTIVE FOR THE ANALYSIS, DESIGN AND CONTROL OF MECHANICAL/STRUCTURAL SYSTEMS

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Summary The classical (Lyapunov) concept of local stability is reconsidered with the aim of reducing the gap between the theoretical predictions and the critical instability thresholds observed in practice or in experiments. The key idea is that the attractor must be robust with respect to *finite*, although small, perturbations, and not only with respect to *infinitesimal* perturbations, as in classical stability. This requires considering several theoretical issues, like safe basin, integrity measure, etc., which are particularly involved in the presence of fractality, a common situation in nonlinear dynamics. Furthermore, a local analysis is no longer sufficient, and a detailed study of the global dynamics is required, even in small, but finite, neighbourhoods of the attractor. Finally, these ideas lead toward new approaches to the control of the nonlinear dynamics of mechanical and structural systems. These topics are deeply discussed in this paper, since they are particularly important in load carrying capacity problems, although they are useful in many other fields of science and engineering.

ABSTRACT

In the last 50 years, the achievements of complexity have entailed a substantial change of perspective within the nonlinear dynamics community, and they are now ready to meaningfully affect engineering science and technology. This paper aims at highlighting and discussing the important, yet still overlooked, role played by some relevant paradigmatic concepts in the analysis, design and control of mechanical/structural systems for engineering applications. Attention is paid, in particular, to the evolution and update of the old concept of stability, as ensuing from consideration of global dynamics effects.

The problem of "practical stability" of attractors of dynamical systems is addressed starting from the Thompson idea [1] that classical (Lyapunov) local stability is not enough for practical uses, an issue which bridges the gap between theoretical investigations and practical applications, as recently reconsidered by the authors [2, 3]. The very basic idea is that, since in nature/technology there are always imperfections of static or dynamic nature, the system must be able to sustain changes both in control parameters and in initial conditions (i.c.) without changing the desired outcome.

In the mechanical community, the effects of changes of parameters were deeply studied by Koiter, co-workers and followers, whereas in the mathematical community their analysis led to the development of the structural stability theory. Yet this theory, when applied to classical mechanical engineering problems (such as, e.g., buckling), has still a substantially local character, with the main abstract result being that pitchfork and transcritical bifurcations perturb to a saddle-node bifurcation; thus, they cannot be observed in real world, unless some constraints (e.g. symmetry) force their occurrence. Hopf and period doubling bifurcations, on the other hand, are structurally stable, and this agrees with the fact that they are observed experimentally in both laboratories and practical applications.

Historically, the persistence of system response with respect to changes in i.c. was analyzed earlier, and led to the concept of Lyapunov local stability, which played a major role in the solutions of engineering problems ensuing from technological developments. It is so important that, contrary to structural stability, to date it is taught in any engineering program. The mathematical definition of stability roughly says that under infinitesimal changes in initial conditions the system must keep the reference response.

Recently, without overlooking the fundamental value of the classical concept of stability, the need to properly update it for practical purposes has become apparent. In fact, considering only infinitesimal changes in initial conditions, which makes sense from the mathematical viewpoint, appears weak from a mechanical/practical viewpoint, especially close to bifurcation points where small (but finite) perturbations may significantly change the system response. This is the theoretical justification behind the fact that in experiments it is not possible to exactly detect the bifurcation points. Indeed, finite, though small, changes of i.c. have to be taken into account, this leading to the concept of "practical stability."

In spite of its conceptual simplicity, this is a paramount enhancement, full of consequences and problems to be accurately addressed. The most important one is concerned with the need to definitely abandon a merely local perspective and pass to a global one, in both time-independent and time-dependent problems, by considering the whole dynamical behaviour of the system even if being actually interested in only a small (but finite!) neighbourhood of the considered solution. This has both theoretical and practical implications. In fact, in an actually dynamic environment, it requires on one side to analyze the effects of non-classical solutions such as homoclinic/heteroclinic orbits, quasi-periodic responses, chaotic attractors, etc., while, on the other side, it calls for the development of numerical tools able to perform the global analysis of systems with large number of degrees of freedom (d.o.f.) in a reasonable time.

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The starting point for an accurate analysis is the correct definition of a “safe basin” [2], which is the set of i.c. sharing a common dynamical property. Safe basins are usually considered to coincide with classical basins of attraction, i.e. with sets of initial conditions leading to given attractors. This is also the point of view necessary for studying their “practical stability”. However, nowadays, it is more apparent that this “common property” may be not only the convergence toward an attractor, but also, e.g., the non-escape from a potential well (irrespective of what happens within the well), the behaviour in transient instead of in steady regime, or even the non-sensitivity to i.c. Also this point involves theoretical and practical issues. In fact, for certain definitions of safe basin (essentially when they are basins of attraction or union of them) the basin boundaries, which play a major role, are stable manifolds of given saddles, and have a well-defined behaviour and properties, not shared by other safe basin definitions. From a practical point of view the issue is that, apart from few cases, safe basins can be determined only numerically, and to date this is still computationally onerous for systems with more than, say, four d.o.f.
Once the safe basin has been properly defined, one can consider an attractor as “practically stable” if its safe basin is large enough. This correlation is correct, but the meaning of “large enough” is not trivial and needs a proper definition of “magnitude”. The most natural one refers to the geometrical hyper-volume of the safe basin and is technically formalized by the so-called Global Integrity Measure (GIM, [2]), which is a dimensionless measure suitable for parametric studies. Although GIM is sufficient in various cases, there are many others, mainly related to possible fractality of the basin of attraction, where it is inadequate. Fractal basin boundaries may spread the basin all over the phase space, destroying – or strongly reducing – the compact subset around the attractor, which is the sole one allowing for safe finite changes in i.c.
Thus, one must introduce and use measures of the integer (or compact) part of the basin like, e.g., the Local Integrity Measure (LIM) or the Integrity Factor (IF), which is the normalized radius of the largest hyper-sphere belonging to the safe basin [2]. They allow to rule out the fractal part of the basin from the measure of its magnitude, and thus they are required when fractality is a drawback (but not when it is desired, as in mixing problems!). The advantages and disadvantages of distinct measures are discussed in the paper.
The analysis of the variable topology of basins permits to dwell on the concept of robustness of a dynamic solution against i.c. as the result of a proper magnitude of its safe, i.e. integer, basin. In fact, in a time-independent environment or in the absence of fractality of the basin boundary of, e.g., a dynamic solution existing and possibly competing with another one within a potential well, robustness of the solution solely depends on the magnitude of its (certainly integer) safe basin. However, in the presence of either in-well or induced from out-of-well fractality, robustness of the solution also depends on the integrity (compactness) of its basin, which becomes thus an essential ingredient of the very definition of safe basin.
Moreover, still having in mind the “practical stability” concept, it is necessary to investigate how attractor robustness and basin integrity evolve with a varying control parameter. This is done via the so called erosion profiles, which are useful graphical tools allowing for an immediate feeling of the relevant loss (or even increase), to be possibly explained in terms of global phenomena (homo/heteroclinic bifurcations, crises, etc.) occurring in the system. In this respect, the most important result is that robustness and integrity become residual well before the disappearance of the attractor by local bifurcations: it is just this item which makes the concept of “practical stability,” and the associated global analysis, necessary in applications [3].
The overall transition from a local stability perspective to a global safety concept has also major implications as regards the feasibility and effectiveness of a control of the nonlinear dynamics of engineering systems based just on exploiting some associated global bifurcational event [4]. In fact, the control approach and techniques may drastically change when the goal is increasing the dynamical integrity of the system, and no longer controlling (i.e. stabilizing) a single trajectory, as done by classical chaos control techniques. Here in fact, one needs to control the overall dynamics, and methods based on shifting homoclinic or heteroclinic bifurcations triggering the fractalization can help.
The previous ideas are illustrated in the paper by referring to several models describing the behavior of discrete systems (mechanical pendulums in different configurations) or representing reduced order approximations of continuous systems (post-buckled cylindrical shells, micro- or nano-beams). It is also shown how these concepts permit to explain some experimental results where the observed excitation threshold of existence of attractors is well below the one theoretically predicted by classical, local, stability.

References

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