RECTILINEAR MOTION OF A TWO-MASS POINT SYSTEM IN A RESISTIVE MEDIUM

Klaus Zimmermann, Igor Zeidis[a)] & Mikhail Pivovarov
Faculty of Mechanical Engineering, Ilmenau University of Technology,
Ilmenau, D-98693, Germany

Summary: The problem of motion of two interconnected mass points in a resistive medium under periodic change of distance between them is considered. It is shown that a necessary condition for the locomotion (displacement of the center of mass) is a nonlinear law of resistance (dry or nonlinear viscous friction). If the mass points are identical, any periodic control law leads to a displacement of the center of mass only in the presence of anisotropic (asymmetrical) friction. If the masses are different in magnitude, the motion is possible also in a medium with isotropic friction law under the action of a periodic control law. In this paper, such special laws for the control of the velocity and the direction of motion are presented. The friction is assumed to be small and the investigations are based on the method of averaging. Analytical expressions for the velocity of the motion are obtained.

INTRODUCTION

The rectilinear motion of a system of two identical bodies (point masses) in a resistive medium, excited by periodic (harmonic) change in the distance between the bodies, is considered in [1]. In the same monograph, the motion of a chain of identical bodies, the distances between which change according to the same harmonic law, is studied. It is shown that in this case, the progressive motion of the center of mass on the average is possible only if the friction (dry or viscous) is anisotropic, i.e., the coefficient of friction depends on the direction of motion; the displacement of the center of mass being possible only in the direction of lower friction. The motion of a system of three bodies when the laws of change in the distances between the bodies are shifted in phase relative to one another is investigated in [2]. In this case, the motion along a rough plane is possible and, moreover, the system can move even in the direction of higher friction if the friction law is anisotropic. The motion of two interacting bodies in a dry-friction environment is considered in [3]. A periodic relative motion of the bodies with piecewise constant velocity that generates the motion of the entire system with periodically changing velocity is constructed. The expressions for the average velocity of the system and for the energy consumption per unit path are obtained. The optimal parameters maximizing the average velocity are determined.

MATHEMATICAL MODEL

We consider the motion of two different mass points along a horizontal straight line OX in a resistive medium (see figure 1, left). Let m_1 and m_2 denote the masses of each mass point and x_1 and x_2 denote their coordinates. The motion is excited and controlled by changing the distances between the mass points. The medium acts on each point with the force of friction $F(\dot{x}_i)$, depending on the velocity $\dot{x}_i$ ($i = 1,2$) of this mass point relative to the environment.

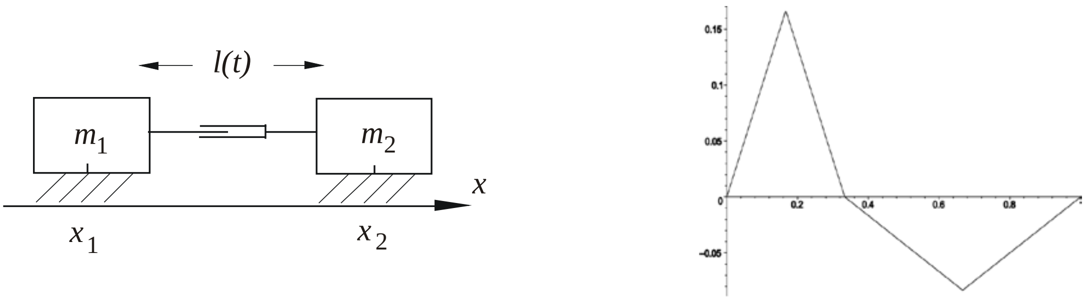

Figure 1. Mechanical model (left) and u vs. t for $\tau = 1/3$ (right).

We assume that the distance between the mass points $l(t)$ is a periodic function with period T, i.e.,

$$x_2 - x_1 = l(t), \qquad l(t+T) = l(t). \tag{1}$$

The coordinate X of the center of mass of the system and its velocity V are defined by

$$X = \mu_1 x_1 + \mu_2 x_2, \quad V = \dot{X}, \quad \mu_1 = m_1/M, \quad \mu_2 = m_2/M, \quad M = m_1 + m_2, \quad \mu_1 + \mu_2 = 1. \tag{2}$$

The motion of the center of mass is governed by the equation $M\dot{V} = F(\dot{x}_1) + F(\dot{x}_2)$.
Using Eqs. (1) and (2), we obtain the equation of the motion in the form

$$\dot{V} = \left(F(V - \mu_2 u) + F(V + \mu_1 u)\right)/M, \qquad u(t) = \dot{l}(t). \tag{3}$$

Since $l(t)$ is a periodic function with period T, the average value of the function $u(t)$ for this period is zero.

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a) Corresponding author. Email: igor.zeidis@tu-ilmenau.de.

Linear resistance law

In this case, $F(z) = -kz$, where k is the coefficient of friction, and Eq. (3) becomes $\dot{V} = -k\big(2V + (\mu_1 - \mu_2)u\big)/M$. Integrate this equation over n periods (from 0 to nT) to obtain $\int_0^{nT} \dot{V}(t)dt = -\frac{2k}{M}\int_0^{nT} V(t)dt - \frac{k}{M}(\mu_1 - \mu_2)\int_0^{nT} u(t)dt$.

The average velocity $\overline{V}$ on the interval between 0 and nT is defined by $\overline{V} = \frac{1}{nT}\int_0^{nT} V(t)dt$. Taking into account that the average value of function $u(t)$ for the period T is zero, we obtain $\overline{V} = M(V(0) - V(nT))/(2kTn)$. As $n \to \infty$, the value $V(nT)$ is bounded (this follows from the equation of motion), and hence $\overline{V} \to 0$.

Coulomb's dry friction

Dry friction is the resistance characterized by Coulomb's law: $F(\dot{x}_i) = k_- m_i g$, if $\dot{x}_i < 0$, $F(\dot{x}_i) = -k_+ m_i g$, if $\dot{x}_i > 0$, $F(\dot{x}_i) \in [-k_+ m_i g, k_- m_i g]$, if $\dot{x}_i = 0$, $i = 1, 2$. Here k_- and k_+ are positive quantities.

Introduce the dimensionless variables (denoted by the asterisks): $t^* = t/T$, $l^* = l/L$, $V^* = VT/L$, $F^* = F/F_0$, where L is a length characteristic of the change in the distances between the mass points, and F_0 is a quantity characterizing the magnitude of the friction force. For example, we can define $F_0 = \max(k_-, k_+)Mg$. The nondimensionalized Eq. (3) can be represented as (the asterisks are omitted)

$$\dot{V} = \varepsilon G(V,t), \quad G(V,t) = F(V - \mu_2 u) + F(V + \mu_1 u), \quad \varepsilon = F_0 T^2/(ML). \tag{4}$$

Analytical investigations: method of averaging

Consider a smooth periodic control $l(t)$. The function $l(t)$ is formed by parabolic segments so that in the intervals $0 \le t < \tau/2$ and $\tau/2 \le t < \tau$, the absolute value of the second derivative of this function is constant and is equal to a, whereas in the intervals $\tau \le t < 1/2 + \tau/2$ and $1/2 + \tau/2 \le t \le 1$, the absolute value of the second derivative is also constant but is different in magnitude ($L_0 = 2aT^2$). The parameter τ characterizes the asymmetry degree. The dimensionless derivative $u(t)$ of the function $l(t)$ is shown in figure 1, right.

We assume that the parameter ε is small ($\varepsilon << 1$) and apply the method of averaging to Eq. (4) to obtain the time-invariant averaged equation

$$\dot{v} = \varepsilon \overline{G}(v), \quad \overline{G}(v) = \int_0^1 \big(F(v - \mu_2 u) + F(v + \mu_1 u)\big)dt. \tag{5}$$

We consider the case of dry friction and are interested in the steady-state motions, i.e. in the motions with constant velocity. The steady-state velocity, defined as the solution of the equation $\overline{G}(v) = 0$, is expressed as follows:

$$v_* = \frac{\mu_1\mu_2\tau^2\big(\mu_2 - \kappa\mu_1 - \tau(1+\kappa)(\mu_2 - \mu_1)\big)}{2(1+\kappa)\big(\mu_1^2\tau^2 + \mu_2^2(1-\tau)^2\big)}, \quad \kappa = \kappa_+/\kappa_-. \tag{6}$$

If the friction is isotropic ($\kappa = 1$), expression (6) becomes

$$v_* = \frac{\mu_1\mu_2\tau^2(\mu_2 - \mu_1)(1 - 2\tau)}{4\big(\mu_1^2\tau^2 + \mu_2^2(1-\tau)^2\big)}.$$

In the case of isotropic friction, necessary and sufficient conditions for locomotion are a difference in masses of the mass points ($\mu_1 \ne \mu_2$) and an asymmetry in the control law ($\tau \ne 1/2$).

CONCLUSIONS

The rectilinear motion of two different point masses in a resistive environment was studied. The motion is excited and controlled by a periodic change in the distance between the bodies. It is shown that for the linear resistance law, the average velocity is zero for any periodic law of changing the distance. For low anisotropic dry friction and a particular periodic asymmetric control law, analytical expressions for the average velocity of the entire system were obtained on the basis of the method of averaging. For the system to be able to move in the environment characterized by isotropic friction, it is necessary and sufficient that the point masses have different magnitudes and the control law be asymmetric.

References

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