RESEARCH ADVANCES OF EXACT SOLUTIONS FOR FREE VIBRATIONS OF PLATES AND SHELLS

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Summary This paper reviews the recent research advances of exact solutions for the transverse free vibrations of isotropic and orthotropic Kirchhoff and Mindlin plates and Donnell shells, and for the in-plane free vibrations of isotropic and orthotropic plates. Most new exact solutions were obtained by the authors of present paper using the direct separation of variable method.

FREE VIBRATION OF THIN PLATES

The free vibration of thin plate is a classical elastic problem which has been extensively studied using different methods. The exact normal modes and frequency equations of isotropic rectangular thin plates with at least two simply supported opposite edges were tabulated [1], these exact solutions were solved by inverse method or the Levy's type of method in which a pair of simple support boundary conditions are satisfied by the constructing eigenvalue and corresponding eigenfunction. The essence of the inverse method is to transform the double spatial-eigenvalue problem to the single spatial-eigenvalue problem, it is only applicable to the plate with at least two simply supported opposite edges. Gorman [2] has done many works on vibration analysis of plates by the superposition method, but his analytical solutions are in series expansion form, not in closed form. Until 2009, Xing and Liu [3, 4] made some progress and solved the exact solutions for free vibration of isotropic and orthotropic thin rectangular plates with any combinations of simply supported and clamped boundary conditions using direct separation of variable method, not inverse method. The exact solutions [3, 4] include the six available solutions [1] and the solutions of thin plate with CCSS (C denotes clamp support, S denotes simple support), CCCS and CCCC boundaries which were regarded not solvable before.

FREE VIBRATION OF MINDLIN PLATES

It is more difficult to obtain the exact solutions for the free vibrations of Mindlin plate compared with thin plate. Until 1994, Huang et al. [5] solved the exact solutions for free vibrations of isotropic thick sectorial plates with simply supported radial edges. In 2005, Hashemi and Arsanjani [6] solved the exact characteristic equations for rectangular isotropic Mindlin plates with two opposite edges simply supported. Recently, Xing and Liu, using direct separation of variables, made some progress and achieved a few new results for this problem, including: (1) A new two-eigenfunction theory [7], using the amplitude deflection and the generalized curvature as two fundamental eigenfunctions, was proposed for the free vibration solutions of isotropic rectangular Mindlin plates. The three classical eigenvalue differential equations of Mindlin plate were reformulated to arrive at two new eigenvalue differential equations for the new theory. The closed form eigensolutions, which were solved from the two differential equations, are identical with those based on Kirchhoff plate theory for thin plate, and can be employed to predict frequencies for any combinations of simply supported and clamped edge conditions. (2) the closed-form analytical solutions, which satisfy two of the three governing equations and all boundary conditions and are accurate for isotropic rectangular plates with moderate thickness, were obtained [8] for any combinations of simply supported and clamped edges, the free edges can also be dealt with if the other pair of opposite edges is simply supported; Three different characteristic equations were derived by using three different methods, it was found that the deflection could be expressed by four characteristic roots and the two rotations by all six characteristic roots, this particularity of Mindlin plate theory was revealed for the first time. (3) Exact closed-form solutions were obtained [9] for free vibrations of orthotropic rectangular Mindlin plates with two opposite edges simply supported and all possible combinations of classical boundary conditions at the other two edges. The exact solutions were validated through both mathematical proof and numerical comparisons with available p-Ritz solutions and the differential quadrature finite element method (DQFEM [10]) solutions calculated by the authors.

IN-PLANE FREE VIBRATION OF PLATES

There is an on-going interest in the in-plane vibration of plates due to a realization of its importance in recent years. The in-plane vibration modes can be of very practical significance in aerospace structures, piezoelectric crystals, etc. It is noteworthy that there have been some exact solutions for the free in-plane of clamped circular plates by Park [11] and rectangular plates with at least two opposite edges simply supported by Gorman [12] who employed a quarter-plate approach, and assumed the two non-simply supported edges were either free or clamped. In Gorman's elegant work, only one quarter of the rectangular plate was analyzed to avoid the interpretation of the computed mode shapes, the

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probability of missing an eigenvalue and the problem of repeated eigenvalues. Using the separation of variables and analyzing the entire plate, Xing and Liu [13] formulated all classical boundary conditions including two distinct types of simple support boundary conditions using the Rayleigh quotient variational principle and obtained exact solutions for in-plane vibrations of isotropic rectangular plates with two simply supported opposite edges and any classical boundary conditions at the other two edges, and some of the exact solutions were not available before. Recently, Liu and Xing [14] obtained all possible exact solutions in terms of 10 sets of distinct eigensolutions for the free in-plane vibration of orthotropic rectangular plates with boundary conditions as [13].

FREE VIBRATION OF SHELLS

Exact solutions to the differential equations of motion for shells are available only for a limited number of conditions. In most papers using the exact solutions, only shells with all four boundaries having the shear diaphragm (SD) boundaries are used. It is known that the exact procedure is complicated for shells and this complexity has prevented most researchers from getting exact results. Recently, Liu and Xing [15] presented an analytical procedure and closed-form vibration solutions with analytically determined coefficients for orthotropic circular cylindrical shells having classical boundary conditions. This analysis was based upon the Donnell–Mushtari shell theory. This is the simplest thin shell theory and its results for the lowest frequencies of a closed cylinder may not be as accurate. Using the separation of variables method, the closed-form natural frequencies in a compact form were successfully obtained. The exact solutions were validated through numerical comparisons with available solutions in literatures and the semi-analytical differential quadrature finite element method (S-DQFEM) solutions calculated by the authors. The exact solutions for free vibrations of thick shells are limited to shells with all four boundaries having the shear diaphragm boundaries so far.

CONCLUSIONS

This paper reviewed the research advances of exact solutions for free vibrations of plates and shells. It was regarded that most of the new exact solutions were impossible or hard to be solved. Besides the above results, we also obtained some exact solutions for the free vibration of open and closed cylinder shells, and submitted them to journal for consideration of publication. In the future, there are many relative works need to be studied, as the free vibration of thin plate with two free adjacent edges, and the eigenbuckling problems of plates and shells.

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