PROBABILISTIC SOLUTION OF THE STOCHASTIC OSCILLATORS WITH NONZERO MEAN RESPONSE

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Summary The probabilistic solution of the nonlinear stochastic oscillators with even nonlinearity and constant term is investigated with exponential-polynomial closure (EPC) method. Numerical results show that the results obtained with the exponential-polynomial closure method agree well with the simulated solution in the presented case even if the mean of displacement is nonzero and the probability density function of displacement is nonsymmetric about its mean.

INTRODUCTION

The investigation on the probabilistic solutions of nonlinear stochastic oscillators was attractive in the last decades because other statistic or reliability analysis is based on the probabilistic solutions of the oscillator responses. Excited by Gaussian white noise, the probability density function (PDF) of the oscillator responses is governed by Fokker-Planck-Kolmogorov (FPK) equation. Therefore, various methods were proposed for the solution of the FPK equation or the moment equations resulted from the FPK equation. Equivalent linearization method (EQL) is frequently employed to obtain the approximate mean and second moment of the responses of weakly nonlinear stochastic dynamic (NSD) systems [1, 2, 3]. The advantage of the EQL method is that it is applicable for multi-degree-of-freedom systems. Monte Carlo simulation is another method that is applicable for the numerical solution of multi-degree-of-freedom NSD systems, but the computational efficiency, numerical stability, convergence, round-off error, and requirement for large sample size can be challenges for Monte Carlo simulation in analyzing large systems [4, 5]. Exponential polynomial closure (EPC) method was proposed and employed to analyze the solution of the reduced FPK equations about various stochastic oscillators with odd nonlinearity [6, 7], the oscillators with odd nonlinearity and constant term [8], and 2-degree-of-freedom NSD systems [9]. Recently, a new method named state-space-split-EPC (3S-EPC) was proposed for the probabilistic solutions of some multi-degree-of-freedom NSD systems or the solutions of some reduced FPK equations in high dimensions [10, 11]. With the 3S-EPC method, the FPK equations in high dimensions is reduced to the FPK equations in low dimensions which are solvable with the EPC solution procedure. Hence, the the investigation on the effectiveness of the EPC method in various cases is fundamental for analyzing the multi-degree-of-freedom NSD systems. In this paper, the PDF solution of the oscillators with the terms with odd and even nonlinearities in displacement, the term with odd nonlinearity in velocity, and constant term is investigated with the EPC method. Numerical results obtained from the EPC, EQL, and Monte Carlo simulation methods are given to investigate and show the effectiveness of the EPC method in analyzing the probabilistic solutions of the nonlinear stochastic oscillators in this case.

NONLINEAR STOCHASTIC THE OSCILLATORS AND ANALYSIS

EPC solution procedure

Considering the following oscillator with nonlinearity in both displacement and velocity besides constant term.

$$\ddot{X} + \alpha\dot{X} + \beta\dot{X}^3 + \gamma X + \varepsilon_1 X^2 + \varepsilon_2 X^3 + m = W(t) \tag{1}$$

where X and $\dot{X}$ denote the displacement and velocity of the oscillator, respectively; $W(t)$ is zero-mean Gaussian white noise which autocorrelation function is

$$E[W(t)W(t+\tau)] = \delta(\tau) \tag{2}$$

Set the values of system parameters as $\alpha = 1, \beta = 0.3, \gamma = 1, \varepsilon_1 = 0.5$, $\varepsilon_2 = 0.3$, and $m = 0.3$. In this case, the oscillator is strongly nonlinear and the exact solution is not available at the moment. The EPC method is employed to obtain the approximate PDF of the oscillator responses. The PDFs and mean up-crossing rate (MCR) of displacement from Monte Carlo simulation with 10^8 samples are used to test the solution obtained from EPC method. The results are presented in Figs. (a)-(d).

Numerical Analysis and Discussions

From Figs. (a)-(d), it is seen that the results obtained from EPC with n=6 agree well with simulation while the PDFs and MCRs of displacement given by EQL deviate a lot from the simulation, especially in the tail regions of the PDF and MCR. It is also seen that the mean value of displacement is nonzero, and the PDF of displacement is nonsymmetric about the mean of displacement. It is observed from numerical analysis that the mean value and the variance of displacement are influenced by the values of all the nonlinear parameters $\beta, \varepsilon_1, \varepsilon_2$ and constant term m. The mean value of displacement is more sensitive to constant term m than to ε_1,especially to β. As the constant term m or the parameter ε_1 increases, the absolute value of the mean of displacement increases while the variance of displacement decreases. Numerical results

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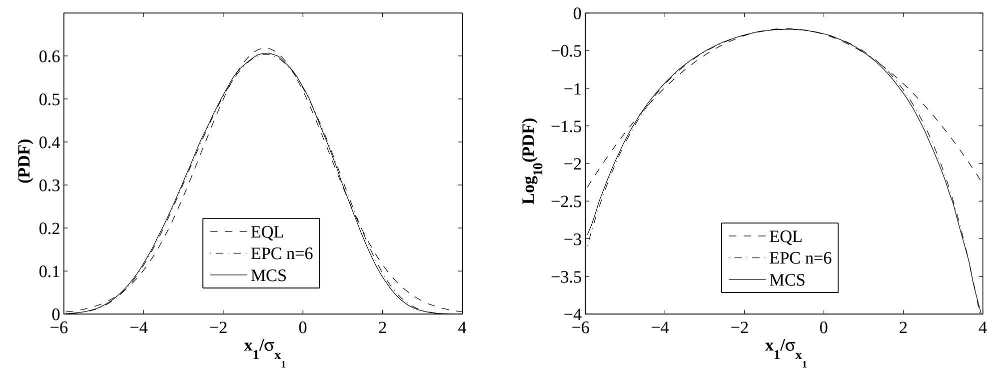
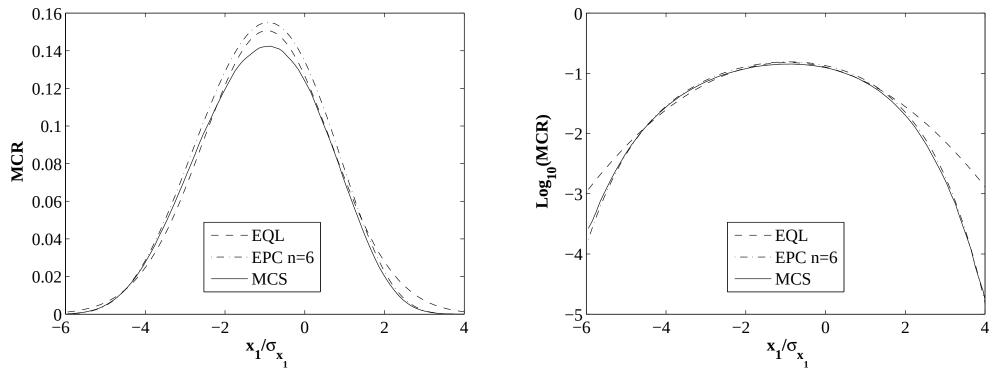


(a) PDF of displacement

(b) Log(PDF) of displacement



(c) MCR of displacement

(d) Log(MCR) of displacement

also show that the variance of velocity always keeps the same value when there are changes in the nonlinear parameters or constant term.

CONCLUSIONS

The probabilistic solutions of the oscillator with both odd and even nonlinearity in displacement, odd nonlinearity in velocity, and constant term are analyzed with EPC method, EQL method, and MCS method. The numerical analysis shows that the EPC method is effective in analyzing the PDF and MCR solutions of this type of oscillators.

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References

[1] Booton R. C.: Nonlinear Control Systems with Random Inputs. *IRE Trans. Circuit Theory 1(1)*:9-19, 1954.
[2] Lin Y. K. and Cai G. Q.: Probabilistic Structural Dynamics. McGraw-Hill, International Editions 1995.
[3] Socha L.: Linearization Methods for Stochastic Dynamic Systems. Springer-Verlag, Berlin 2008.
[4] Harris C. J.: Simulation of Multivariate Nonlinear Stochastic System. *Int. J. for Num. Mech. in Eng. 14*:37-50, 1979.
[5] Kloeden P. E. and Platen E.: Numerical Solution of Stochastic Differential Equations. Springer-Verlag, Berlin 1995.
[6] Er G. K.: A New Non-Gaussian Closure Method for the PDF Solution of Nonlinear Random Vibrations. Murakami, H., Luco, J. E., ed., *Engineering Mechanics: A Force for the 21st Century, ASCE 12th Engineering Mechanics Conference*, Reston, 1400-1406, 1998.
[7] Er G. K.: An Improved Non-Gaussian Closure Method for Randomly Excited Nonlinear Stochastic Systems. *Nonlinear Dyn. 17(3)*:285-297, 1998.
[8] Er G. K., Zhu H. T., Iu V. P., and Kou K. P.: Nonzero Mean PDF Solution of Nonlinear Oscillators under External Gaussian White Noise. *Nonlinear Dyn. 62*:743-750, 2011.
[9] Er G. K.: The Probabilistic Solution to Non-linear Random Vibrations of Multi-degree-of-freedom Systems. *ASME J. Appl. Mech. 67*:355-359, 2000.
[10] Er G. K.: Methodology for the Solutions of Some Reduced Fokker-Planck Equations in High Dimensions. *Annalen der Physik 523(3)*:247-258, 2011.
[11] Er G. K. and Iu V. P.: A New Method for the Probabilistic Solutions of Large-scale Nonlinear Stochastic Dynamic Systems. W. Q. Zhu, Y. K. Lin, and G. Q. Cai, ed., *Proceedings of the IUTAM Symposium on Nonlinear Stochastic Dynamics and Control 2010*, Springer, Berlin, 25-34, 2011.