NONLINEAR VIBRATIONS OF AXIALLY MOVING VISCOELASTIC BEAMS WITH SUPERCRITICAL TRANSPORT SPEED

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Summary Nonlinear vibration of supercritically transporting viscoelastic beams is studied. The natural frequencies and modes are analyzed via the Galerkin method for the linear standard form with space-dependent coefficients. Multiple scales method and a finite difference scheme are respectively applied to determine steady-state responses for harmonic excitation. The present procedure can also be extended to tackle other complex dynamical systems such as rotating plates, rings, or shafts, pipes conveying fluid and buckled beams.

INTRODUCTION

The class of travelling continua encompasses such apparently diverse mechanical systems as high-speed magnetic, elevator cables, power transmission band, aerial cable tramways and pipes that contain fluid. The transporting tensioned Euler-Bernoulli beam is the most prototypical model of these travelling systems. Meanwhile, transporting beam is a typical gyroscopic continuum because the governing equations contain a term of the mixed partial derivative with respect to the space and the time coordinates. The related investigations also have theoretical significance. The problem of viscoelasticity effect has not been addressed for vibration of transporting continua in the supercritical regime. And there was no approximate analytical approach to deal with space-dependent coefficients for gyroscopic continua.
The transport speed greatly affects the dynamic behaviour of the travelling systems. At the critical speed, the first frequency becomes zero and the translating element experiences divergence-type instability. In this case, one must resort to a nonlinear analysis. The nonlinear dimensionless governing equation and hybrid boundary conditions are

$$\begin{gathered} v_{,tt}+2\gamma v_{,xt}+\left(\gamma^2-1\right)v_{,xx}+k_{\mathrm{f}}^2 v_{,xxxx}+\alpha v_{,xxxxt}=\frac{k_1^2}{2}v_{,xx}\int_0^1 v_{,x}^2\mathrm{d}x+\frac{\alpha k_1^2}{k_{\mathrm{f}}^2}v_{,xx}\int_0^1 v_{,x}v_{,xt}\mathrm{d}x+b\cos\left(\omega t\right) \\ v\left(0,t\right)=v\left(1,t\right)=0, v''\left(0,t\right)=\eta v'\left(0,t\right), v''\left(1,t\right)=-\eta v'\left(1,t\right) \end{gathered} \tag{1}$$

where a comma denotes partial differentiation, the dimensionless t is the time, x is the spatial coordinate along the axis of the beam, $v(x,t)$ is the transverse displacement, γ is the constant axial speed, k_{f} is the flexural stiffness, k_1 is the nonlinear coefficient, α is the viscosity coefficient, b is the excitation amplitude, and η is the dimensionless stiffness constant of torsion springs at ends. Letting $\eta\to\infty$ and $\eta=0$ in respectively yields fixed ends and simply supported ends.

EQUILIBRIA

Equilibria of travelling beams are a prerequisite for supercritical vibration. The equilibrium solutions $\hat{v}(x)$ satisfy [1,2]

$$\left(\gamma^2-1-\frac{1}{2}k_1^2\int_0^1\hat{v}'^2\,\mathrm{d}x\right)\hat{v}''+k_{\mathrm{f}}^{\,2}\hat{v}''''=0 \tag{2}$$

where the prime indicates differentiation with respect to x and the superscript indicates the sense of the equilibrium displacement. After obtaining ω and θ via $\eta=\omega\tan\theta$ and $\omega=(2k-1)+2\theta$, the equilibria and the critical speeds as follows

$$\hat{v}(x)=\pm\frac{2}{k_1}\sqrt{\frac{\gamma^2-1-\omega^2 k_{\mathrm{f}}^{\,2}}{\omega^2-\omega\sin\omega}}\left[\sin(\omega x-\theta)+\sin\theta\right];\gamma=\sqrt{1+k_{\mathrm{f}}^{\,2}\omega^2} \tag{3}$$

NATURAL FREQUENCIES AND MODES

Ding et al. have investigated frequencies of nonlinear planar vibration for travelling beams in the supercritical regime [3]. Galerkin method is used to solve the linear elastic equations v_0 correspondingly nonlinear equations for the natural frequencies and modes [4]. Under simply supported boundary conditions, suppose that the solution takes the form

$$v_0\left(x,T_0,T_2\right)=\varphi_n\left(x\right)Y_n\left(T_2\right)\mathrm{e}^{\mathrm{i}\omega_n T_0}+cc;\varphi_n\left(x\right)=\sum_{k=1}^{N}C_{nk}\sin(k\pi x) \tag{4}$$

where $T_0=\varepsilon^0 t$ is the fast scale characterizing motions occurring at ω_n, $T_1=\varepsilon^1 t$ is a slow scale and $T_2=\varepsilon^2 t$ is a slower sale characterizing the modulation of the amplitudes and phases due to viscoelasticity and possible resonance, ε is a small parameter, Y_n denote complex functions to be determined later, ω_n and φ_n are respectively the nth modal functions and frequency, and cc stands for complex conjugate of the preceding term. Galerkin procedure leads to

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$$\left[-\omega_n^2\mathbf{M}+\mathrm{i}\,\omega_n\mathbf{G}+\mathbf{K}\right]\begin{pmatrix}C_{n1} & C_{n2} & C_{n3} & C_{n4} & \cdots & \cdots & C_{n(N-1)} & C_{nN}\end{pmatrix}^{\mathrm{T}}=\mathbf{0} \tag{5}$$

For a non-trivial solution, the nth natural frequencies ω_n and the modal function φ_n can be obtained. For N=1, the steady-state responses and the instability boundary can be determined via the Lyapunov linearized stability theory [5].

STEADY-STATE RESPONSE

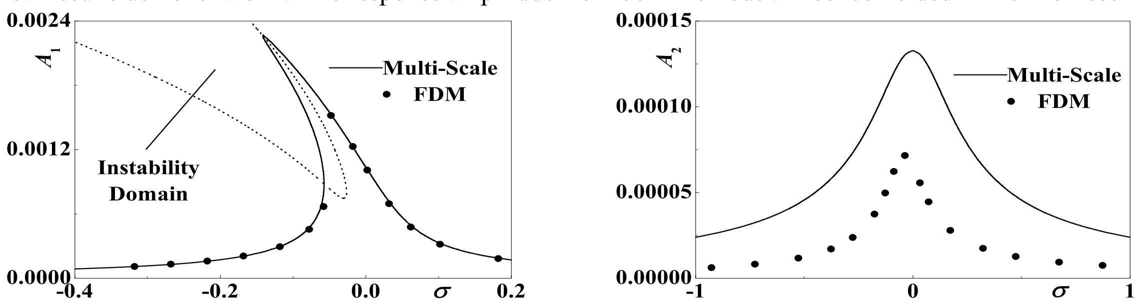
Introducing $\varepsilon^2\alpha$ and $\varepsilon^3 b$ instead of α and b. One assumes an expansion of dimensionless displacement

$$v(x,t)=\varepsilon^1 v_0\left(x,T_0,T_2\right)+\varepsilon^2 v_1\left(x,T_0,T_2\right)+\varepsilon^3 v_2\left(x,T_0,T_2\right)+O\left(\varepsilon^4\right);v_1=\frac{1}{A_S}\left[\phi_{n1}\left(x\right)Y_n^2\mathrm{e}^{2\mathrm{i}\omega_n T_0}+\phi_{n2}\left(x\right)Y_n\bar{Y}_n\right]+cc \tag{6}$$

where $\phi_1\left(x\right)$ and $\phi_2\left(x\right)$ are complex function remain to be determined and satisfy

$$\begin{aligned}&-4\omega_n^2\phi_{n1}+4\mathrm{i}\omega_n\gamma\phi'_{n1}+\pi^2k_{\mathrm{f}}^2\phi''_{n1}+k_{\mathrm{f}}^2\phi''''_{n1}+\pi^4k_1^2A_S^2\sin\left(\pi x\right)\int_0^1\phi_{n1}\sin\left(\pi x\right)\mathrm{d}\,x=\frac{1}{2}\pi^2k_1^2A_S^2\left[\varphi''_n-\sin\left(\pi x\right)\int_0^1\left(\varphi'_n\right)^2\mathrm{d}\,x\right]\\&\pi^2k_{\mathrm{f}}^2\phi''_{n2}+k_{\mathrm{f}}^2\phi''''_{n2}+\pi^4k_1^2A_S^2\sin\left(\pi x\right)\int_0^1\phi_{n2}\sin\left(\pi x\right)\mathrm{d}\,x=\frac{1}{2}\pi^2k_1^2A_S^2\left[\varphi''_n-\sin\left(\pi x\right)\int_0^1\left(\varphi'_n\bar{\varphi}'_n\right)\mathrm{d}\,x\right]\end{aligned} \tag{7}$$

After determining v_0 and v_1, the steady-state responses in transverse vibration under periodic transverse loads can be obtained via solvability condition. Resonance occurs if the external load frequency approaches the natural frequencies. The comparisons of results via the multiple scales method and the finite difference method are shown in Fig. 1. Numerical results demonstrate that the response amplitude from both methods almost coincided in the first resonance.



(a) The first resonance (b) The second resonance

Fig. 1 The first two resonances versus detuning parameter with b=0.001, α=0.0001, k_1=100, k_{f}=0.8 and γ=4.0

CONCLUSIONS

The straight configuration of traveling tensioned beams experiences divergence instability at critical speed. The equilibrium solutions for both fixed ends and the simply supported ends can be reduced from solutions for the hybrid boundary conditions as special cases. The numerical results demonstrate that the Galerkin method can be used to obtain the natural frequencies and modes from the equation with space-dependent coefficients. Based on the natural frequencies and modes, the relation between the load frequency and the response amplitude are determined via the multiple scales method. The finite difference schemes are developed to verify the approximate analytical results. The relation between the response amplitude and the external excitation frequency also reveal that increasing nonlinearity clearly softens the system and the steady-state amplitude under the periodic transverse loads is more sensitive to the dynamic viscosity in the supercritical range. The present work enlarges the knowledge of the dynamics of supercritically travelling systems.

ACKNOWLEDGMENTS

This work was supported by the National Science Foundation of China (Project No. 10902064), Shanghai Rising-Star Program (No. 11QA1402300), and Innovation Program of Shanghai Municipal Education Commission (No. 12YZ028).

References

[1] Ding H., Chen L.Q.: Equilibria of axially moving beams in the supercritical regime. *Arch Appl Mech* **81**:51-64, 2011.

[2] Ding H., Zhang G.C., Chen L.Q.: Supercritical equilibrium solutions of axially moving beams with hybrid boundary conditions. *Mech Res Commun* **38**(1):52-56, 2011.

[3] Ding H., Zhang G.C., Chen L.Q.: Supercritical vibration of nonlinear coupled moving beams based on discrete Fourier transform. *Int J Nonlin Mech* doi:10.1016/j.ijnonlinmec.2011.09.010, 2011

[4] Ding H., Chen L.Q.: Galerkin methods for natural frequencies of high-speed axially moving beams. *J. Sound Vib* **329**:3484-3494, 2010.

[5] Zhang G.C., Ding H., Chen L.Q.: Galerkin Method for Steady-State Response of Nonlinear Forced Vibration of Axially Moving Beams at Supercritical Speeds. *J. Sound Vib* doi:10.1016/j.jsv.2011.12.004, 2011.